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IEEE TASK FORCE ON IDENTIFICATION OF ELECTROMECHANICAL MODES

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INTRODUCTION

Authors: Juan Sanchez-Gasca and Daniel Trudnowski

This special publication describes the mathematical background and the application of several modal identification methods used in the study of lightly damped electromechanical modes in power systems. The publication also includes a highly detailed survey on the assessment of the introduction, performance and usage of real-time modal estimation systems in practice. The report presents a current perspective on different modal identification methods, their analytical formulations, and their application to elementary examples and to practical scenarios.

The objective of the document is to provide a compendium of information regarding the theory and application of modal identification techniques in power systems. The report is intended as a reference for a broad audience including newcomers into the field of system identification in power systems as well as more experienced readers. For neophytes the document provides an overall view of techniques currently in use. For more experienced users with a background in a particular type of identification methods, the document provides descriptions of different types of analyses and new trends of investigation. The identification methods covered in the report have been used to compute the damping, frequency, and mode shapes associated with lightly damped electromechanical modes in a power system from time-domain data (as opposed to frequency response data). The provenance of the time-domain data may be either measurements or digital simulations. It is recognized that the application of these methods extends to other fields of analysis where knowledge of the modal content is relevant; however, this document is confined to power system applications.

Several modern power-system advances are making oscillations a more wide-spread concern. High-gain fast-acting excitation systems and higher bandwidth speed governor systems are well-known for reducing damping torque contributions from synchronous generators. Power System Stabilizer (PSS) control systems improve the damping; but, they often have limited controllability of the lower-frequency interarea modes. Larger power transfers across the grid often decrease the damping of the interarea modes. Also, the increasing use of renewable energy with uncontrolled prime-mover sources adds to the challenges of maintaining oscillatory stability. All of these issues are contributing to the decreasing oscillatory stability of many modern systems.

Lightly damped electromechanical oscillations are a frequent concern in the power industry. Their presence can have the undesirable effect of reducing the power transfer capabilities of a system with the associated detrimental economic and operational consequences. As utilities and generating companies increase power transfers, often over essentially fixed transmission grids, the study of these oscillations becomes an important aspect of power system planning and operation. The study of electromechanical oscillations is often done in the context of small signal stability analysis using linear analysis techniques. These techniques can be applied very effectively to complement the information obtained from nonlinear time domain simulations and field measurements; their value often resides in allowing for a better understanding of the dynamic characteristics of a system than that obtained from the inspection of time records alone. Of particular relevance is the computation of important modes (often lightly damped modes) and, depending on the study objectives, mode shapes and transfer functions.
Two different approaches exist for computing the modes and associated mode shapes in a power system. One approach is based on the linearization of the power system equations; a second approach is based on the computation of modal information from time domain data (identification). Which approach to use depends on different factors including data availability, computational resources, and study objectives.

The computation of system modes using the linearization approach is performed by first computing the system state matrices, followed by the application of a suitable eigen-analysis method. This approach has the desirable characteristics that it allows for the accurate computation of any system modes and their associated mode shapes for a given operating condition. For small systems, all the modes and associated eigenvectors can be readily computed using numerical methods designed to deal with full matrices; for large systems, typically only a small number of modes are computed using methods tailored to process large sparse matrices [1].

System identification methods provide an alternative analysis approach to linearization methods for computing power system modal information. These methods are designed to process time domain records such as simulation results or measured data and are not limited by the system size; rather, their performance depends on the availability and the quality of the time-domain signals. Identification methods are the only alternative for computing power system modes when the linearized computation of a power system is not a viable option or when only time domain data is available. Furthermore, identification methods are often the most direct means of computing lightly damped modes and allow for the efficient computation of relatively low-order linear systems from time domain data. Their practical value has been demonstrated in different studies; current applications include model validation, stability analysis, and control design.

As with all analysis tools, system identification methods have their own limitations. Whereas the linear representation of a power system allows, in principle, for the accurate computation of any system mode at a given operating condition, identification methods only allow for the computation of those modes present in the signals used to perform the identification. Furthermore, relatively well damped modes and/or modes embedded in noisy signals may not be easily detected. Practical issues such as the selection of a suitable system order, the presence of spurious modes, and the effect of nonlinearities are also important practical issues that need to be addressed when using identification methods.

The application of identification methods for studying power system electromechanical oscillations in power systems was initiated by the work of Dr. John Hauer in the early 1980s. Among other contributions, these efforts led to the pioneering paper [2] which introduced Prony analysis to the power industry. To this date, the Prony method is perhaps the method for which more extensive results and applications have been reported in the power system literature. Its application to system analysis and control of electromechanical oscillations has shown the value of deriving system modal characteristics from time domain simulations and measured data. Following those initial works, other identification methods have been introduced and their applications to practical problems have been described in different venues. One objective of this special publication is to facilitate the access to this information. To this end, identification methods that share common features are described in the first three chapters; furthermore, the order of the chapters reflects the evolution of the identification methods.

Chapter 1, *Linear Ringdown Analysis Methods*, describes methods typically used to process ringdown signals, i.e., signals that arise following a transient event. Examples of such events are faults, switching actions, and step inputs into control systems. Three methods are covered: the Prony method, ERA (Eigensystem Realization Algorithm), and the Pencil Method. Although variations and extensions to these methods continue to be published, these
methods have been used extensively in a variety of applications and can be considered mature analysis techniques.

Chapter 2, *Mode-Meter Analysis Methods*, focuses on methods primarily tailored to ambient data. That is, the system is in a steady-state condition with the primary excitation being the random variations of small load switchings. Many of the ambient algorithms also provide accurate mode information for transient conditions. The first reported result for ambient-data estimation is [3] which used a Yule Walker based algorithm. Since then, many papers have been published on the subject. Algorithm classes discussed in the chapter include Yule Walker, Least-Squares, and Least Mean Squares approaches. Also, approaches for estimating mode shape are summarized. Other conditions discussed in the chapter include low-level probing to improve the accuracy of the mode estimation.

Chapter 3, *Nonlinear and Non-stationary Analysis Methods*, summarizes relatively new research in the application of time-varying nonlinear analysis to power system transients. Major transients within a power system are well known for having nonlinear and non-stationary components. The foundation for the analyses approaches in the chapter is the Hilbert-Huang Transform (HHT). The HHT decomposes a signal into a set of nearly orthogonal basis functions with a set of associated instantaneous attributes. First, a signal is decomposed into time-scale components, and then extracting associated with the oscillating components. Several examples demonstrate the approaches.

Chapter 4, *Assessment of the Introduction, Performance, and Usage of Mode Estimators in Practice*, concludes the main body of the document by presenting the results of an extensive questionnaire on the application of modal estimation in the electric supply industry. Responses from eleven entities from around the world are included in the survey. The questionnaire includes very detailed responses on the application of modal estimation of grid operations, future plans to integrate modal estimation into grid operations, and testing/assessment approaches.

The Appendices are important contributions that the Editors chose to keep as separate entities to preserve the continuity of the document. The Appendices and their relation to individual chapters are as follows:

- **Appendix 1** (broken into 1.A and 1.B) consists of two contributions from Dr. John Hauer.
  - 1.A, *Application Examples*, is a collection of three example modal analysis of major WECC system events and a short example from Australia.
  - 1.B, *Overview of DSI Toolbox*, summarizes a Matlab-based software for modal analysis. It was the policy of the Editors to avoid specific presentation of commercial software. But, the DSI Toolbox was the first software written for such analysis and is a public-domain software. Therefore, we decided to include it as part of the publication.


- **Appendix 3**, *Modal Identification of Transient and Ambient Data Oscillations using Local Empirical Mode Decomposition and Teager-Kaiser Energy Operator*. Introduces an approach that combines the local Empirical Mode Decomposition introduced in Chapter 3 with the Teager-Kaiser energy operator (TKEO) to extract temporal features from measured data. The TKEO is a recent addition to the power system literature as a tool for assessing the energy in signals; Appendices 2 and 3 describe this operator.
References
Although many systems are inherently nonlinear, in some instances they may respond to well-tuned linear controls. In order to implement linear feedback control, the system designer must have an accurate model of sufficiently low order from which to design the control. Several approaches to developing such lower-order models have included dynamic equivalencing, eigenanalysis, and pole/zero cancellation. Frequently, however, the original system is too complex or the parameters are not known with enough accuracy to produce an adequate reduced order model. In practice, the system may have parameters that drift with time or operating condition which compromises the accuracy of the mathematical model. In these cases, it is desirable to extract the modal information directly from the system response to a perturbation. Using this approach, it may be possible to replace the actual dynamic model with an estimated linear model that is derived from the system output waveform. The time-varying dynamic response of a power system to a disturbance may be composed of numerous modes that must be identified. Several methods have been proposed to extract the pertinent modal information from time varying responses. An appropriate method must consider the inclusion of nonlinearities, the size of the model that can be effectively utilized, and the reliability of the results; methods that are applied directly to the nonlinear system simulation or field measurements include the effects of nonlinearities.

It should be noted that in full state eigenvalue analysis, the size of the system model is typically limited to several hundred states with present computing capabilities. This means that a typical system containing several thousand nodes must be reduced using an appropriate reduction method, e.g., dynamic equivalencing. To perform an eigenvalue analysis of a large power system without first reducing it, specialized sparse eigensolvers are required [29, 30]. Modal analysis techniques that operate directly on system output are not limited by system size. This means that standard time-domain-analysis results are directly usable. This eliminates the possibility of losing some of system modal content due to reduction. The estimated linear model may then be used for control design applications or other linear analysis techniques. The estimated model may be chosen to be of lower order than the original model, but still retain the dominant modal characteristics.

1.1.1 Background

The modal analysis problem may be posed such that given a set of measurements that vary with time, it is desired to fit a time-varying waveform of pre-specified form to the actual waveform (i.e., minimize the error between the actual measured waveform and the proposed waveform). The coefficients of the pre-specified waveform yield the dominant modal characteristics of the underlying linear system. Consider the following linear system:

\[ \dot{x} = Ax + Bu \quad x(t_0) = x_0 \]  

(1)
\[ y = C \dot{x} + Du \]  

where \( \dot{x} \) denotes differentiation of \( x \) with respect to time. Variables \( u \) and \( y \) are respectively the input and the output of the system; \( x \), the internal state of the system, is usually taken to be a vector of \( n \) elements (\( n \) being the order of the system differential equation). These equations, and the system matrices within them, can be rearranged in many different ways to serve specific purposes. Each individual element \( x_i \) can be given by:

\[
x_i(t) = \sum_{i=1}^{n} r_i x_{i0} e^{\lambda_i t} = \sum_{i=1}^{n} a_i e^{\sigma_i t} \cos(\omega_i t + \theta_i)
\]

The parameter \( r_i \) is the residue of the mode \( i \), \( x_{i0} \) is derived from influence of the initial conditions, and \( \lambda_i \) represents the (possibly complex) eigenvalues of \( A \). The estimation of these responses yields modal information about the system that can be used to predict possible unstable behavior, controller design, parametric summaries for damping studies, and modal interaction information.

The discrete form of equations (1) and (2) is given by:

\[
x(k+1) = Ax(k) + Bu(k)
\]
\[
y(k) = Cx(k) + Du(k)
\]

where \( k \) represents the discrete time interval. This system of equations is shown in Figure 1-1.

The primary task in modal identification is to determine the system poles of the system transfer function or, equivalently, the eigenvalues of \( A \). Transfer function identification must, in addition to the poles, also determine the zeros and the gains along one or more response paths. The system transfer function involves all of the system matrices in equations (1) and (2):

\[
T(s) = \frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} = \frac{G(s^{m} + a_{m-1}s^{m-1} + \ldots + a_{1}s + a_{0})}{(s^{n} + b_{m-1}s^{m-1} + \ldots + b_{1}s + b_{0})}
\]

\[
= \frac{G(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}
\]

\[
= \sum_{i=1}^{n} \frac{K_i}{s-p_i}
\]
where each pole $p_i$ is identical to an eigenvalue $\lambda_i$ of the matrix $A$ and $K_i$ is the transfer function residue of the associated pole $p_i$.

### 1.1.2 Prony Methods

The core notion in Prony analysis originated in an earlier century [1]. Its practical use was not possible until the advent of the digital computer and numerical methods for ill-conditioned systems were developed. Prony methods and their modern extensions are designed to directly estimate the parameters for the exponential terms in (3), by fitting a function to an observed record for $y(t)$. In doing this it may also be necessary to model offsets, trends, noise, and other extraneous effects in the signal. The Prony method is a “polynomial” method in that it includes the process of finding the roots of a characteristic polynomial.

Let the record for $y(t)$ consist of $N$ samples $y(t_k)$ that are evenly spaced by an amount $\Delta t$. The notation is simplified if (3) is recast in the exponential form

$$\hat{y}(t) = \sum_{i=1}^{n} A_i e^{\sigma_i t} \cos(\omega_i t + \theta_i)$$

where $n \leq N$ is the subset of modes to be determined. At the sample times $t_k$, this can be discretized to

$$\hat{y}(k) = \sum_{i=1}^{n} B_i z_i^k$$

where

$$z_i = \exp(\lambda_i \Delta t)$$

The $z_i$ are the roots of the polynomial

$$z^n - (a_1 z^{n-1} + a_2 z^{n-2} + \ldots + a_{n-1} z + a_n) = 0$$

where the $a_i$ coefficients are unknown and are calculated from the set of measurement vectors.

The strategy for obtaining a Prony solution can be summarized as follows:

1. **Step 1**: Assemble selected elements of the record into a Toeplitz data matrix
2. **Step 2**: Fit the data with a discrete linear prediction model, such as a least squares solution.
3. **Step 3**: Find the roots of the characteristic polynomial (8) associated with the model of step 1.
4. **Step 4**: Using the roots of step 3 as the complex modal frequencies for the signal, determine the amplitude and initial phase for each mode.

A Toeplitz matrix is a matrix with a constant diagonal in which each descending diagonal from left to right is constant. The approach to the Toeplitz (or the closely related Hankel) matrix assembly of Step 1 has received the most attention in the literature. The problem can be formulated in many different ways. If the initial (i.e., $i < 0$) and post (i.e., $i > N$) conditions are assumed to be zero, then the subscript ranges $X_1$ through $X_4$ in equation(6) represent four such formulations. The range $X_1$ is termed the covariance problem. Because the initial and post conditions are not used, no assumption is required on their value. The range $X_4$ is called the correlation problem; it incorporates both initial and post conditions. The remaining problems, $X_2$ and $X_3$, are termed the pre-windowed and post-windowed methods.
In the majority of practical cases, the Toeplitz matrix is non-square with more rows than columns. The system of equations (9) requires a least squares error solution to find the factors \(a_i\) through \(a_n\). After the \(a_i\) coefficients are obtained, the \(n\) roots \(z_i\) of the polynomial in (8) can be found by factoring.

Step 4 is also a linear algebra problem. Once the roots \(z_i\) are obtained from step 3, they are substituted into (7) and written in matrix form as

\[
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
y_0 & y_0 & \cdots & 0 \\
y_1 & y_0 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
y_n & y_{n-1} & \cdots & y_0 \\
y_{n+1} & y_n & \cdots & y_1 \\
\vdots & \vdots & \ddots & \vdots \\
y_N & y_{N-1} & \cdots & y_{N-n-1} \\
0 & 0 & \cdots & 0 \\
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n \\
0 \\
\end{bmatrix}
= 
\begin{bmatrix}
y(0) \\
y(1) \\
\vdots \\
y(N-1) \\
0 \\
\end{bmatrix}
\] (11)

The \(N \times n\) matrix in (12) is a Vandermonde matrix and solving it for the \(B_{ii}\) is called the Vandermonde problem. Once the residue coefficients are found, the estimated signal \(\hat{y}(t)\) can be reconstructed from (7) using the roots of (8). The reconstructed signal \(\hat{y}(t)\) will usually not fit \(y(t)\) exactly. An appropriate measure for the quality of this fit is the signal to noise (SNR) ratio:

\[
\text{SNR} = 20\log\frac{\|\hat{y} - y\|}{\|y\|}
\] (13)

where the SNR is given in decibels (dB).
Example: To understand the basic approach of the Prony method, consider the simple example:

\[
y(t) = e^{-0.1t} \cos(2\pi t) + \frac{1}{4} e^{-0.25t} \cos\left(14\pi t + \frac{\pi}{8}\right)
\]

which yields the waveform shown in Figure 1-2. The waveform has two damped oscillatory modes at 1 and 7 Hz. The sampling rate is chosen to be 8 times the highest frequency or 56 Hz for a Δt = 1/56 = 0.0179 s. The sample number \( N \) is 113. The data sample \( y_k \) is:

<table>
<thead>
<tr>
<th>( k )</th>
<th>( y_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.2310</td>
</tr>
<tr>
<td>1</td>
<td>1.0874</td>
</tr>
<tr>
<td>2</td>
<td>0.8762</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>112</td>
<td>0.9953</td>
</tr>
<tr>
<td>113</td>
<td>0.9986</td>
</tr>
</tbody>
</table>

**Step 1:** Assemble the data elements into a Toeplitz matrix. Since we desire to identify \( n = 4 \) modes, the matrix will be \( n \times n \) or \( 4 \times 4 \). The elements of \( T \) are given by:

\[
T = \begin{bmatrix}
y_3 & y_2 & y_1 & y_0 \\
y_4 & y_3 & y_2 & y_1 \\
y_5 & y_4 & y_3 & y_2 \\
y_6 & y_5 & y_4 & y_3 \\
\vdots & \vdots & \vdots & \vdots \\
y_{110} & y_{109} & y_{108} & y_{107} \\
y_{111} & y_{110} & y_{109} & y_{108} \\
y_{112} & y_{111} & y_{110} & y_{109} \\
\end{bmatrix}
= \begin{bmatrix}
0.7094 & 0.8762 & 1.0874 & 1.2310 \\
0.6656 & 0.7094 & 0.8762 & 1.0874 \\
0.7446 & 0.6656 & 0.7094 & 0.8762 \\
0.8679 & 0.7446 & 0.6656 & 0.7094 \\
\vdots & \vdots & \vdots & \vdots \\
0.7019 & 0.5614 & 0.5176 & 0.5715 \\
0.8759 & 0.7019 & 0.5614 & 0.5176 \\
0.9953 & 0.8759 & 0.7019 & 0.5614
\end{bmatrix}
\]
Step 2: The polynomial coefficients are found by solving

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\end{bmatrix} = \begin{bmatrix}
y_4 \\
y_5 \\
y_6 \\
y_7 \\
y_8 \\
\vdots \\
y_{110} \\
y_{111} \\
y_{112} \\
y_{113} \\
\end{bmatrix} = \begin{bmatrix}
0.6656 \\
0.7446 \\
0.8679 \\
0.9257 \\
\vdots \\
0.7019 \\
0.8759 \\
0.9953 \\
0.9986 \\
\end{bmatrix}
\]

using a pseudo-inverse (for the least squares error solution), yielding:

\[
\left( T^T T \right)^{-1} T^T b = \begin{bmatrix}
3.3949 \\
-4.7914 \\
3.3811 \\
-0.9920 \\
\end{bmatrix}
\]

Step 3: The \( z_i \)s are found from the roots of the polynomial of (8):

\[
z = \begin{bmatrix}
0.7055 + j0.7055 \\
0.7055 - j0.7055 \\
0.9919 + j0.1118 \\
0.9919 - j0.1118 \\
\end{bmatrix}
\]

Step 4: Recall that in a stable system, all of the root will lie within the unit circle. The Vandermonde matrix from equation (12) is:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 + j0.0918 & 1 + j0.8150 & 1 + j0.5519 \\
1 & 1 + j0.0918 & 1 + j0.8150 & 1 + j0.5519 & 1 + j0.5519 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 + j0.0918 & 1 + j0.8150 & 1 + j0.5519 & 1 + j0.5519 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
0.7055 + j0.7055 & 0.7055 - j0.7055 & 0.9919 + j0.1118 & 0.9919 - j0.1118 \\
\end{bmatrix}
\]
The residues $B_i$ are then further calculated using a pseudo-inverse:

$$
B = \begin{bmatrix}
0.1250 \angle 22.5^\circ \\
0.1250 \angle -22.5^\circ \\
0.5000 \\
0.5000
\end{bmatrix}
$$

From the roots and residues, the parameters from (6) can be calculated:

$$
\lambda_j = \frac{\ln(z_j)}{\Delta t} = \begin{bmatrix}
-0.1250 + j43.9823 \\
-0.1250 - j43.9823 \\
-0.1000 + j6.2832 \\
-0.1000 - j6.2832
\end{bmatrix}
= \begin{bmatrix}
-0.1250 + j14\pi \\
-0.1250 - j14\pi \\
-0.1000 + j2\pi \\
-0.1000 - j2\pi
\end{bmatrix}
$$

$$
A = 2|B| = \begin{bmatrix}
0.25 \\
0.25 \\
1.00 \\
1.00
\end{bmatrix}
$$

$$
\theta = \angle B = \begin{bmatrix}
\pi \\
8\pi \\
8\pi \\
0
\end{bmatrix}
$$

From these parameters, it is straightforward to reconstruct the original signal of equation (14).

### 1.1.3 Eigensystem Realization Algorithm

The Eigensystem Realization Algorithm (ERA) is a system identification algorithm introduced by Jer-Nan Juang and Richard S. Pappa in 1985 [11]. The algorithm allows for the modal identification and model reduction of linear systems. The ERA was introduced within the aerospace community where it has been used extensively. Since its inception, numerous references dealing with the theoretical and practical aspects of the method have been published, applications in different disciplines, including power systems [12-13], have also been reported and new developments continue to be made. The popularity of the ERA can be attributed to its straightforward implementation, numerical robustness, and sound mathematical foundation. Furthermore, the basic formulation of the method is for MIMO systems. References [3, 11, 14,15, 31-46] document different aspects and applications of the ERA.

The Eigensystem Realization Algorithm (ERA) is based on the singular value decomposition of the Hankel matrix $H_0$ associated with the linear ringdown of the system. A Hankel matrix is a square matrix with constant skew-diagonals. The Hankel matrices are typically assembled using all of the available data such that the top left-most element of $H_0$ is $y_0$ and the bottom right-most element of $H_1$ is $y_N$. The Hankel matrices are assembled such that:
\[
H_0 = \begin{bmatrix}
y_0 & y_1 & \cdots & y_r \\
y_1 & y_2 & \cdots & y_{r+1} \\
\vdots & \vdots & \ddots & \vdots \\
y_r & y_{r+1} & \cdots & y_{N-1}
\end{bmatrix}
\]
(15)

\[
H_1 = \begin{bmatrix}
y_1 & y_2 & \cdots & y_{r+1} \\
y_2 & y_3 & \cdots & y_{r+2} \\
\vdots & \vdots & \ddots & \vdots \\
y_{r+1} & y_{r+2} & \cdots & y_N
\end{bmatrix}
\]
(16)

and \( r = \frac{N}{2} - 1 \). This choice of \( r \) assumes that the number of data points is sufficient such that \( r > n \).

The ERA formulation begins by separating the singular value decomposition of \( H_0 \) into two components according to the relative size of the singular values:

\[
H_0 = U \Sigma V^T = U_n \Sigma_n \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_z \end{bmatrix} \begin{bmatrix} V_n^T \\ V_z^T \end{bmatrix}
\]
where \( \Sigma_n \) and \( \Sigma_z \) are diagonal matrices with their elements ordered by magnitude:

\[
\Sigma_n = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)
\]

\[
\Sigma_z = \text{diag}(\sigma_{n+1}, \sigma_{n+2}, \ldots, \sigma_N)
\]

and the singular values are ordered by magnitude such that

\[
\sigma_1 > \sigma_2 > \cdots > \sigma_n > \sigma_{n+1} > \sigma_{n+2} > \cdots > \sigma_N
\]

The SVD is a useful tool for determining an appropriate value for \( n \). The ratio of the singular values contained in \( \Sigma \) can determine the best approximation of \( n \). The ratio of each singular value \( \sigma_i \) to the largest singular value \( \sigma_{\text{max}} \) is compared to a threshold value, where \( p \) is the number of significant decimal digits in the data:

\[
\frac{\sigma_i}{\sigma_{\text{max}}} \approx 10^{-p}
\]

An example is to set \( p \) equal to 3 significant digits, thus any singular values with a ratio below 10\(^{-3}\) are assumed to be part of the noise and are not included in the reconstruction of the system. The value of \( n \) should be set to the number of singular values with a ratio above the threshold 10\(^{-p}\). It can be shown that for a linear system of order \( n \), the diagonal elements of \( \Sigma_z \) are zero (assuming that the impulse response is free of noise). The practical significance of this result is that the relative size of the singular values provides an indication of the identified system order. If the singular values exhibit a significant grouping such that \( \sigma_n \gg \sigma_{n+1} \), then from the partitioned representation given in (17), \( H_0 \) can be approximated by

\[
H_0 \approx U_n \Sigma_n V_n^T
\]
(18)

The method for obtaining the eigenvalue realization algorithm solution can be summarized as follows:
**Step 1:** Assemble selected elements of the record into a Hankel data matrices $H_0$ and $H_1$.

**Step 2:** Perform the singular value decomposition of $H_0$ and estimate the system order $n$ based on the magnitude of the singular values.

**Step 3:** Compute the discrete system matrices as follows:

$$A = \Sigma_n^{\frac{\sqrt{}}{2}} U_n H_1 V_n \Sigma_n^{-\frac{\sqrt{}}{2}}$$

$$B = \Sigma_n^{\frac{\sqrt{}}{2}} V_n \left( \mathbf{1} : n \cdot n \right)$$

$$C = U_n \left( \mathbf{1} : N \cdot n \right) \Sigma_n^{\frac{\sqrt{}}{2}}$$

$$D = y_0$$

**Step 4:** Calculate continuous system matrices $A_c, B_c$ assuming a zero order hold and sampling interval $\Delta t$:

$$A_c = \ln \left( \frac{A}{\Delta t} \right)$$

$$B_c = \left[ \int_{0}^{\Delta t} e^{A_c \tau} d\tau \right] B$$

The reduced system response can then be computed from the continuous matrices.

Example: The same example system of (14) is used to illustrate the steps of the ERA method.

**Step 1:** The Hankel matrices $H_0$ and $H_1$ in this example are $56 \times 56$ and are summarized:

$$H_0 = \begin{bmatrix} y_0 & y_1 & \cdots & y_r \\ y_1 & y_2 & \cdots & y_{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_r & y_{r+1} & \cdots & y_{N-1} \end{bmatrix} = \begin{bmatrix} 1.2310 & 1.0874 & 0.8762 & \cdots & 0.9701 & 1.1050 \\ 1.0874 & 0.8762 & 0.7094 & \cdots & 1.1050 & 1.1087 \\ 0.8762 & 0.7094 & 0.6656 & \cdots & 1.1087 & 0.9818 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.9701 & 1.1050 & 1.1087 & \cdots & 0.5614 & 0.7019 \\ 1.1050 & 1.1087 & 0.9818 & \cdots & 0.7019 & 0.8759 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} y_1 & y_2 & \cdots & y_{r+1} \\ y_2 & y_3 & \cdots & y_{r+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{r+1} & y_{r+2} & \cdots & y_N \end{bmatrix} = \begin{bmatrix} 1.0874 & 0.8762 & 0.7094 & \cdots & 1.1050 & 1.1087 \\ 0.8762 & 0.7094 & 0.6656 & \cdots & 1.1087 & 0.9818 \\ 0.7094 & 0.6656 & 0.7446 & \cdots & 0.9818 & 0.7950 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1.1050 & 1.1087 & 0.9818 & \cdots & 0.7019 & 0.8759 \\ 1.1087 & 0.9818 & 0.7950 & \cdots & 0.8759 & 0.9953 \end{bmatrix}$$

**Step 2:** Perform a singular value decomposition of $H_0$ to determine the number of relevant modes $n$. The diagonal matrix $\Sigma$ has four significant diagonal elements:

$$\Sigma = \begin{bmatrix} 25.4711 \\ 25.3681 \\ 6.2149 \\ 6.1993 \\ 0 \ddots \end{bmatrix}$$
thus the number of relevant modes is four. The reduced matrix \( \Sigma \) is the top \( 4 \times 4 \) diagonal block of \( \Sigma_n \). \( \Sigma_z \) is the lower right block diagonal matrix of \( \Sigma \). The reduced matrices \( U_n \) and \( V_n \) are:

\[
U_n = \begin{bmatrix}
-0.1982 & -0.0015 & -0.1961 & -0.0372 \\
-0.1961 & 0.0206 & -0.1104 & -0.1648 \\
-0.1917 & 0.0425 & 0.0397 & -0.1945 \\
\vdots & \vdots & \vdots & \vdots \\
-0.1761 & -0.0415 & -0.0344 & 0.1761 \\
-0.1789 & -0.0215 & -0.1471 & 0.1004 \\
\end{bmatrix}
\]

\[
V_n^T = \begin{bmatrix}
-0.1982 & 0.0015 & -0.1961 & 0.0372 \\
-0.1961 & -0.0206 & -0.1104 & 0.1648 \\
-0.1917 & -0.0425 & 0.0397 & 0.1945 \\
\vdots & \vdots & \vdots & \vdots \\
-0.1761 & 0.0415 & -0.0344 & -0.1761 \\
-0.1789 & 0.0215 & -0.1471 & -0.1004 \\
\end{bmatrix}
\]

**Step 3:** The discrete matrix \( A \) is computed:

\[
A = \Sigma_n^{-1/2} U_n^T H_n V_n \Sigma_n^{-1/2} = \begin{bmatrix}
0.9902 & -0.1118 & -0.0033 & -0.0002 \\
0.1118 & 0.9937 & -0.0005 & -0.0001 \\
-0.0033 & 0.0005 & 0.7035 & 0.7055 \\
0.0002 & -0.0001 & -0.7055 & 0.7076 \\
\end{bmatrix}
\]

**Step 4:** The eigenvalues of \( A \) are:

\[
\text{eig}(A) = \begin{bmatrix}
0.7055 + j0.7055 \\
0.7055 - j0.7055 \\
0.9919 + j0.1118 \\
0.9919 - j0.1118 \\
\end{bmatrix}
\]

which are the same as the poles \( (z) \) from the Prony method. The remainder of the method follows the Prony method.

### 1.1.4 Matrix Pencil Method

The Matrix Pencil approach was introduced by Hua and Sarkar [2] for pole estimation and was initially used for extracting poles from antennas’ electromagnetic transient responses.

The matrix pencil method produces a matrix whose roots provide \( z_i \). The poles are found as the solution of a generalized eigenvalue problem.

The basic process of the Matrix Pencil is similar to that of the ERA method up through Step 3.

**Step 1:** Assemble selected elements of the record into a Hankel data matrix

**Step 2:** Fit the data with a discrete linear prediction model, such as a least squares error solution.

**Step 3:** Define the matrices \( V_1 \) and \( V_2 \) from \( V \) in (17):

\[
[V_1] = [v_1 \ v_2 \cdots v_{n-1} ]
\]

\[
[V_2] = [v_2 \ v_3 \cdots v_n ]
\]

and calculate the matrices \( Y_1 \) and \( Y_2 \):
\[ Y_1 = V_1^T V_1 \]
\[ Y_2 = V_2^T V_2 \]

Step 4: The desired poles \( z \) may be found as the generalized eigenvalues of the matrix pair \( \{ Y_2, \lambda Y_1 \} \). The eigenvalue set \( \lambda(Y_2, Y_1) \) is contained in the square matrices \( Y_1 \) and \( Y_2 \), as the pencil values or roots of \( Y_2 \) relative to \( Y_1 \).

Example: The Hankel matrix \( H_0 \) used for the Pencil method is the same as that for the ERA method. From the singular value decomposition, assemble the matrices \( V_1 \) and \( V_2 \):

\[
V_1^T = \begin{bmatrix}
-0.1982 & 0.0015 & -0.1961 & 0.0372 \\
-0.1961 & -0.0206 & -0.1104 & 0.1648 \\
-0.1917 & -0.0425 & 0.0397 & 0.1945 \\
0 & 0 & 0 & 0 \\
-0.1761 & 0.0415 & -0.0344 & -0.1761
\end{bmatrix}
\]
\[
V_2^T = \begin{bmatrix}
-0.1961 & -0.0206 & -0.1104 & 0.1648 \\
-0.1917 & -0.0425 & 0.0397 & 0.1945 \\
-0.1852 & -0.0637 & 0.1662 & 0.1095 \\
0 & 0 & 0 & 0 \\
-0.1761 & 0.0415 & -0.0993 & -0.1497
\end{bmatrix}
\]

Compute the matrices \( Y_1 \) and \( Y_2 \):

\[
Y_1 = \begin{bmatrix}
0.9680 & 0.0038 & -0.0263 & -0.0180 \\
0.0038 & 0.9995 & 0.0032 & 0.0022 \\
-0.0263 & 0.0032 & 0.9784 & -0.0148 \\
-0.0180 & 0.0022 & -0.0148 & 0.9899
\end{bmatrix}
\]
\[
Y_2 = \begin{bmatrix}
0.9581 & -0.1077 & -0.0280 & -0.0181 \\
0.1122 & 0.9937 & 0.0000 & 0.0001 \\
-0.0376 & 0.0047 & 0.6780 & 0.6873 \\
0.0063 & -0.0009 & -0.7016 & 0.7109
\end{bmatrix}
\]

and calculate the eigenvalues of \( Y_1^{-1} Y_2 \)

\[
eig(Y_1^T Y_2) = \begin{bmatrix}
0.7055 + j0.7055 \\
0.7055 - j0.7055 \\
0.9919 + j0.1118 \\
0.9919 - j0.1118
\end{bmatrix}
\]

which are the same as the poles \( z \) from the Prony method. The remainder of the method follows the Prony method.

From this example, it may appear as if each method produces identical results. For linear systems, this is true, but for nonlinear systems or systems with noise, each method will produce slightly different results. Each of these three methods has its advantages and disadvantages, making the choice of a “best” algorithm dependent on the problem at hand. In the following sections, each method will be developed in greater detail and application examples will be provided.

### 1.2 Prony Method

At a more abstract level, Prony analysis is sometimes characterized as a projection method in which steps 1 and 2 define a modal basis onto which the observed data \( y(k) \) are projected at step 3. Once this basis is determined, some or all of it can be reused in subsequent “repeat” solutions. Conditions under which this is useful include the following:
• mode shapes are desired for a larger number of signals than can be processed in one tandem analysis
• modal residues are desired at locations where the signal to noise ratio is adverse
• the signal to be analyzed consists of a fast transient imposed upon a much slower one

This last situation is illustrated in Figure 1-3. The first step would be to model the trend, perhaps on a time frame ranging from 3 seconds to 30 seconds. Then, retaining this estimate, another repeat solution would be performed from 3 seconds to about 15 seconds. This two-step approach permits better separation of the trend from the swing dynamics, while avoiding inclusion of the “noise tail” in the final estimate of modal parameters.

![Figure 1-3: Transient signal with slow exponential trend.](image)

### 1.2.1 Menu of Solution Techniques in the DOE Prony Analysis Tool

The Prony analysis software reported in [3] and [4] offers 288 different solution paths for solving the Prony analysis problem. These primary options, listed in Table 1-I, are retained in the Graphical User Interface (the Ringdown GUI) that has since been built around that initial software and its various extensions [4, 5]. All of these options are major extensions to the classical Prony Method, which is not included among these solution alternatives.

Thorough comparisons of these solution alternatives and their associated processing controls are provided in [6] and [7]. For each solution technique, close attention is given to issues (a-f) from the following list:

a) assumed model order
b) sampling frequency of the data
c) length of the data record
d) level and coloration of additive noise
e) closely spaced poles
f) closely spaced pole-zero pairs
g) highly damped interarea modes  
h) weak saturations at the start of ringdowns

The cited comparisons involve single-output analyses of realistic but linearized power system models containing up to 14 oscillatory modes. Similar comparisons for multi-output analysis can be found in [8] and various analysis reports for the Western North America Interconnection [9]. Issues (g-h) are new ones produced by trends in western system dynamics.

Table 1-I: Primary Solution Techniques in the DOE Prony Analysis Tool

<table>
<thead>
<tr>
<th>Options for formulating the Toeplitz data matrix (termed the LP method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
</tr>
<tr>
<td>pre-windowed</td>
</tr>
<tr>
<td>Covariance</td>
</tr>
<tr>
<td>post-windowed</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Options for formulating the LP polynomial (termed the FB method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward</td>
</tr>
<tr>
<td>Backward</td>
</tr>
<tr>
<td>forward-backward</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Options for LP solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>singular value decomposition (SVD)</td>
</tr>
<tr>
<td>standard QR</td>
</tr>
<tr>
<td>total least squares (TLS)</td>
</tr>
<tr>
<td>fast QR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Options for solving the Vandermonde problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard QR</td>
</tr>
<tr>
<td>fast real</td>
</tr>
<tr>
<td>fast complex</td>
</tr>
</tbody>
</table>

1.2.1.1 Computer Platform Issues

Prony analysis and perhaps modal identification generally, involves complex calculations that can become numerically sensitive for large or difficult data sets. The finding of polynomial roots is a source of special concern in this regard, and its known sensitivity is not avoided by such simple expedients as posing an equivalent eigenvalue problem. The initial DOE Prony analysis tool provided direct factoring with a choice of either double or quadruple precision arithmetic on a high performance VAX computer. An attractive alternative, still under investigation, is the FFT-like rooting algorithm reported in [10].

Many derivatives of the DOE tool are in use, on a wide range of computer platforms, and with a progressive shift from the original FORTRAN modules to nominally equivalent ones coded in MATLAB or other languages. The computational precision of these platforms and codes changes with the technology, and should not be taken for granted. References [6] and [7] examine these matters briefly, but a comprehensive and ongoing review of computer platform issues is badly needed.

1.2.1.2 General Capabilities of the DOE Prony Analysis Tool

In very broad terms, the capabilities of the DOE Prony Analysis Tool are the following:
- Tandem fitting can be applied to a maximum of 20 signals.
- The data matrix can accept a maximum of 8190 points. When this size is not sufficient the data can be decimated, and/or selected signals can be truncated.
- Modes can be estimated by any of the techniques listed in Table 1-I. Additional techniques can be added as needed.
- Estimated modes can be sorted according to energy, frequency, damping, and other parameters. User then determines which modes to retain in candidate models.
- Several metrics are used to compare candidate models against original data, in time domain and in frequency domain.
- Selected modes can be used in repeat solutions, with a different choice of signals if desired.

Though it can operate in a stand-alone mode, the Tool works best as the “Ringdown GUI” within the DSI Toolbox. This facilitates such secondary tasks as repair and filtering of signals, and it provides display logic for mode shapes and other materials for report generation.

Documentation for the DSI Toolbox, though extensive, is somewhat dated [4, 5]. It is augmented by a set of self-executing tutorial examples, plus occasional workshops on power systems analysis.

**1.2.2 Validation and Evaluation of Prony Methods**

The validation of modal estimators is guided by some basic rules:

**Rule 1.** When the data are perfect the results should also be perfect.

**Rule 2.** Small changes in the estimation procedure should produce only small changes in the results.

**Rule 3.** Results obtained with different procedures, or with different but closely related data, should be compatible.

Application of Rule 1 requires an ample collection of synthetic data. Such data, with parent models and options for systemic noise, are readily available as accessories to the DOE modal analysis tools.

Rule 2 permits many variations. The most common ones are to compare results obtained with different signal sets, with somewhat different record segments, or with different decimation rates of the initial data. A model fit to the middle of a “clean” ringdown should approximately predict the observations to either side of the fitting window (Figure 1-4).
1.3 Eigensystem Realization Algorithm

The ERA fits a discrete state space model to the impulse response of a linear system. From the state space model, modal frequencies and damping coefficients can be calculated. Succinctly, the algorithm consists of building a Hankel matrix whose elements are the Markov parameters of the system under study; from this matrix, the state space matrices are derived using the singular value decomposition (SVD) of the Hankel matrix. Lastly, the system modes are computed from the realized system matrices. Figure 1-5 is a conceptual view of the ERA identification process.

![Figure 1-4: Model prediction outside the fitting window](image)

**Figure 1-4: Model prediction outside the fitting window**

1.3.1 ERA Formulation

The ERA is based on the singular value decomposition of a Hankel matrix constructed from the impulse response of the system to be identified. The Hankel matrix provides the “bridge” between the sampled data and the system state space realization. In principle, the identification of a minimum order system can be obtained by retaining the largest singular values of the Hankel matrix. The theoretical basis for the derivation resides on the seminal work of Ho and Kalman dealing with system realizations [16].

It can be shown that for a linear system of order $n$, the diagonal elements of $\Sigma_z$ are zero (assuming that the impulse response is free of noise). The practical significance of this result...
is that the relative size of the singular values provides an indication of the identified system order. For example, Figure 1-6 shows the thirty singular values of $H(0)$ associated with the unit pulse response of a simple linear sixth-order system consisting of three oscillatory modes. The figure clearly shows a gap of nearly seven orders of magnitude separating the largest six singular values from the rest. Thus, the system order, $n$, is six. It must be noted that in practical applications, either due to noise or system nonlinearities, making the distinction between small and large singular values is often not straightforward. This is illustrated in Figure 1-7, which shows the thirty singular values of $H(0)$, plotted in order of decreasing magnitude, associated with a generator output power in response to a step input in the voltage regulator of its excitation system. In this case, the singular values are not as clearly divided into two sets.

![Figure 1-6: Singular Values – Linear System](image1)

![Figure 1-7: Singular Values – Nonlinear System](image2)
1.3.2 ERA Illustrative Example

The objective of the example is to illustrate the application of the ERA using the linearized representation of the power system shown in Figure 1-8. Although the system in this example is very simple, it suffices to illustrate the application of the ERA and highlight some associated practical issues.

The system consists of an equivalent 1000 MVA generator (Gen1) connected to a much larger system via a step-up transformer and a two-circuit transmission line. The system is essentially a single machine infinite bus (SMIB) system. The Gen1 generator is modeled using the classical generator model.

The stimulus is a pulse in the mechanical torque. It should be emphasized that an ideal pulse of mechanical torque is used here merely to illustrate the application of the ERA and does not represent a realistic event.

Using the classical model to represent Gen 1, the SMIB system can be redrawn as:

The reactance $X_{eq}$ represents the equivalent reactance of the transformer and transmission line; $X_d$ is the generator synchronous reactance, and $E'$ is the internal voltage of generator Gen 1.

The system operating conditions are as follows:

---

**Figure 1-8: ERA example power system.**

**Figure 1-9: Single machine infinite bus representation**
The dynamic performance of the power system in Figure 1-9 is described by the power transfer equation between the generators and by the swing equation of generator Gen 1. The linearized representation of the system can be written as:

\[
\begin{bmatrix} \Delta \omega \\ \Delta \delta \end{bmatrix} = \begin{bmatrix} - \frac{K_p}{2H} & - \frac{K_v}{2H} \\ \frac{1}{2H} & 0 \end{bmatrix} \begin{bmatrix} \Delta \omega \\ \Delta \delta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta T_m
\]

where the synchronizing coefficient, \( K_s \), is given by

\[
K_s = \frac{E'V_i}{X_d' + X_{eq}} \cos(\delta_0)
\]

If the output is chosen to be the rotor angle, then the output equation is given by

\[
[\Delta \delta] = \begin{bmatrix} 0 & 1 \\ \Delta \omega \\ \Delta \delta \end{bmatrix}
\]

The block diagram for the linearized system is drawn below

![Figure 1-10: Block diagram of linearized system.](image-url)

Using the values the values of the initial conditions in (21) yields

\[
[\Delta \omega] = \begin{bmatrix} -0.25 \\ 376.99 \end{bmatrix} \begin{bmatrix} \Delta \omega \\ \Delta \delta \end{bmatrix} + \begin{bmatrix} 0.1250 \\ 0 \end{bmatrix} \Delta T_m
\]

\[
[\Delta \delta] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta \omega \\ \Delta \delta \end{bmatrix}
\]

The response of the system to the input shown in Figure 1-11 is the decaying sinusoidal shown in Figure 1-12.
Figure 1-11: Unit impulse.

Figure 1-12: System response.
The first ten output points following the application of the input are found in Table 1-II:

<table>
<thead>
<tr>
<th>Time</th>
<th>$\Delta\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0100</td>
<td>0.0024</td>
</tr>
<tr>
<td>1.0200</td>
<td>0.0070</td>
</tr>
<tr>
<td>1.0300</td>
<td>0.0116</td>
</tr>
<tr>
<td>1.0400</td>
<td>0.0161</td>
</tr>
<tr>
<td>1.0500</td>
<td>0.0205</td>
</tr>
<tr>
<td>1.0600</td>
<td>0.0247</td>
</tr>
<tr>
<td>1.0700</td>
<td>0.0287</td>
</tr>
<tr>
<td>1.0800</td>
<td>0.0324</td>
</tr>
<tr>
<td>1.0900</td>
<td>0.0359</td>
</tr>
</tbody>
</table>

The corresponding Hankel matrices $H(0)$ and $H(1)$ are given by:

$$ H(0) = \begin{bmatrix} 0.0024 & 0.0070 & 0.0116 & 0.0161 \\ 0.0070 & 0.0116 & 0.0161 & 0.0205 \\ 0.0116 & 0.0161 & 0.0205 & 0.0247 \\ 0.0161 & 0.0205 & 0.0247 & 0.0287 \end{bmatrix} $$

$$ H(1) = \begin{bmatrix} 0.0070 & 0.0116 & 0.0161 & 0.0205 \\ 0.0116 & 0.0161 & 0.0205 & 0.0247 \\ 0.0161 & 0.0205 & 0.0247 & 0.0287 \\ 0.0205 & 0.0247 & 0.0287 & 0.0324 \end{bmatrix} $$

The singular value decomposition of $H(0)$ is as follows:

$$ H(0) = U \Sigma V^T = \begin{bmatrix} \Sigma & 0 & 0 & 0 \\ 0 & 0.0694 & 0.0062 & 0 \\ 0 & 0 & 0.0062 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_n & V \end{bmatrix} $$

Since the test system is a second order linear system only two singular values are non-zero. The matrices $U$ and $V$ are given by:

$$ U = \begin{bmatrix} -0.2978 & -0.7809 & 0.5084 & 0.2075 \\ -0.4228 & -0.3511 & -0.8291 & 0.1032 \\ -0.5441 & 0.0805 & 0.1409 & -0.8232 \\ -0.6607 & 0.5103 & 0.1854 & 0.5183 \end{bmatrix} $$
The discrete state space representation of the identified system is given by

\[ V = \begin{bmatrix}
-0.2978 & 0.7809 & 0.3701 & 0.4056 \\
-0.4228 & 0.3511 & -0.7931 & -0.2626 \\
-0.5441 & -0.0805 & 0.4806 & -0.6831 \\
-0.6607 & -0.5103 & -0.0550 & 0.5477
\end{bmatrix} \]

The corresponding continuous system matrices \( A_c, B_c \) are as follows:

\[ A_c = \log_e \left( \frac{1}{h} A \right) = \begin{bmatrix}
22.9983 & -24.7894 \\
24.7894 & -23.2483
\end{bmatrix} \]

\[ B_c = \left[ \frac{h}{1} e^{A_c \tau} d\tau \right]^{-1} B = \begin{bmatrix}
-6.1741 \\
7.8544
\end{bmatrix} \]

For this simple linear system the eigenvalues of the identified state matrix \( A_c \) exactly match those of the original state matrix:

\[ \Lambda \begin{bmatrix}
22.9983 & -24.7894 \\
24.7894 & -23.2483
\end{bmatrix} = \Lambda \begin{bmatrix}
-0.25 & -0.2118 \\
376.99 & 0
\end{bmatrix} = \begin{bmatrix}
-0.1250 + j8.9838 \\
-0.1250 - j8.9348
\end{bmatrix} \]

and the identified transfer function between the input (torque) and the output (rotor angle) also exactly matches the transfer function of the original system as shown in Figure 1-13.
The speed measurements were collected with a sampling rate of 2 KHz following an out-of-phase synchronization test performed when the phase angle difference between the generator and the power system was approximately -10°.

Figure 1-14 provides a conceptual representation of the modal computation procedure. Subsequent paragraphs summarize the steps leading to the identification of the torsional modes.
1.3.3.1 Signal Processing and Fourier Analysis

Prior to performing the modal identification, the speed measurements shown in Figure 1-15 are filtered to reduce noise and high frequency components, this is followed by a Fourier analysis which consists of computing the power spectral density of the measured speeds to ascertain the torsional modes present in the signals.

The plots of the power spectral density shown in Figure 1-16 clearly show that there are four subsynchronous torsional modes at 8.2 Hz, 14.7 Hz, 19.5 Hz and 21.7 Hz. The lower three subsynchronous torsional modes are clearly discernible in both signals; the 21.7 Hz mode is only observable in the front standard speed. There is also a local mode at approximately 1.5 Hz. The number of modes discernible in the power spectral density plots provides the basis for the selection of the order of the system to be identified.
Figure 1-15: Front and rear standard speeds.

Figure 1-16: Power spectral densities of front (top) and rear (bottom) standard speeds.
1.3.3.2 Era Computation

The Hankel matrix for this case is built using speed data samples as follows:

\[
H_0 = \begin{bmatrix}
y(1) & y(2) & \cdots & y(m) \\
y(2) & y(3) & \cdots & y(m+1) \\
\vdots & \vdots & \ddots & \vdots \\
y(n) & y(n+1) & \cdots & y(n+m-1)
\end{bmatrix}
\]

where the matrix elements \(y(k)\) are \(2\times1\) vectors obtained from the filtered measured data:

\[
y = \begin{bmatrix}
\omega_p(1) & \omega_p(2) & \cdots & \omega_p(N_p) \\
\omega_n(1) & \omega_n(2) & \cdots & \omega_n(N_n)
\end{bmatrix}
\]

For this example, different time frames following the switching event are used to verify the consistency of the identified modal frequencies and damping; the last two or three seconds of data were used to further verify the validity of the identified model. Table 1-III summarizes the results obtained. The first and second columns show the time frame processed, \(T_{fr}\), and the number of points used, \(N_p\), respectively. The third column shows the sampling time \(\Delta t\), and the fourth column shows the dimensions of the Hankel matrix \(H(0)\), \(\text{dim } H\). The remaining columns, labeled 8.2 Hz, 14.7 Hz, 19.5 Hz and 21.7 Hz, show the frequencies, \(f\), and damping ratios, \(\xi\), of the torsional modes computed.

<table>
<thead>
<tr>
<th>(T_{fr}) (sec)</th>
<th>(N_p) (msec)</th>
<th>(\Delta t) (sec)</th>
<th>Dim</th>
<th>8.2 Hz mode</th>
<th>14.7 Hz mode</th>
<th>19.5 Hz mode</th>
<th>21.7 Hz mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>728</td>
<td>11</td>
<td>256 × 600</td>
<td>8.18 .0030</td>
<td>14.66 .0014</td>
<td>19.49 .0009</td>
<td>21.71 .0005</td>
</tr>
<tr>
<td>8.0</td>
<td>1067</td>
<td>7.5</td>
<td>534 × 800</td>
<td>8.18 .0031</td>
<td>14.66 .0014</td>
<td>19.49 .0009</td>
<td>21.71 .0006</td>
</tr>
<tr>
<td>7.0</td>
<td>934</td>
<td>7.5</td>
<td>368 × 750</td>
<td>8.18 .0030</td>
<td>14.66 .0014</td>
<td>19.48 .0009</td>
<td>21.71 .0007</td>
</tr>
<tr>
<td>7.0</td>
<td>637</td>
<td>11</td>
<td>274 × 500</td>
<td>8.18 .0030</td>
<td>14.66 .0014</td>
<td>19.48 .0009</td>
<td>21.71 .0007</td>
</tr>
<tr>
<td>6.0</td>
<td>801</td>
<td>7.5</td>
<td>302 × 650</td>
<td>8.18 .0031</td>
<td>14.66 .0014</td>
<td>19.48 .0009</td>
<td>21.72 .0007</td>
</tr>
<tr>
<td>5.0</td>
<td>667</td>
<td>7.5</td>
<td>334 × 500</td>
<td>8.18 .0030</td>
<td>14.66 .0014</td>
<td>19.48 .0009</td>
<td>21.72 .0003</td>
</tr>
</tbody>
</table>

The results listed in Table 1-III show that the ERA produces consistent results. The largest discrepancy occurs for the damping of the 21.7 Hz mode in the last case. However, only five seconds of measured data are used in this case. The modal frequencies are in agreement with those observed in the power spectral density plots.

The identified modal damping coefficients are essentially the same as to those obtained using band-passed filtered signals. To illustrate the latter point, Figure 1-17 shows the band-pass filtered front standard speed at 8.18 Hz. The exponential decay associated with this signal yields a damping ratio of 0.0032. The damping ratios for the other torsional modes can be computed in a similar manner and are listed in Table 1-III. The absence of 21.7 Hz modal content in the rear standard speed prevents the computation of the associated damping.

The results listed in Table 1-III and Table 1-IV show that the modal damping values computed using the ERA are similar to those obtained using an exponential fit. However, the latter approach requires the extraction of each modal component from each signal, whereas...
the ERA processes all signals, without the need for band-pass filtering and subsequent exponential curve fitting.

### Table 1-IV

<table>
<thead>
<tr>
<th>Mode</th>
<th>8.2 Hz mode</th>
<th>14.7 Hz mode</th>
<th>19.5 Hz mode</th>
<th>21.7 Hz mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Std. Speed</td>
<td>0.0031</td>
<td>0.0015</td>
<td>0.0009</td>
<td>0.0007</td>
</tr>
<tr>
<td>Rear Std. Speed</td>
<td>0.0028</td>
<td>0.0016</td>
<td>0.0010</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 1-17: Front standard band-pass filtered at 81.19 Hz.

A comparison of the impulse response of the identified system and the measured signals further verifies the validity of the identified modes. This is illustrated in Figure 1-18 and Figure 1-19, which compare $\omega_{fs}$ and $\omega_{rs}$, against the impulse response of the identified system. As these figures show, the match is very good over the time frame depicted. This time frame corresponds to 10 seconds of measurements following the switching event. The ERA was applied to the first seven seconds of data, the last three seconds verify the validity of the identified model. This type of match is an indication of a successful identification.
An additional perspective into the modal characteristics of the measured signals is provided by Figure 1-20. This figure shows the individual torsional components during the first second of data. This figure clearly shows their relative magnitude of each modal component; the absence of the 21.7 Hz mode from the front standard speed is also evident.
1.4 Eigensystem Realization Algorithm Illustration – Identification of Dominant Inter-Area Modes in the Eastern Interconnection from PMU Data of the FRCC 2008 Disturbance

In this section two of the main electromechanical modes in the US Eastern Interconnection (EI) are identified and the nature of power system oscillations emerging after a system-wide disturbance is examined. The disturbance in question was a major event that occurred on February 26, 2008 within the service area of the Florida Reliability Coordinating Council (FRCC) Bulk Electrical system [17, 18]. The disturbance data as measured by several PMUs across the Eastern Interconnection were made available for analysis by several operators and were obtained from TVA’s Super Phasor Data Concentrator (SPDC).

The application of the ERA here does have the goal of obtaining a system realization of the same order of the system from where the measurements where obtained, which would be impractical given the dimension of the US EI. It should also be noted that obtaining a system realization that exactly reproduces the sequence of data measured by the PMUs is not the goal of this work. Rather, the true goal is to obtain a system realization that closely represents the underlying electromechanical dynamics of the system. To this aim, the important issue of model order estimation is relevant. We provide general guidelines that aid in this respect.
The ERA is applied to decompose the PMU measurements into the different modal components contained within the available PMU measurements, and examine the dominant features of inter-area mode distribution in power networks. Two dominant oscillatory modes are identified and the individual behavior of each mode component is analyzed.

Major disturbances involving significant amounts of generation and load losses are precursors of power system oscillations that propagate throughout the whole system via the high-voltage transmission network. One of such events took place on February 26, 2008 when the Florida Reliability Coordinating Council (FRCC) Bulk Electrical system experienced a disturbance whose effect was spread across the U.S. Eastern Interconnection. The inception of the disturbance was the result of delayed clearing of a three-phase fault on a 138 kV switch in one of the substations of Florida Power & Light near Miami. Local primary protection and local back-up breaker failure protection were previously disabled for troubleshooting. As a result, remote backup protection relays performed delayed clearing of the 138 kV fault.

In summary [19, 20], the outcome of this disturbance was a generation loss of approximately 2,500 MW near the fault and 1,800 MW in the FRCC system. In addition, 2,300 MW of load were shed between under-frequency load shedding zones [19, 20]. With the exception of the trip of 25 transmissions lines involved in the remote clearing, and other minor transmission line trips, the FRCC system did not experience additional line outages and remained intact. The FRCC system remained connected to the EI during the course of the disturbance. As a result, this large disturbance excited several interarea oscillatory modes across the Eastern Interconnection [17, 18, 21].

Next, the main features of the Florida disturbance are studied by analyzing archived phasor measurement data. The PMUs considered in this analysis are: Manitoba, near the city of Winnipeg, Canada, Main near Bangor, Maine, Florida near Jacksonville, Florida, West Tennessee (W. Tenn.), near Memphis, Tennessee, and East Tennessee (E. Tenn.), near Knoxville, Tennessee, as shown in Figure 1-21(a). Figure 1-21(b), is the plot of the bus frequency measured during the disturbance by the PMUs, showing the wide-area impact of the disturbance. The steady state frequency deviation is approximately $\Delta f = 30$ mHz, while the electromechanical swing is propagated from Florida, to East Tennessee and West Tennessee, and subsequently to Manitoba, and finally Maine. The goal is to analyze the oscillatory components and characteristics contained in these measurements.

Figure 1-21: PMU locations and recorded measurements during the 2008 Florida disturbance.
1.5 System Realization

The ERA was applied to the voltage magnitude, voltage angle, and active power flow measurements available from each PMU. A general outline of the different steps used in the system realization process is the following:

Step 1. Data pre-processing and conditioning to generate appropriate signals for ERA
Step 2. Convert the pre-processed data (impulse response signals) into Markov parameters
Step 3. Construct a block Hankel matrix \( H(0) \) by arranging the Markov parameters into blocks
Step 4. Determine the order of the system by computing the singular values of the Hankel matrix \( H(0) \)
Step 5. Using a Hankel matrix shifted at \( k=1 \), i.e. \( H(1) \), obtain a system realization
Step 6. Quantify the system noise and select modes for a reduced system realization.
Step 7. Obtain a reduced model of the system using the eigenvalues selected Step 6, compute the impulse response of the reduced model and compare them with the pre-processed data.

Repeat Step 6 and Step 7 until an acceptable reduced model has been obtained. Subsequent paragraphs explain the steps above through an application example using the Florida disturbance data.

**Step 1.** Data pre-processing and conditioning to generate appropriate signals for ERA

Data pre-processing and conditioning is a crucial step for a successful application of different realization techniques. This step has not been entirely documented in different studies and may be an unintended aspect affecting the estimations obtained from the subsequent realizations. Here we perform two simple steps for pre-processing the data prior to applying the ERA, for this the voltage magnitude measurements are used at the different buses shown in Figure 1-22(a) (which have already been per unitized), and the voltage phase angles (not shown). To detrend the data, the mean of each measurement is computed and subtracted from each corresponding voltage, as a result the pre-processed data in Figure 1-22(b) is obtained.

![Figure 1-22: Detrending the PMU measurements of the voltage magnitude at each bus.](image-url)
Perhaps the most difficult step in pre-processing the data is to select a proper window of data that contains the ringdown which will be used for system realization, and at the same time, does not longer contain non-linearities which are evident after the on-set of the disturbance. This is more of an art than a science, and the selection of this window is done by careful judgment of the analyst. For this data set, the window shown in Figure 1-23 was selected, this window produced good estimates, as discussed below.

**Step 2. Convert the pre-processed data (impulse response signals) into Markov parameters**

Markov parameters can be obtained by different means [3, 22]; here they are obtained them from the response of the power system conceptualizing it as an “impulse response.” This conceptualization is important because from the Markov parameters, the embedded system matrices $A$, $B$, $C$, and $D$ can be extracted. The impulse response used for this example is a measurement matrix $y$ which contains the bus voltage magnitudes and the bus voltage angles shown in Figure 1-24, hence $y$ is of size $361 \times 10$. The corresponding time vector is of size $1 \times 361$, the number of inputs is 1, and the number of outputs is 10.
Let the Markov parameters, $\beta_0^{(i)}$, be given by

$$
\begin{align*}
Y_Q &= \beta \\
Y_{10} &= CB \beta^{(1)} \\
Y_{20} &= CAB \beta^{(2)}
\end{align*}
$$

where $Y_k$ is the corresponding measurements from the impulse response matrix $y$, whose four first rows ($k = [0-3]$) are given by

$$
\begin{bmatrix}
0.6362 & -0.4942 & 0.4804 & 0.1891 & 0.1874 & 0.0044 & -0.1037 & -0.0065 & -0.0062 & 0.0044 \\
0.6442 & -0.4875 & 0.4986 & 0.1638 & 0.1694 & 0.0034 & -0.0935 & -0.0067 & -0.0061 & 0.0071 \\
0.6507 & -0.4800 & 0.5166 & 0.1382 & 0.1502 & 0.0019 & -0.0832 & -0.0081 & -0.0056 & 0.0071 \\
0.6555 & -0.4715 & 0.5332 & 0.1123 & 0.1302 & 0.0007 & -0.0729 & -0.0091 & -0.0053(23) & 0.0072 \\
0.6588 & -0.4625 & 0.5495 & 0.0864 & 0.1099 & -0.0006 & -0.0626 & -0.0096 & -0.0049 & 0.0074
\end{bmatrix}
$$
Markov parameters in $\beta_0$, can be obtained directly from the impulse response at $k=0$, these correspond directly to the $D$ matrix:

$$D \bar{\beta}_0 = y_0 = \begin{bmatrix} 0.6362 & -0.4942 & 0.4804 & 0.1891 & 0.1874 & 0.0044 & -0.1037 & -0.0065 & -0.0062 & 0.0069 \end{bmatrix}^T$$

where the superscript {} indicates the input channel from the $y$ measurement matrix.

The next set of Markov parameters, $\beta_0^{(1)}$, are obtained from the impulse response at $k=1$, these being:

$$y_0 \bar{\beta}_0^{(1)} = \begin{bmatrix} 0.6442 & -0.4875 & 0.4986 & 0.1638 & 0.1694 & 0.0034 & -0.0935 & -0.0067 & -0.0061 & 0.0071 \end{bmatrix}^T$$

Similarly, for $k=2$ the parameters are:

$$y_0 \bar{\beta}_0^{(2)} = \begin{bmatrix} 0.6507 & -0.4800 & 0.5166 & 0.1382 & 0.1502 & 0.0019 & -0.0832 & -0.0081 & -0.0056 & 0.0071 \end{bmatrix}^T$$

For $\beta_0^{(3)}$ at $k=3$ the parameters are:

$$y_0 \bar{\beta}_0^{(3)} = \begin{bmatrix} 0.6555 & -0.4715 & 0.5332 & 0.1123 & 0.1302 & 0.0007 & -0.0729 & -0.0091 & -0.0053 & 0.0072 \end{bmatrix}^T$$

Finally, for $\beta_0^{(4)}$, the Markov parameters are given by:

$$y_0 \bar{\beta}_0^{(4)} = \begin{bmatrix} 0.6588 & -0.4625 & 0.5495 & 0.0864 & 0.1099 & 0.0006 & -0.0626 & -0.0096 & -0.0049 & 0.0074 \end{bmatrix}^T$$

Markov parameters continue to be determined until all measurements in $y$ have been processed. These first few parameters can be used to illustrate other parts of the ERA algorithm.

**Step 3.** Construct a block Hankel matrix $H(0)$ by arranging the Markov parameters into blocks

A Hankel matrix composed by the Markov parameters is now constructed; in general, this Hankel matrix has the following form

$$H(0) = \begin{bmatrix} Y_{12} & Y & Y_p \\ Y_2 & Y_3 & Y_{p-1} \\ Y_p & Y_{p+1} & Y_{p+r} \end{bmatrix}$$

where $p$ and $\gamma$ are integers such that $\gamma p \geq pm$. Note that $m$ is the number of outputs and $r$ is the number of inputs. Observe that $Y_0 = D$ is not included in $H(0)$.

For this example, using the Markov parameters, the four upper right vectors of the Hankel matrix are
Step 4. Determine the order of the system by computing the singular values of the Hankel matrix $H(0)$

The ERA algorithm uses Hankel matrices composed of the Markov parameters to compute the system matrices $A$, $B$, and $C$. Note that matrix $D$ is readily computed from the Markov parameters $\hat{\beta}_0$ in (20). Firstly, the ERA starts by factorizing the Hankel matrix $H(0)$ by performing singular value decomposition

$$H(0) = R \Sigma S^T$$

where the columns of $R$ and $S$ are orthonormal and $\Sigma$ is a rectangular matrix containing the singular values $\sigma_i, \ i = 1, 2, \cdots, n$, meaning that $H(0)$ is of rank $n$ which is the order of the system, in the example in this Section the rank is 40. A plot of the computed singular values is shown in Figure 1-25.

Although the matrix $H(0)$ is of full rank, which generally speaking would be the order of the system, this might not be desirable for many purposes. Measurement noise, nonlinearities, and computer round off contribute for $H(0)$ to be of full rank. The goal should not be to construct a realization that reproduces the input sequence of data exactly, but, that encapsulates the underlying dynamics of the system. For the purpose of analyzing inter-area modes this is relevant; a realization of a large dimension might fail to capture the true nature of the overall inter-area dynamics, and thus, the insight gained from the system identification would not be very useful. The approach that we have used to accomplish a reasonable system realization described below.

$$H(0) = \begin{bmatrix}
0.6442 & 0.6507 \\
-0.4875 & -0.4800 \\
0.4986 & 0.5166 \\
0.1638 & 0.1382 \\
0.1694 & 0.1502 \\
0.0034 & 0.0019 \\
-0.0935 & -0.0832 \\
-0.0067 & -0.0081 \\
-0.0061 & -0.0056 \\
0.0071 & 0.0071 \\
0.6507 & 0.6555 \\
-0.4800 & -0.4715 \\
0.5166 & 0.5332 \\
0.1382 & 0.1123 \\
0.1502 & 0.1302 \\
0.0019 & 0.0007 \\
-0.0832 & -0.0729 \\
-0.0081 & -0.0091 \\
-0.0056 & -0.0053 \\
0.0071 & 0.0072
\end{bmatrix}$$
Step 5. Using a Hankel matrix shifted at \( k=1 \), i.e. \( H(1) \), obtain a system realization of large order

A shifted Hankel matrix at \( k=1 \) is calculated as

\[
H(1) = \begin{bmatrix}
Y_2 & Y_3 & Y_{r+1} \\
Y_3 & Y_4 & Y_{r+2} \\
& Y_{r+12} & Y_{r+7}
\end{bmatrix}
\]

where the Markov parameters of the four upper left corner elements are given by (26)-(28).

By writing the Hankel matrix in terms of matrices \( A, B \), and \( C \) the following relationship is obtained

\[
H(1) = R_n \Sigma_n^{1b} A \Sigma_n^{1b} S_n
\]

where the \( \Sigma \) indicates a realization of not necessarily order \( n \), but of large order. Defining the null matrix of order \( i \), \( O_i \), and the identity matrix \( I_i \), of order \( i \) also, then \( E_r^T = [I_r, O_r, \cdots O_r] \) and \( E_m^T = [I_m, O_m, \cdots O_m] \), refer to [3] for details. In general, it can be said that \( B \) is formed by the first \( r \) columns of \( \Sigma_n^{1/2} S_n^T \), and \( C \) is formed by the first \( m \) rows of \( R_n \Sigma_n^{1/2} \) [22].

For this example, a large order realization of order 40 is obtained. It is possible to compare the impulse response of the realization against the pre-processed data, as shown in Figure 1-26 where only the voltage magnitude components are shown. Although this realization fits well the data, a model of order 40 might have too many eigenvalues close to each other which arise when the method is trying to fit the noise exactly. To obtain a better realization we turn to the discussion in the next step.

Figure 1-25: Singular values obtained from SVD.
Step 6. Quantify the system noise and select modes for a reduced system realization.

There are many available techniques for this step, however, a simple technique is to choose an appropriate number of eigenvalues to be included in a reduced order realization. The discrete model can be transformed to a continuous time model; then mode damping and frequencies can be computed from the eigenvalues of the continuous time state matrix $A$. When this is done for the large order realization, the continuous time state matrix is of order 40 and hence there are 40 eigenvalues. In Table 1-V only a subset of those that are relevant to the discussion is presented. The complex pairs are ordered in terms of their energy, it can be observed from Mode 1 that one of the issues of using a large model is noise over fitting, this mode with negative damping has been added to fit the data exactly, but a realization as such would impose difficulties if the model were to be used for feedback control design. In addition, for inter-area mode analysis the frequency range of analysis is between 0.1-2 Hz, for this reason Mode 2-3 is not necessary. Therefore, only the complex pairs in Mode 4-7 have been used to obtain a two-mode realization of the inter-area behavior in the US Eastern Interconnection.

Table 1-V: A Subset of the Estimated Mode Damping and Frequencies from the Large Order Realization (the values highlighted in green are used to construct the reduced order realization)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq. (rad/s)</th>
<th>Freq. (Hz)</th>
<th>Damping</th>
<th>Residue</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0865</td>
<td>0.0000</td>
<td>-1.0000</td>
<td>0.4286</td>
<td>40.4745</td>
</tr>
<tr>
<td>2</td>
<td>0.5919</td>
<td>0.0272</td>
<td>0.9575</td>
<td>1.8385</td>
<td>2.0756</td>
</tr>
<tr>
<td>3</td>
<td>0.5919</td>
<td>0.0272</td>
<td>0.9575</td>
<td>1.8385</td>
<td>2.0756</td>
</tr>
<tr>
<td>4</td>
<td>1.4290</td>
<td>0.2227</td>
<td>0.2026</td>
<td>5.6925</td>
<td>1.0214</td>
</tr>
<tr>
<td>5</td>
<td>1.4290</td>
<td>0.2227</td>
<td>0.2026</td>
<td>5.6925</td>
<td>1.0214</td>
</tr>
<tr>
<td>6</td>
<td>3.0703</td>
<td>0.4868</td>
<td>0.0877</td>
<td>0.1893</td>
<td>0.9725</td>
</tr>
<tr>
<td>7</td>
<td>3.0703</td>
<td>0.4868</td>
<td>0.0877</td>
<td>0.1893</td>
<td>0.9725</td>
</tr>
</tbody>
</table>

Step 7. Obtain a reduced model of the system using the eigenvalues selected Step 6, compute the impulse response of the reduced model and compare them with the pre-processed data.

Using the eigenvalues in Table 1-IV, a reduced order realization is obtained, the continuous time state matrices for this realization are:
It should be noted that the mode frequencies and damping are those of the selected modes in Table 1, moreover, eigenvectors can be also computed from (35).

The impulse response of (36) overlaid to the pre-processed data is shown in Figure 1-27. Observe that this second order realization captures the main dynamics of interest in inter-area oscillations phenomena; this will be further discussed in the next section.

![Figure 1-27: Impulse Response of the Large Order Realization overlaid with the pre-processed data.](image)

### 1.5.1 Analysis of the Identified Electromechanical Modes

The frequencies of the identified low-frequency interarea modes are 0.22 Hz and 0.49 Hz. Similar mode frequencies for this event have been identified by other analysts using other techniques [17, 21, 18]. In Figure 1-28 and Figure 1-29, the voltage measurements of each PMU are shown along with their ERA approximation for the 0.22 and 0.49 Hz components of the signals. Note that the 0.22 Hz component is prominent in Maine, Florida and Manitoba, while the 0.49 Hz component has much more influence in West Tennessee's and East Tennessee's voltage.
All identified components for the 0.22 Hz mode in the voltage angle are shown in Figure 1-30(a), and for the 0.49 Hz mode they are shown in Figure 1-30(b). A snapshot of the voltage angle components is taken at $t=3$ sec., and is used for the projection of the 0.22 Hz mode shown in Figure 1-30(c), and for the 0.49 Hz mode a snapshot of the voltage angle components is taken at $t=1.93$ sec., and used for the projection in Figure 1-30(d). The starting time $t=0$ sec corresponds to 18:10:04.333 hrs. From the voltage angles of the 0.22 Hz mode it can be observed that Florida oscillates against Maine and Dorsey, that is, it is a North vs. South mode.
The 0.49 Hz mode is more difficult to analyze from this limited data set. However, it can be noted that the voltage angle at West Tennessee, East Tennessee, and Florida have the largest oscillations while Dorsey and Maine have a less significant contribution. More important, West Tennessee and East Tennessee are in anti-phase suggesting that the pivot of the oscillation occurs somewhere between those locations.

The single most important observation that could be made about the oscillations discussed above is the following: for all of the network variables, the independent modal components do not peak at the same instants. This is in fact a time delay between the modal components. This time delay can be rationalized as a phase shift between the modal components in the phase domain. In [47] the origin of this phase shift is explained by analyzing the mode shapes of a test network and by providing analytically closed form expressions.

### 1.6 Matrix Pencil Method

This section examines the Matrix Pencil approach, which was introduced for extracting poles from antennas' electromagnetic transient responses [2]. The Matrix Pencil method is fairly new to the field of power, but has some advantages over polynomial methods in solving these challenging initialization problems when dealing with unknown signal characteristics. The Matrix Pencil’s inherent ability to accurately analyze noisy signals makes it a promising technique that should be further developed in this field. Some advantages of the Matrix Pencil
technique include easier pre-tuning with fewer variable restrictions, little sensitivity to sample size changes, and a built in method for pre-determining the number of modes in an undetermined system [23].

The Matrix Pencil approach was introduced by Hua and Sarkar [2] for pole estimation and was initially used for extracting poles from antennas’ electromagnetic transient responses. The advantage of the Matrix Pencil method is that the signal poles can be found directly from the eigenvalues of a single developed matrix in contrast to polynomial methods which require a two-step process [2]. This method is designed to directly estimate the parameters for the exponential terms by fitting the function (2) to an observed measurement for \( y(t) \) in (1), where \( y(t) \) consists of \( N \) samples that are evenly spaced by a time interval \( \Delta t \). Since the measurement signal \( y(t) \) may contain noise or dc offset, it may have to be conditioned before the fitting process is applied.

By using the generalized eigenvalue solution to find \( z_i \), the Matrix Pencil method removes the limitation on the number of poles \( M \), whereas, the polynomial method has difficulties obtaining roots of a polynomial if \( M \) is greater than 50 [24]. This results in the estimates of \( z_i \) having better statistical properties [25]. The Matrix Pencil method produces a matrix whose roots provide the system poles, \( z_i \) given by (3)

\[
\begin{bmatrix} Y_1 & -\lambda Y_1 \end{bmatrix} = [Z_R \, \mathbb{I}] [Z_a] - \lambda [Z_R] [Z_a] \tag{37}
\]

where

\[
[Z_a] = \text{diag}[z_1, z_2, \ldots, z_M] \tag{38}
\]

\[
[Z_i] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{(s-L-1)} & z_2^{(s-L-1)} & \cdots & z_M^{(s-L-1)} \end{bmatrix} \tag{39}
\]

\[
[Z_j] = \begin{bmatrix} 1 & \cdots & z_i^{L-1} \\ 1 & \cdots & z_i^{L-1} \\ \vdots & \ddots & \vdots \\ 1 & \cdots & z_M^{L-1} \end{bmatrix} \tag{40}
\]

The eigenvalue set \( \lambda(Y_2, Y_1) \), is contained in the square matrices \( Y_1 \) and \( Y_2 \), as the pencil values or roots of \( Y_2 \) relative to \( Y_1 \). The parameters in (35)-(38) include: \( M \) the desired number of eigenvalues, \( [R] \) the matrix of residuals, \( [I] \) an \( M \times M \) identity matrix, and \( L \) the pencil parameter such that \( M \leq L \leq M \). To determine the eigenvectors and eigenvalues of the signal, a Hankel matrix is formed.

The variable \( L \), known as the pencil parameter, is useful for eliminating some effects of noise in the data [25]. If the pencil parameter \( L \) is chosen such that \( L = N/2 \), then the performance of the method is close to the optimal Cramer-Rao bound [26]. The Cramer-Rao bound is essentially an estimate of the “absolute” best result that a technique can achieve in a certain “noisy” environment. In studies by Hua and Sarkar [26], the Matrix Pencil method reaches the approximate Cramer-Rao bound when the pencil parameter is set to \( L = N/2 \). The basic theory behind this phenomenon states that the dimension of the column subspace of \( [Y] \) is largest for this value of \( L \), indicating that more noise can be filtered out by throwing away the largest noise subspace in the decomposition of \( [Y] \).
1.6.1 The Matrix Pencil Method Without Noise

The two nonlinear examples, referred to as NL1 and NL2, come from the dynamic response of a Power System Simulator for Engineering (PSS/E) simulation of a large Midwestern utility system. The waveforms used for this study are from tie line power flows in response to a short circuit on the system. The signals have been normalized for easier manipulation by the Matrix Pencil method. The number of dominant modes $M$ is unknown, so the Matrix Pencil method will be analyzed on its ability not only to extract the dominant modes from the nonlinear signals, but also in how it determines the best settings for $M$ using SVD.

1.6.1.1 Matrix Pencil Simulation Results for NL1

The Matrix Pencil identification method is applied to the nonlinear power system example NL1 where the modal content is initially unknown, with the intent of finding the most relevant eigenvalues and corresponding modes. In examining the individual normalized singular values (9) for the nonlinear waveform NL1, included in Table 1-VI, there are several possible groupings in which to determine the number of significant modes. Examples of feasible significant mode groupings include: (1-6) where all values are greater than 0.5, (1-11) with all values greater than 0.1, (1-18) with all values greater than 0.005, and (1-24) includes the most dominant modes in $[Y]$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\sigma_i$</th>
<th>$i$</th>
<th>$\sigma_i$</th>
<th>$i$</th>
<th>$\sigma_i$</th>
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<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>9</td>
<td>0.2984</td>
<td>17</td>
<td>0.0133</td>
</tr>
<tr>
<td>2</td>
<td>0.9953</td>
<td>10</td>
<td>0.2207</td>
<td>18</td>
<td>0.0094</td>
</tr>
<tr>
<td>3</td>
<td>0.8333</td>
<td>11</td>
<td>0.1509</td>
<td>19</td>
<td>0.0047</td>
</tr>
<tr>
<td>4</td>
<td>0.8223</td>
<td>12</td>
<td>0.0964</td>
<td>20</td>
<td>0.0041</td>
</tr>
<tr>
<td>5</td>
<td>0.8099</td>
<td>13</td>
<td>0.0515</td>
<td>21</td>
<td>0.0028</td>
</tr>
<tr>
<td>6</td>
<td>0.7688</td>
<td>14</td>
<td>0.0235</td>
<td>22</td>
<td>0.0015</td>
</tr>
<tr>
<td>7</td>
<td>0.4204</td>
<td>15</td>
<td>0.0199</td>
<td>23</td>
<td>0.0014</td>
</tr>
<tr>
<td>8</td>
<td>0.3622</td>
<td>16</td>
<td>0.0146</td>
<td>24</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

The four groupings of singular values produce eigenvalues with similar frequencies. Figure 1-31 contains the reconstructed signals $\hat{y}_k$ for the different modal groupings. The results in Figure 1-31 show that using a value of 18 for $M$ produces a waveform composed of 10 modes with a SNR of 50.85, which is close to the original signal with only a small amount of oscillation in the first 0.5 seconds. A SNR of 40 dB or greater is an acceptable level of performance for this type of signal and modal technique as compared to the work in [27] and [28]. This is confirmed in the statistical results presented in Table 1-VII.
Increasing the number of modes past 10 only marginally improves precision, but the computation time is increased. A subroutine can be written into the Matrix Pencil method to determine $M$ based on the ratio of singular values compared to a precision threshold in (41). When $p$ in the threshold parameter is set to a precision of 3, the value of $M$ becomes 24 and when $p$ is set to a precision of 2, the Matrix Pencil routine chooses $M$ to be 18 from the significant modes.

$$\frac{\sigma_i}{\sigma_{\text{max}}} = 10^{-p}$$  \hspace{1cm} (41)

Table 1-VII: Statistical results for Matrix Pencil Solution of NL1

<table>
<thead>
<tr>
<th>$M$</th>
<th>6</th>
<th>11</th>
<th>18</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>SNR</td>
<td>13.01</td>
<td>26.81</td>
<td>50.85</td>
<td>51.25</td>
</tr>
</tbody>
</table>

1.6.1.2 Matrix Pencil Simulation Results for NL2

Results for the second nonlinear example NL2 are shown in Figure 1-32. For this example, the singular values shown in Table 1-VIII are grouped into the following categories: (1-4) are the singular values greater than 0.75, (1-12) have values greater than 0.01, (1-19) are greater than 0.001, and (1-26) are the most dominant values of matrix $[Y]$. As more modes are extracted, the frequency range of available modes is increased, leading to a better reconstruction of the signal. This will be even more beneficial for the high frequency noise studies presented in the following section. Increasing the number of extracted modes can improve accuracy to an extent, but a balance needs to be maintained to prevent over-estimation. The number of desired modes affects the size of the SVD matrices, which can increase computation time and computer memory resources.
Table 1-VIII: Singular Values of Matrix Y

<table>
<thead>
<tr>
<th>i</th>
<th>$\sigma_i$</th>
<th>i</th>
<th>$\sigma_i$</th>
<th>i</th>
<th>$\sigma_i$</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>0.1388</td>
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<td>2</td>
<td>0.9215</td>
<td>11</td>
<td>0.1049</td>
<td>20</td>
<td>0.0048</td>
</tr>
<tr>
<td>3</td>
<td>0.8317</td>
<td>12</td>
<td>0.1044</td>
<td>21</td>
<td>0.0047</td>
</tr>
<tr>
<td>4</td>
<td>0.8191</td>
<td>13</td>
<td>0.0863</td>
<td>22</td>
<td>0.0047</td>
</tr>
<tr>
<td>5</td>
<td>0.5146</td>
<td>14</td>
<td>0.0651</td>
<td>23</td>
<td>0.0044</td>
</tr>
<tr>
<td>6</td>
<td>0.4695</td>
<td>15</td>
<td>0.0497</td>
<td>24</td>
<td>0.0035</td>
</tr>
<tr>
<td>7</td>
<td>0.3280</td>
<td>16</td>
<td>0.0444</td>
<td>25</td>
<td>0.0025</td>
</tr>
<tr>
<td>8</td>
<td>0.2768</td>
<td>17</td>
<td>0.0356</td>
<td>26</td>
<td>0.0022</td>
</tr>
<tr>
<td>9</td>
<td>0.1517</td>
<td>18</td>
<td>0.0165</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1-IX: Statistical Results for Matrix Pencil Solution of NL2

<table>
<thead>
<tr>
<th>M</th>
<th>4</th>
<th>12</th>
<th>19</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>SNR</td>
<td>10.89</td>
<td>27.52</td>
<td>51.72</td>
<td>52.17</td>
</tr>
</tbody>
</table>

Figure 1-32: Reconstruction of the NL2 using different numbers of modes.

The statistical data contained in Table 1-IX indicates that 10 or more extracted modes provide the most accurate results using the Matrix Pencil method. As with the previous nonlinear example, once a reasonable level of accuracy is obtained, increasing the number of modes used to recreate the signal provides limited improvement on the results, along with an increase in computational time and resources needed to compute the result. Modal content for the Matrix Pencil results using 26 singular values and 14 modes is presented in Table 1-VII.

1.6.2 The Matrix Pencil with Noise

A Gaussian random walk signal was added to the nonlinear waveforms to test the ability of the Matrix Pencil technique at handling noise. The type of signal used to create the noise is referred to as Brownian Motion [29]. Brownian motion is a real-valued stochastic process...
where a random variable $W(t)$ depending continuously on $t \in [0, T]$ satisfies the following conditions:

- $W(0) = 0$ (with probability 1)
- For $0 \leq s < t \leq T$, the random variable given by increment $W(t) - W(s)$ is normally distributed with mean zero and variance $t - s$.
- $W(t) - W(s) = \sqrt{t - s} \times WGN(0, 1)$ where $WGN(0, 1)$ is real white Gaussian noise.
- For $0 \leq s < t < u < v \leq T$, the increments $W(t) - W(s)$ and $W(v) - W(u)$ are independent.

For this purpose, the random walk has a step size that varies according to a Gaussian distribution with zero mean. The SNRs of the noisy signals are 27.84 dB and 8.97 dB, respectively, indicating that signal NL2 has a higher level of noise penetration than signal NL1.

### 1.6.2.1 Matrix Pencil Simulation Results for NL1

The Matrix Pencil solution for the first nonlinear noisy signal is shown in Figure 1-33. The Matrix Pencil method can extract the noisy signal using 13 extracted modes with a reconstruction SNR of 43.05 dB. If the reconstructed signal is compared to the original signal without noise, the SNR drops to 27.09 dB.

Better accuracy can be obtained if the SVD is used to determine the number of modes in the noisy signal from the most dominant singular values. In (41), the value for $p$ is set to 1, 1.5, 2, and 3 to determine what precision and value for $M$ is the best fit for this signal. Statistical results are shown in Table 1-X. In this case, a precision of 2 with $M$ equal to 52 is probably the best fit, because increasing the number of modes from 27 to 257 doubles the computation time, for the gain of only a very small improvement in SNR.

---

**Table 1-X: Results for NL1 with Noise for Different Values of M**

<table>
<thead>
<tr>
<th>$p$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>13</td>
<td>20</td>
<td>52</td>
<td>509</td>
</tr>
<tr>
<td>Mode</td>
<td>7</td>
<td>11</td>
<td>27</td>
<td>257</td>
</tr>
<tr>
<td>SNR</td>
<td>26.64</td>
<td>43.86</td>
<td>48.12</td>
<td>50.08</td>
</tr>
</tbody>
</table>
1.6.2.2 Matrix Pencil Simulation Results for NL2

The second nonlinear example has a greater presence of noise with a SNR of 8.97 dB. The Matrix Pencil reconstruction in Figure 1-34 contains 14 modes and is able to follow the general shape of the noisy signal with a reconstruction SNR of 29.57 dB.

![Matrix Pencil results for signal NL2 with noise.](image)

When the SVD is used to determine the best value for $M$, the Matrix Pencil solution improves, which can be seen from the statistical data in Table 1-XI. For the Matrix Pencil solution of NL2 with noise, when 22 modes are extracted (SNR=34.49 dB), the highest modal frequency is 49.91 Hz; whereas, when 51 modes are extracted (SNR=44.03 dB), the highest frequency is 137.78 Hz. Extracting more modes results in a better fit of the noisy signal, since higher frequencies go into the reconstruction of the signal.

<table>
<thead>
<tr>
<th>$p$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>14</td>
<td>42</td>
<td>101</td>
</tr>
<tr>
<td>Mode</td>
<td>7</td>
<td>22</td>
<td>51</td>
</tr>
<tr>
<td>SNR</td>
<td>22.63</td>
<td>34.49</td>
<td>44.03</td>
</tr>
</tbody>
</table>

The comparison of results raises some questions on the quality of modal content being extracted for the noisy signals. The goal of the modal analysis methods is to identify the underlying signal hidden in the noise for adequate control or damping applications and the high frequency modes extracted by the Matrix Pencil technique, show an overestimation of the signals by extracting extraneous noisy modes.

For example, the original version of signal NL1 can be accurately recreated using the 13 modes in Table 1-VI by the Matrix Pencil technique with a reconstruction SNR of 51.25 dB. When noise is added to this signal, the Matrix Pencil method must extract an increased number of 27 modes to accurately reconstruct the noisy signal with a noise reconstruction SNR of 48.12 dB. The extra 14 modes extracted from the noisy version of the signal may not be necessary for some control or damping applications. It would be beneficial to see if the
Matrix Pencil method is able to extract the most dominant 13 modes found by the study of the noiseless case in a subset of the 27 noisy modes found when analyzing the noisy version of the signal.

There are three logical subsets of the 27 noisy modes that would best suit the most dominant 13 original signal modes including: the 13 modes of largest magnitude, the 13 modes of smallest frequency, and the 13 modes with frequencies most similar to the frequencies found in Table 1-IV. These three sets of 13 modes were reconstructed into Matrix Pencil solutions and compared to the noiseless NL1 signal to determine if accurate modal content can be extracted regardless of the additionally extracted noisy high frequency modes. The results show a good fit to the original signal when using subsets of the noisy modes obtained by the Matrix Pencil method. The dominant modes of the underlying signal are found by the Matrix Pencil method and deductive reasoning can be applied to separate the dominant modal content from the noise generated modal content. The error caused by the added noise could be improved by advanced filtering of the noisy signal before the modal extraction method is applied.

Another method for finding the most dominant modes to recreate the original noiseless signals from the extracted noisy modes is to use a Genetic Algorithm (GA) to select the modes [30]. The GA was able to extract a better combination of the noisy modes of the signals. Table 1-XII contains the SNR of all modal combinations tested, compared with the noiseless and noisy versions of NL1 and NL2, respectively.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Largest Magnitude</th>
<th>Smallest Frequency</th>
<th>Similar Frequency</th>
<th>Genetic Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>19.25</td>
<td>27.80</td>
<td>27.82</td>
<td>34.51</td>
</tr>
<tr>
<td>Noise</td>
<td>21.70</td>
<td>31.25</td>
<td>31.07</td>
<td>29.19</td>
</tr>
<tr>
<td>NL2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>9.86</td>
<td>8.98</td>
<td>9.09</td>
<td>19.42</td>
</tr>
<tr>
<td>Noise</td>
<td>18.42</td>
<td>16.26</td>
<td>16.18</td>
<td>16.85</td>
</tr>
</tbody>
</table>

Any technique used for power system modal analysis must have the ability to function in the presence of noise. The Matrix Pencil method has been demonstrated to have the capabilities of extracting the dominant modes of a signal regardless of noise. This method also has a built-in method for determining the desired number of modes by using SVD which is very desirable in power applications where there are often many unknowns about the signal. Performing a SVD on the data matrix actually filters out some of the system noise while discarding the non-principal singular vectors improving the signal reconstruction. Once the modes of the noisy signal are found, deductive reasoning or simple optimization tools such as Genetic Algorithm can be used to discard the extraneous noisy modes allowing for only the most pertinent modes to be applied for damping or control purposes.

1.7 Chapter 1 References


APPENDIX 1 A

Application Examples

Author: John Hauer

A1.1 Comments on Application Examples

The application examples to follow illustrate the results obtainable from modal analysis in general. There is a strong focus upon modal identification, but it is used together with complimentary or supporting tools such as Fourier analysis.

The examples draw upon WAMS data collected on the Western Interconnection in North America [1]. Figure A1-1 shows the geography involved.

The dynamic behavior of any power system is strongly determined by its topology. The “WECC doughnut,” a circular structure with in the grid south of the US-Canada border, is clearly apparent in Figure A1-1. The implications of this structure are well known, and they have received much attention in the literature.

Less attention has been paid to the “crown” of radial connections that route power into the doughnut from large remote plants in Western Canada and Eastern Montana. In several cases the characteristics of a major interarea mode are strongly influenced by the controls at a single plant, and the very existence of the mode depends on the operational status of a single transmission line.
A1.2 Example 1: The U.S.-Canada Blackout of August 14, 2003

U.S.-Canada Blackout on August 14, 2003, was immediately notable for its extent, complexity, and impact [2, 3]. Background information concerning the event was gathered together by a group of utilities that, collectively, had been developing a WAMS for the eastern interconnection [4]. Like the WECC WAMS in the western interconnection, "WAMS East" had a primary backbone of synchronized phasor measurement units (PMUs) that continuously stream data to Phasor Data Concentrators (PDCs) at central locations for integration, recording, and further distribution. Both WAMS also employed Portable Power System Monitor (PPSM) units as a secondary backbone, to continuously record analog transducer signals on a local basis [5].

Though oscillation problems were not a significant factor in the August 14 Blackout, straightforward Fourier analysis provides oscillation signatures that contribute to situational awareness for real time grid operation.

Figure A1-2 shows a waterfall plot for Fourier spectra of bus frequency fluctuations at the American Electric Power (AEP) Kanawha River substation. Frequency of the spectral peaks shows a general downward trend, plus sharp discontinuities that are associated with system events. This behavior
suggests that the swing frequencies associated with interarea modes were declining through increasing stress and network failures on the power system [6]. As illustrated in Figure A1-3, such trends are not typical [7].

The modal activity shown in Figure A1-2 is typical of data that were collected as far away as Entergy's Waterford substation near New Orleans, LA. The coherency waterfall of Figure 4 shows that ambient measurements at the two well separated substations were highly correlated, even though the Entergy data were acquired with analog transducers located near New Orleans LA. Distance between the two substations is estimated to be about 800 miles.

**Figure A1-2:** Spectral History for US-Canada Blackout of August 14, 2003: AEP Kanawha River bus frequency, 12:00-16:10 EDT. Data provided by Navin Bhatt, AEP
Figure A1-3: Spectral History for AEP Jackson’s Ferry bus frequency on December 20, 2002.
Data provided by Navin Bhatt, AEP

Figure A1-4: Coherency Waterfall for Entergy Waterford frequency vs. AEP Callaway frequency
US-Canada Blackout of August 14, 2003 (bandpassed signals)
A1.3 Example 2: The Western System Breakup of August 10, 1996

Data archives for the breakup of August 10, 1996 are extensive, and the event is a popular test bed for modal identification algorithms of all kinds. Users are warned that the instrument technology was rudimentary, however, and the data should be examined with close attention to the issues reported in [8].

The abundance of data reflects ongoing utility concerns about the realism of their power system models and their modeling practices [9]. Figure A1-5 shows results for the first system wide model validation test performed by the western utilities. Model studies of that time showed transient 0.7 Hz oscillations as a major threat to the system, despite little evidence of this in observed system behavior. This test, together with subsequent investigations, indicated that the 0.7 Hz threat was largely due to modeling artifacts. The evidence also indicated that models understated the dangers posed by the North-South and Alberta modes. While these differences are apparent in the figure, it should be recognized that activity peaks at the higher frequencies represent clusters of modes and lack the resolution needed to sharply identify specific interactions.

![Figure A1-5: Model vs. actual response of AC Intertie power to Chief Joseph brake power on May 16, 1989.](image)

These modeling uncertainties came into sharp focus with the breakup of August 10, 1996 [10]. Figure A1-6 shows one of the many signals recorded for this event and Figure 7 shows its spectral waterfall.

Modal estimates for ringdown events during this and earlier records are compared in Table A1-I. It is notable that loss of the Keeler-Allston line abruptly shifted the North-South mode to the unusually
low frequency of 0.264 Hz, with abnormally low damping. The John Day-Marion line produced similar values, but the system recovered from what may have been a “near miss.”

Figure 6 is annotated with results from an early ambient mode meter. Analysis of the several minutes of the signal prior to the Keeler-Allston oscillation places the N-S mode at 0.270 Hz with 7.0% damping. Similar analysis after the Keeler-Allston transient indicates the mode is near 0.252 Hz with 1.2% damping. Figure A1-8 and Figure A1-9 show similar results for a recent version of the DOE ModeMeter.

Figure A1-6: Oscillation buildup for the WSCC breakup of August 10, 1996
Figure A1-7: Oscillation spectra for theWSCC breakup of August 10, 1996.
Table A1- I: Observed Behavior of the PACI Mode via Ringdown Analysis

<table>
<thead>
<tr>
<th>Before August 10, 1996</th>
<th>Frequency</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/08/92 (Palo Verde trip)</td>
<td>0.28 Hz</td>
<td>7.5 %</td>
</tr>
<tr>
<td>03/14/93 (Palo Verde trip)</td>
<td>0.33 Hz</td>
<td>4.5 %</td>
</tr>
<tr>
<td>07/11/95 (brake insertion)</td>
<td>0.28 Hz</td>
<td>10.6 %</td>
</tr>
<tr>
<td>07/02/96 (system breakup)</td>
<td>0.22 Hz</td>
<td>1.2 %</td>
</tr>
</tbody>
</table>

**On August 10, 1996**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:52:19 (brake insertion)</td>
<td>0.285 Hz</td>
</tr>
<tr>
<td>14:52:37 (John Day-Marion)</td>
<td>0.264 Hz</td>
</tr>
<tr>
<td>15:18 (ringing) 0.276 Hz</td>
<td></td>
</tr>
<tr>
<td>15:42:03 (Keeler-Allston)</td>
<td>0.264 Hz</td>
</tr>
<tr>
<td>15:45 (ringing) 0.252 Hz</td>
<td></td>
</tr>
<tr>
<td>15:47:40 (oscillation start)</td>
<td>0.238 Hz</td>
</tr>
<tr>
<td>15:48:50 (oscillation finish)</td>
<td>0.216 Hz</td>
</tr>
</tbody>
</table>

Figure A1-8: ModeMeter display 160 seconds after Keeler-Alston line trip
A1.4 Example 3: Northwest Oscillations on June 4, 2003

On June 4, 2003 the western interconnection experienced an unusual oscillation at a steady frequency of 0.584 Hz. Though fairly small, it was widely observable throughout the WECC WAMS.

Phasor data collected at BPA, BCH, and Alberta later revealed this as an interaction between the Colstrip plant in eastern Montana, and the Kemano plant in west-central British Columbia. The cause of the interaction apparently involved weak voltage support in Grand Coulee area of central Washington State.

A time domain view of the oscillations is shown in Figure A1-10; the corresponding spectral waterfall in Figure A1-11 is representative of both plants.

Dominant modes for these two plants are usually well separated, at frequencies around 0.75 Hz and 0.63 Hz respectively. Mode shape, however, indicated that these two modes had effectively merged at 0.584 Hz. This was determined by a tandem Prony fit to 18 frequency signals around the grid, with results that are provided in Table A1-II and Figure A1-12. In both cases the Kemano frequency was metered by a PMU at Williston, to which the Kemano plant is radially connected.

Williston merges generated power from several major plants and routes it to Kelly Lake, where it is forwarded toward Vancouver Canada and eventually the Ingledow connection into the United States. The Williston-Kelly Lake line is a major interaction path for interarea modes, similar in many ways to the California-Oregon Interconnection.
Correlation against MW swings on the Williston-Kelly Lake line revealed corresponding power oscillations on key tielines throughout the western interconnection, including very small ones on the Pacific HVDC Intertie. Figure A1-13 shows that the interaction was clearly apparent in the coherency function for the Palo Verde-Devers line (Arizona to Southern California), even though this line is some 1400 miles from Williston and the signal is barely visible in time domain data.

![Figure A1-10: Key frequency signals for oscillation event on June 4, 2003](image-url)
Figure A1-11: Waterfall plot for oscillation on June 4, 2003 – local frequency at Williston

Table A1-II: Prony solution for tandem fit to 18 frequency signals (bandpass filtered)

<table>
<thead>
<tr>
<th>Signal</th>
<th>Res Mag</th>
<th>Res Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>GC50 Grand Coulee Hanford</td>
<td>0.00255613</td>
<td>94.508</td>
</tr>
<tr>
<td>MALN Malin N.Bus Voltage</td>
<td>0.00216967</td>
<td>99.949</td>
</tr>
<tr>
<td>SCE1 Vincent Voltage</td>
<td>0.00013265</td>
<td>-14.848</td>
</tr>
<tr>
<td>SCE1 Devers 500 Bus Voltage</td>
<td>0.00105099</td>
<td>-82.371</td>
</tr>
<tr>
<td>COLS Colstrip Bus Voltage</td>
<td>0.01525229</td>
<td>-160.754</td>
</tr>
<tr>
<td>MPLV Maple Valley Bus Voltage</td>
<td>0.00298496</td>
<td>70.830</td>
</tr>
<tr>
<td>SLAT Slatt 500 kV Voltage-W</td>
<td>0.00217843</td>
<td>112.630</td>
</tr>
<tr>
<td>SUML Summer Lake 500 kV VoltageN</td>
<td>0.00214484</td>
<td>104.623</td>
</tr>
<tr>
<td>CPJK Capt Jack 500 kV VoltageN</td>
<td>0.00219423</td>
<td>99.947</td>
</tr>
<tr>
<td>JDAY John Day Bus Voltage</td>
<td>0.00220184</td>
<td>108.900</td>
</tr>
<tr>
<td>BE23 Big Eddy 230 Bus3 Voltage</td>
<td>0.00223918</td>
<td>106.851</td>
</tr>
<tr>
<td>BE50 Big Eddy 500 Bus Voltage</td>
<td>0.00222430</td>
<td>107.214</td>
</tr>
<tr>
<td>SYLM Sylmar Bus Voltage</td>
<td>0.00073248</td>
<td>-81.350</td>
</tr>
<tr>
<td>ING1 5L52 Custer Voltage DeOsc</td>
<td>0.00535420</td>
<td>4.926</td>
</tr>
<tr>
<td>DMR1 5L29 Malaspina Voltage DeOsc</td>
<td>0.00650348</td>
<td>0.031</td>
</tr>
<tr>
<td>NIC1 5L81 Ingledow Voltage DeOsc</td>
<td>0.00599687</td>
<td>0.284</td>
</tr>
<tr>
<td>WSN1 5L1 Williston Voltage DeOsc</td>
<td>0.00875887</td>
<td>33.289</td>
</tr>
<tr>
<td>ALTA PMU N1 kV</td>
<td>0.00130610</td>
<td>-77.739</td>
</tr>
</tbody>
</table>
Figure A1-12: Mode shape for 0.584 Hz oscillation in local frequency (scaled)

Figure A1-13: Correlation functions for MW oscillations on June 4, 2003
A1.5 Analysis Examples in Australia

This Section provides a collection of examples that show modal analysis results for a collection of events recorded in the South East Australian power system. Analytical details are provided in other sections of this Report.

The South East Australian system extends some 5,000 km from Pt. Lincoln in South Australia around the south and east coasts of the continent through the states of Victoria and New South Wales terminating near Cairns in the far north of the state of Queensland (see Figure 1). Historically, the power systems of the Australian states evolved independently before being successively interconnected over a period of several decades. The majority of the system loads are in the vicinity of the capital cities while generation is located on the major coal fields or close to sources of natural gas. The interconnections between the states are relatively weak compared to generation capacities of the respective regions. Without stabilizers, the system is inherently unstable. Power system stabilizers (PSSs) installed on generators and power oscillation dampers (PODs) fitted to SVCs are essential to the secure and stable operation of this system. The island state of Tasmania was asynchronously connected to Victoria by a 600 MW, 290 km submarine HVDC link (Basslink) in 2005.

The three dominant inter-area oscillatory modes on the mainland system and their indicative mode shapes are shown in Figure A1-14; these modes are relatively close in frequency. Although these modes are of primary interest there are also significant intra-regional modes, such as that between generators in the southern and central area of Queensland. Two modal estimation systems are in operation, the Oscillatory System Monitor (OSM) developed by TransGrid and the University of Queensland (see Section 2.8.5), and a commercial system. The voltage-angle signals employed by OSM are derived from simultaneous, GPS-adjusted measurements from the four capital cities. The voltage-angle differences are then processed to eliminate modes close in frequency to the mode of interest. The ISO, AEMO, uses the OSM system mainly for longer-term estimation, especially to validate and calibrate small signal models of the system which are used for power system planning and operational purposes. The commercial system is used for shorter-term estimation for the purposes of security assessment and alarming, etc. Both systems are used to verify the performance of damping controllers, especially PODs.
Figure A1-14: Indicative frequencies and shapes of the three inter-area modes of oscillation of the South East Australian power system.
A1.6 Appendix 1 A References


Overview of the DSI Toolbox

Author: John Hauer

The Dynamic System Identification (DSI) Toolbox is the latest Matlab version of BPA systems analysis tools that trace their origins to wide area control projects in the mid 1970's. Core elements of the DSI Toolbox are provided to the public as open source software.

This distribution, and considerable software development, was funded as a reliability outreach activity of the U.S. Department of Energy (DOE). Though authored by BPA and PNNL, the DSI Toolbox is regarded as DOE property.

As illustrated in Figure B1-1, the DSI Toolbox can accept and integrate data from all major monitor types now operating within the western power system, and from all important transient stability programs used there. Many of these data sources are widely used in the eastern interconnection as well.

Figure B1-1: Integration of multi-source data with the DSI Toolbox.

The DSI Toolbox is an "interactive batch" tool that is optimized for dynamic systems analysis in a planning or control engineering environment. Nearly all of its functionalities can be adapted to real-time operation, and this has become a routine matter.

1 Available at BPA WAMS website (ftp://ftp.bpa.gov/pub/WAMS_Information/), the NASPI website (https://www.naspi.org/), from the developing institutions, or from WAMS workshops.
The DSI Toolbox consists of PSMtools, the Ringdown GUI, and various add-on or special toolsets that are usually reserved to special users. In its present form, PSMtools provides the overall processing framework with processing functionalities indicated in Figure B1-2.

Placing the response data in the PSM Tools standard format involves a number of tasks that are not indicated in the figure. Chief among these are the following:

- Initiation of a processing log
- Signal extraction and joining
- Repair of record defects (e.g., lost PMU messages)
- Calculation and naming of derived signals (e.g., rms signals from phasor data)

Add-on toolsets are often temporary. Examples include various ModeMeters, general purpose identification tools such as N4SID\(^2\), and event detection logic.

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\(^2\) Numerical algorithms for Subspace State Space System IDentification
CHAPTER 2
Mode-Meter Analysis Methods

Editors: Daniel Trudnowski and John Pierre

Authors: Claudio Canizares, Luke Dosiek, Hassan Ghasemi, Mike Gibbard, Emil Johansson, Gerard Ledwich, Russell Martin, Enrique Martinez, Luigi Vanfretti, David Vowles, Richard Wies, Ning Zhou

2.1 Introduction

Time-synchronized measurements provide rich information for estimating a power- system’s electromechanical modal properties via advanced signal processing. This information is becoming critical for the improved operational reliability of interconnected grids. A given mode’s properties are described by its frequency, damping, and shape. Modal frequencies and damping are useful indicators of power-system stress, usually declining with increased load or reduced grid capacity. Mode shape provides critical information for operational control actions. Over the past two decades, many signal-processing techniques have been developed and tested to conduct modal analysis using only time-synchronized actual-system measurements. Some techniques are appropriate for transient signals while others are for ambient signal conditions.

We classify the oscillatory dynamic response of a power system as one of two types: transient (sometimes termed a ringdown); and ambient. The basic assumption for the ambient case is that the system is excited by low-amplitude random variations typically assumed to be load variations. This results in a system response that is colored by the system dynamics. A transient response is typically larger in amplitude and is caused by a sudden switching or fault. The resulting time-domain response is a multi-modal oscillation superimposed on the underlying ambient response.

In terms of application, we classify modal frequency and damping estimation algorithms into two categories: 1) ringdown analyzers; and 2) mode meters. A ringdown analysis tool operates specifically on the ringdown portion of the response; typically the first several cycles of the oscillation (5 to 20 seconds). This is the subject of Chapter 1. Alternatively, a mode meter is applied to any portion of the response: ambient; transient; or combined ambient/transient. Ultimately, a mode meter is an automated tool that estimates modal properties continuously, and without reference to any exogenous system input. This is the subject of this chapter.

Near real-time operational knowledge of a power system’s modal properties may provide critical information for control decisions and thus enable reliable grid operation at higher loading levels. For example, modal shape may someday be used to optimally determine generator and/or load-tripping schemes to improve the damping of a dangerously low-damped mode. The optimization involves minimizing load shedding and maximizing improved damping. The two enabling technologies for such real-time applications are a reliable real-time synchronized measurement system and accurate modal analysis signal-processing algorithms. Modal identification also has applications which do not require real-time operation. These applications include using measurement-based techniques, as described in this chapter, to validate mode estimates from large-scale system models. Also, modal identification methods can be used to perform baseline studies on archived measurements.
Many different (modal identification) analysis methods may be employed. These can be categorized in a number of different ways. The methods can be parametric or non-parametric. Parametric methods assume an underlying model while non-parametric methods make few assumptions. The algorithms can also be categorized as time-domain or frequency-domain methods depending on whether they use time-domain or frequency-domain data. The algorithms can also be categorized as block or recursive processing. Block processing algorithms work on an entire record length of data to generate modal estimates. Recursive algorithms provide updates of the modal estimates based on the most recent data and the previous estimates of the modal properties.

Modal identification algorithms have their origin in signal processing and system identification literature. The concept of identifying oscillatory modes is by no means new but has extensive applications in other engineering disciplines such as structures. What is new over the past twenty years is the growing application and customization of modal identification to power systems. This chapter explores these topics.

Pierre and others [5] published the first result on applying parametric mode estimation to ambient data. Since then, many papers have been published and commercial software is now being offered. The goal of the following chapter is to introduce the reader to the subject and summarize the most successful signal-processing approaches. This includes several application examples.

### 2.2 System Model, Conditions, and Assumptions

Analyzing and estimating power-system electromechanical dynamic effects are a challenging problem because the system:

- is nonlinear, high order, and time varying;
- contains many electromechanical modes of oscillation close in frequency; and
- is primarily stochastic in nature.

Design of signal-processing algorithms requires that one address each of these issues. Fortunately, the system behaves in a relatively linear fashion when at a steady-state operating point [1].

As has been established in one of the many excellent books that address the properties and nature of electromechanical dynamics in power systems (e.g., see [2], [3]), electromechanical modes are typically classified as either local or inter-area in nature. Local modes occur when a single generator or plant swings against the system while an inter-area mode occurs when several generators in an area swing against generators in another area. Because local modes are characterized by larger inertias and lower impedance paths, their frequencies tend to be higher. In general, local modes tend to be in the 1 Hz to 2 Hz range while inter-area modes tend to be in the 0.2 Hz to 1.0 Hz range. Typically, the inter-area modes are more troublesome.

Consistent with power-system dynamic theory, we assume that a power system can be linearized about an operating point [2], [3]. The underlying assumption is that small motions of the power system can be described by a set of ordinary differential equations of the form

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B隈(t) + B隈(t) \\
y(t) &= Cx(t) + D隈(t) + D隈(t) + \mu(t)
\end{align*}
\]
where \( q \) is a hypothetical random vector perturbing the system, vector \( x \) contains all system states including generator angles and speeds, and \( t \) is time. Control actions that can be described as smooth functions of the state \( x \) are embedded in the system \( A \) matrix, and all other actions are represented by the exogenous input vector \( u_{E} \). These include set-point changes, low-level probing signals (e.g., a low-level probing signal into a DC converter), and load pulses that are applied to examine system dynamics. Measurable signals are represented by \( y \) which contains measurement noise \( \mu \) that includes effects from instruments, communication channels, recording systems, and similar devices. In general, measurement noise has a relatively small amplitude when quality instrumentation is employed. Changes which are breaker actuated may produce system topology changes that alter the system \( A \) matrix to various degrees.

The assumption for \( q \) is that it is a vector of small-amplitude random perturbations typically conceptualized as noise-produced load switching. It has been hypothesized that the load switching is primarily random noise with frequency content in the range of the electromechanical modes with each element of \( q \) independent [4]. This assumption is certainly open to more research.

An expanded perspective of the system is shown in Figure 2-1 where \( y_i \) is the \( i \)th element of \( y \). MIMO system \( G \) is assumed to be linear. Network topology changes are represented by switches in dynamic gain matrices \( K \) and \( K' \), which may or may not be deliberate.

We classify the response of the system in Figure 2-1 as one of two types: transient (sometimes termed a ringdown); and ambient. The basic assumption for the ambient case is that the system is excited by low-amplitude variations at \( q \) and \( u_{E} \) and that the variations are typically random or pseudo-random in nature. This results in a response at \( y \) that is colored by the dynamics \( G \). A transient response is typically larger in amplitude and is caused by a sudden switch at \( s \) or \( s' \), or a sudden step or pulse input at \( u_{E} \). The resulting time-domain response is a multi-modal oscillation superimposed with the underlying ambient response.

![Figure 2-1: A structure for information sources in process identification.](image)

The different types of responses are shown in Figure 2-2, which shows a widely published plot of the real-power flowing on a major transmission line during a breakup of the western North American power system (wNAPS) in 1996. Prior to the transient at the 400-second point, the system is in an ambient condition. After the ringdown at the 400-second point, the
system returns to an ambient condition. The next event in the system initiates an unstable oscillation.

In developing and applying measurement-based modal analysis algorithms, it is imperative that one consider the stochastic nature of the problem. Power systems are continually excited by random inputs with a high level of independence. This is modeled by \( q(t) \) in our formulation. Because of this stochastic nature, no algorithm can exactly estimate the modal properties of the system from finite-time measurements. **There will always be an error associated with the estimate.** When evaluating estimation algorithms, one must address these error properties. This includes the bias error as well as the variance of the estimate.

![Figure 2-2: Real power flowing on a major transmission line during the western North American power system breakup of 1996.](image)

### 2.3 System Probing

It is imperative to understand that because of the stochastic nature of the system, the accuracy of any mode estimation is limited. It is possible to significantly improve the estimation by exciting the system with a known probing signal. In Figure 2-1, it is assumed that the probing enters at \( u_E(t) \). A signal may be injected into the power system using a number of different actuators such as resistive brakes, generator excitation, or modulation of DC intertie signals. For example, operators of the western North American power system (wNAPS) use both the 1400-MW Chief Joseph dynamic brake and modulation of the Pacific DC intertie (PDCI) to inject known probing signals into the system. The wNAPS is shown in Figure 3 with the PDCI being the DC line flowing from Oregon to southern California. The PDCI has been modulated with a number of different signals including short duration mid-level probing resulting in transient responses and long-duration low-level probing that result in measured signals only slightly above the system ambient noise floor. Low-level probing should be carried out at a level low enough to not be a significant disturbance.

The wNAPS has a long history in the use of probing signals for electromechanical mode identification [40, 41]. During the 1980s and 1990s, the Chief Joseph brake was frequently used to benchmark system characteristics. In the late 1990s, with synchronized wide-area measurements becoming readily available, modulating the PDCI became more common.
Starting in 1999, mid-level probing signals were used to characterize the mode damping. In 2000, low-level pseudo-random noise was injected into the system. The application of system identification methods to the input and output data from that test showed great promise for mode estimation [37].

In 2005, 2006, 2008, 2009, and 2011 a number of extensive tests were carried out using low-level multi-sine probing signals modulated at the PDCI. The multi-sine input was optimized to maximize the amount of energy placed into the modes and minimize the peak-to-peak movement of the system outputs [25, 60, and 62]. The synchronized measurements of the system response to those tests proved to be rich in information about the system’s dynamic characteristics.

With known input signals, not only can the electromechanical modes be identified with improved performance, but complete input/output system models, such as transfer functions and state-space models, can be estimated from the input location to the measured output locations. Many different system identification methods can be used as shown in [9]. There is a tremendous amount of literature on system identification given measured or known inputs and measured outputs. Some of these algorithms work on the time-domain data while other algorithms utilize the frequency-domain data. The literature is too extensive to review here; the reader is referred to one of many textbooks (e.g., see [14]). Classical nonparametric methods such as Empirical Transfer Function Estimation (ETFE) and spectral methods [12, 14] may be used to estimate the system magnitude and phase response. The advantage of the non-parametric methods is that they make very few assumptions about the underlying system model. Thus, they play an important role in validating parametric system models where one looks for consistency from the frequency response identified from a parametric method and the non-parametric methods. The parametric methods provide much more information about the system such as a state-space model or a transfer-function equation. It is important that the parametric algorithm chosen matches well with the underlying condition. For example, if an algorithm designed to analyze a transient response (i.e. a sum of damped sinusoids) is applied to ambient data, which is not the sum of damped sinusoids, then poor results are expected.
2.4 Mode Estimation Approaches and Algorithms

This section describes many of the modal identification techniques for estimating the mode frequency and damping, while Section 2.5 discusses the algorithms for mode shape estimation. Modal estimation algorithms draw extensively from the wealth of literature from the signal processing and system identification communities. Signal processing books (e.g., [12, 53, 54, 55]) on spectral estimation describe many ambient methods. Also system identification books such as [14, 56, 57, 58] provide additional useful information for modal analysis. This section only scratches the surface of the wealth of knowledge in the signal processing and system ID literature. The algorithms described in this section represent the algorithms, which to-date, have been the most applied to the electromechanical mode estimation problem. Not all possible algorithms are described in detail. The first two subsections do not actually describe specific algorithms but describe general classification of algorithms. Section 2.4.1 describes the classification of algorithms as either parametric or non-parametric. Section 2.4.2 discusses block processing versus recursive algorithms. Algorithms can also be categorized as iterative or closed form. Closed form algorithms directly lead to an estimate while iterative algorithms refine the mode estimate through multiple refinements (iterations). Section 2.4.3 gives an example of a block processing, parametric method, namely the Yule-Walker (YW) method and its variations. Section 2.4.4 and 2.4.5 describe recursive parametric algorithms. Section 2.4.6 gives an example of a subspace method [14, 56] by using Numerical algorithm for Subspace State Space System.
2.4.1 Parametric vs. Nonparametric

Modal identification methods can be broken into two categories – parametric and nonparametric. Algorithms in both of these general categories have important applications in modal identification. Parametric methods assume an underlying model where the parameters of the model are estimated from the data. After a model is chosen, many different algorithms can be used to estimate the model parameters. A simple example is to assume an autoregressive (AR) model for the data and then to apply an algorithm to estimate the AR coefficient where the roots of the AR polynomial correspond to the modes. Parametric methods usually result in great amounts of data reduction, i.e. one starts with a long record length of measurement samples and ends with a handful of estimated parameters. On the other hand, non-parametric methods do not assume an underlying system model. Instead they make very few assumptions about data. For example, estimating the frequency content of ambient data using a Welch Periodogram is a non-parametric method as is estimating the autocorrelation or cross-correlation functions. Often a non-parametric method such as a Welch periodogram is a good way to first examine measured data. It helps to discover if there is signal content in the frequency band of interest, i.e. the frequency range of the interarea modes. Non-parametric methods can serve as a good cross check of parametric methods. For example, using the two examples given above, how similar is the AR spectrum to the non-parametric Welch spectrum? If the AR spectrum is vastly different than the Welch spectrum then that is a red flag that the parametric method needs to be refined such as refining the model order or changing to a different model, e.g. autoregressive moving-average (ARMA). In general, it is always good practice to compare the estimated parametric spectrum to the non-parametric spectrum. Most of this section focuses on parametric methods. Later in Section 2.5, a non-parametric method is presented for mode shape estimation.

2.4.2 Block Processing vs. Recursive

Beside parametric and non-parametric algorithms, modal identification algorithms can also be divided into two other categories – block and recursive processing. Block processing algorithms operate on an entire segment of data to provide an estimate of the modes. With these algorithms, a certain block size is frequently specified. For an example, a block processing algorithm might use an entire 15-minute segment. If another estimate is desired 1 minute later, a block processing algorithm would use that new minute along with 14 minutes of the previous segment. Conversely, recursive algorithms are capable of updating the mode estimate with each new sample of data. They achieve this by retaining information of the previous estimate and using the new data in an algorithm to update the mode estimate. Usually with recursive algorithms, a fixed block size is not given. Instead, a time constant is defined as to how quickly the recursive algorithm forgets older data. Consequently, the most recent sample is given the greatest weighting with exponential forgetting of older data. Thus, care needs to be taken when comparing the results of recursive and block processing algorithms. Clearly recursive algorithms should have computational speed advantages over block processing if frequent updates of mode estimates are required in real-time applications. Because the sampling rates for synchrophasor measurements are relatively low compared to processor speeds, both block and recursive algorithms are viable in most real-time applications. The methods described in the following subsections are just a few of the many spectral estimation and system identification methods that can be applied to this problem.
2.4.3 Yule-Walker (YW) and Least-Squares (LS) Based Methods

The first published result on applying parametric mode estimation to ambient data [5] used a YW-based method. Since then, several variations of YW-based methods have been successfully applied. This includes an extensive comparison in [9 and 39] to other methods.

The fundamental idea behind correlation methods is that, for a good model, the prediction error at time $k$ should be independent of past data up to time $k - 1$, i.e. if the errors are correlated with past data, there will be more information available in past data about $y[k]$ than picked by $f[k]$. Normally, similar to the prediction error methods (PEM) [14], correlation methods require an iterative search technique to solve a nonlinear equation. However, Instrumental Variable (IV) methods, as an application of the correlation methods, result in a linear regression. The YW method is a special case of IV methods [11], and is essentially based on a set of linear equations that can be solved by means of fast and efficient algorithms such as Levinson’s.

Several approaches can be utilized to estimate the parameters of the coefficients of the polynomial AR($\mathcal{P}$) in an AR or ARMA model:

$$A(q) = \sum_{i=0}^{p} a_i q^{-i} ; \quad a_0 = 1$$

(2)

They all result in a set of linear equations that relates the parameters with the help of the auto correlation function (ACF). The basic YW equation is explained next, together with some variations that overcome the YW method shortcomings.

2.4.3.1 Yule-Walker (YW)

A strictly AR process can be rewritten as:

$$y[k] = -\sum_{i=1}^{p} a_i y[k-i] + b_0 e[k]$$

(3)

where $k$ is the integer time index. Multiplying both sides of (3.3.1) by $y[k-j]$ and taking the expectation yields:

$$r_{yy}[j] = \begin{cases} 
-\sum_{i=1}^{p} a_i r_{yy}[j-i] & \text{for } j \geq 1 \\
-\sum_{i=1}^{p} a_i r_{yy}[-i] + b_0 \sigma^2 & \text{for } j = 0
\end{cases}$$

(4)

This equation is known as the Yule-Walker (YW) equation, and defines a relationship between the ACF and the parameters of an AR($\mathcal{P}$) process. By solving the following set of linear equations, one can determine the parameters of an AR($\mathcal{P}$):
The autocorrelation matrix $R_{yy}$ in (5) is Hermitian ($R_{yy}^H = R_{yy}$), and Toeplitz since the diagonal elements (top-left to lower-right) are the same; in this case, the model yields stable poles [12]. Equation (5) can be solved by means of an efficient algorithm such as Levinson’s, and it depends on the ACF. A reasonable approximation can be obtained by calculating the elements of $R_{yy}$ from limited available data.

2.4.3.2 Modified Yule-Walker (MYW)

A similar procedure previously used to derive the YW equation can be applied to an ARMA($p,d$) model, leading to the following set of linear equations [12]:

$$
\begin{bmatrix}
\tau_{yy}[d] & \tau_{yy}[d-1] & \cdots & \tau_{yy}[d-p+1] \\
\tau_{yy}[d+1] & \tau_{yy}[d] & \cdots & \tau_{yy}[d-p+2] \\
\vdots & \vdots & \ddots & \vdots \\
\tau_{yy}[d+p-1] & \tau_{yy}[d+p-2] & \cdots & \tau_{yy}[d]
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_p
\end{bmatrix}
= 
\begin{bmatrix}
\tau_{yy}[d+1] \\
\tau_{yy}[d+2] \\
\vdots \\
\tau_{yy}[d+p]
\end{bmatrix}
$$

(6)

where $p$ is the order of the AR component and $d$ is the order of the MA component. The matrix $R_{yy}$ is Toeplitz but not Hermitian; hence, the stability of the poles of AR($p$) is not guaranteed [12].

2.4.3.3 Least Square Modified Yule-Walker (LSMYW)

Equation (6) could also be rewritten as:

$$
\tau_{yy}[k] = -\sum_{i=1}^{p} a_i \tau_{yy}[k-i] \quad \text{for} \quad k \geq q + 1
$$

(7)

Since there is information in the ACF with higher order lags, it is also possible to calculate the $a_i$’s by incorporating these ACFs. Thus [12]:

$$
\begin{bmatrix}
\tau_{yy}[d] & \tau_{yy}[d-1] & \cdots & \tau_{yy}[d-p+1] \\
\tau_{yy}[d+1] & \tau_{yy}[d] & \cdots & \tau_{yy}[d-p+2] \\
\vdots & \vdots & \ddots & \vdots \\
\tau_{yy}[M-1] & \tau_{yy}[M-2] & \cdots & \tau_{yy}[M-p]
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_p
\end{bmatrix}
= 
\begin{bmatrix}
\tau_{yy}[d+1] \\
\tau_{yy}[d+2] \\
\vdots \\
\tau_{yy}[M]
\end{bmatrix}
$$

(8)

In this case, $M - d \geq p$, so that $R_{yy}'' \in \mathbb{R}^{(M-d)\times p}$ is a nonsquare matrix, as opposed to $R' \in \mathbb{R}^{p\times p}$; hence, the least square method has to be employed to calculate the $a_i$’s.
In [12], it is mentioned that for large data records, the estimates obtained by solving LSMYW equations are asymptotically unbiased, and that its variance decreases as the number of equations \((M - d)\) increases, as long as \(N \gg M - d\), where \(N\) is the data record length. In general, LSMYW offers better performance compared to MYW if the number of equations is not too large. As reported in [12, 53], LSMYW yields the most accurate estimates when the discrete-time poles are near the unit circle. This is usually the case in a power system with poorly damped oscillations, which typically have damping ratios of less than 10\%. LSMYW was first successfully applied to power system data in [6] and extended to multichannel in [7].

2.4.3.4 Comparison

The previously explained YW method and its variations are compared here with the PEM for the estimation of the electromechanical modes in the two-area benchmark power system depicted in Figure 2-4 [3], using an AR(10) and an ARMA(10,5). The generators are modeled using subtransient models and simple exciters equipped with PSSs. The corresponding static and dynamic data are given in [42]; the total base loading level is 2734 MW and 200 MVar, and loads are modeled as constant PQ loads.

A two-minute recorded power signal \(P_{G2}\) is used as the system output \(y[k]\); the input is simulated as random load switching. The results depicted in Figure 2-5 correspond to 20 simulations with independent realizations of the noise. An AR(10) is selected for the YW method, and an ARMA(10,5) is assumed for both MYW and LSMYW methods, resulting in the number of linear equations in (6) and (8) being 10 and 15, respectively. Observe that the YW method does the poorest job in estimating the electromechanical mode, showing a high variance but providing stable poles, whereas some of poles estimated with the MYW and the LSMYW methods are unstable. The LSMYW performance is better than both the YW and the MYW methods, and comparable to the prediction error method (PEM) results in terms of accuracy and variance, with the PEM being the most accurate; however, the large CPU times and numerical issues associated with the PEM optimization routine are its main drawbacks.
2.4.4 Regularized Robust Recursive Least Squares (R3LS) Method

Zhou et al., [13] introduces a regularized robust recursive least squares (R3LS) method to estimate power-system modes based on synchronized phasor measurement unit (PMU) data. To continuously monitor power system modes, a method should process different categories of measurement data properly and automatically. The R3LS method uses an autoregressive moving average exogenous (ARMAX) model to account for typical measurement data, which includes low-level pseudo-random probing, ambient, and ringdown data. A robust objective function is used to reduce the negative influence from outliers. A dynamic regularization method is used to help include a priori knowledge about the system and reduce the influence of under-determined problems. A recursive computation method is used to improve computational efficiency. The method is designed to work on-line for monitoring power system modes in real-time.

The R3LS method defines the mode estimation problem by setting up an objective function using an ARMAX model. Let $y(k)$ represent the system response at time step $k$. Signal $u(k)$ represents the injection into the system, $e(k)$ is the process noise. The ARMAX [14] model that is used by R3LS method to describe dynamic behavior of a power system is defined as following:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + \cdots + a_{na} y(k-na)$$
$$= b_1 u(k-1) + b_2 u(k-2) + \cdots + b_{nb} u(k-nb) + e(k) + c_1 e(k-1) + c_2 e(k-2) + \cdots + c_{nc} e(k-nc)$$

for $k = n+1, n+2, \cdots, t$, where $n = \max\{na, nb, nc\}$. Here, $t$ is the current time index. The vector

$$\theta = [a_1, a_2, \cdots, a_{na}, b_1, b_2, \cdots, b_{nb}, c_1, c_2, \cdots, c_{nc}]^T$$

is the parameter vector to be determined, while $na$ is the order of autoregressive (AR) model, $nb$ is the order of X(input) model, and $nc$ is the order of moving average (MA) model. Based on this ARMAX model, system discrete modes ($z_k$) can be identified as the roots of the characteristic polynomial $1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na}$. 

Figure 2-5: The performance of YW, MYW, LSMY, and PEM in one electromechanical mode estimation from ambient data.
Figure 2-6 illustrates the procedure of estimating parameters in an ARMAX model using PMU data. The parameters are determined to minimize the prediction errors of the model. Adding the regularization term, the objective function of the R3LS method is defined as

$$J(\theta(t))=\frac{1}{2} (\theta(t)-\bar{\theta})^T \Pi(t) (\theta(t)-\bar{\theta}) + \sum_{k=n+1}^{T} \lambda^{-1} \epsilon(k|\theta(t))$$

(11)

The $\theta$ can be estimated as [15]

$$\hat{\theta}(t) = \arg \min_{\theta} \{ J(\theta(t)) \} \quad \text{for } t \geq n+1$$

(12)

The first term, $\frac{1}{2} (\theta(t)-\bar{\theta})^T \Pi(t) (\theta(t)-\bar{\theta})$, in the objective function of (11) is a regularization term that is used to include prior knowledge of the model. In Equation (11), $\bar{\theta}$ is a constant vector that represents the prior knowledge of parameter vector $\theta$. The prior knowledge may be gained from either model eigenvalue analysis or expert’s experience. In [16], a method is proposed to include prior knowledge of a mode into the regularization term. The matrix $\Pi(t)$ is a weighting matrix, which is a positive definite matrix [15]. It describes the confidence level in this initial guess at time $t$. A large $\Pi(t)$ indicates high confidence while a small $\Pi(t)$ indicates lower confidence. In addition to including prior knowledge, another major benefit of including this regularized term is that it resolves the under-determined problem associated with a normal LS estimation.

The second term, $\sum_{k=n+1}^{T} \lambda^{-1} \epsilon(k|\theta(t))$, in Equation (11) is a prediction error term, which is used to derive modeling parameters from data. The $\epsilon(k|\theta(t))$ is prediction error at time $k$, which is defined as

$$\epsilon(k|\theta(t)) = y(k) - \hat{y}(k|\theta(t))$$

(13)
Here \( \hat{y}(k | \theta(t)) \) is the prediction of \( y(k) \). It is defined as

\[
\hat{y}(k | \theta(t)) = -a_1 y(k-1) - a_2 y(k-2) - \cdots - a_{na} y(k-na) + b_1 u(k-1) + b_2 u(k-2) + \cdots + b_{nb} u(k-nb) + c_1 e(k-1) + \cdots + c_m e(k-nc | \theta(k-nc))
\]  

(14)

Note that process noise \( e(k | \hat{\theta}(k)) \) cannot be measured directly but can be estimated from prediction errors. The pseudo process noises at time \( k \) can be estimated through

\[
e(k | \hat{\theta}(k)) = y(k) - \hat{y}(k | \hat{\theta}(k))
\]  

(15)

The \( \rho(\varepsilon) \) is a loss function. For the conventional LS criterion, \( \rho(\varepsilon) \) is defined as \( \rho(\varepsilon) = \varepsilon^2 / 2 \). Using this quadratic function, there is a closed-form solution and it yields an optimal solution when the process noises are Gaussian white noises. Yet, the quadratic function is very sensitive to large prediction errors. Thus, even when there is an only very small fraction of large prediction errors caused by outliers, the mode estimates can be changed significantly and become unreliable. To overcome this difficulty, a robust loss function (i.e. the ‘hard’ function as in [17, 18]) is used in R3LS method to reduce the negative influence of outliers. A “Hard” function is defined as

\[
\rho(\varepsilon) = \begin{cases} 
\frac{1}{2} \varepsilon^2 & \text{if } |\varepsilon| \leq \Delta \sigma \\
\frac{1}{2} (\Delta \sigma)^2 & \text{if } |\varepsilon| > \Delta \sigma 
\end{cases}
\]  

(16)

Here, \( \sigma \) is the standard deviation of the \( \varepsilon \). The value \( \Delta \) is a user-chosen positive constant to balance the efficiency and robustness of the algorithm. Thus, \( \frac{1}{2} (\Delta \sigma)^2 \) is a constant. It can be observed that a smaller \( \Delta \) means more robustness and less efficiency. When a very large prediction error occurs, the error’s influences on the loss function is confined to \((\Delta \sigma)^2 / 2\). Thus, the negative influence from some outliers can be reduced. Note that the influences of some normal measurement data with prediction errors larger than \( \Delta \sigma \) are also reduced. Thus, for a data set with no outliers, the efficiency of the algorithm may be reduced with a small \( \Delta \). A common practice is to set \( \Delta = 3 \). Note that the ‘hard’ loss function behaves exactly the same as a conventional LS algorithm when the amplitude of the prediction error \( \varepsilon \) is smaller than \( \Delta \sigma \). The robust processing is only initiated when \( \varepsilon \) exceeds \( \Delta \sigma \).

The constant \( \lambda \) is the forgetting factor, which is positive and slightly smaller than 1. It determines how fast the past data are forgotten. To accommodate the time varying properties of a power system, it assigns smaller weights for the older data because the older data are considered to be less representative of the current power system than the more recent data. A smaller \( \lambda \) gives the algorithm a faster way to track system changes. Yet, with less data to average, variances of parameter estimation increase.

With the robust loss function defined in Equation (16), there is no general closed-form solution for the problem defined by Equations (11) and (12). A recursive numerical solution derived by [18] using the Newton-Raphson-type method is used by R3LS method. The recursive implementation is documented in detail in [16]. Compared to a normal non-recursive solution, the recursive solution efficiently utilizes the information summarized from the previous calculation step. Applying the matrix inversion lemma [14], the recursive algorithm also eliminates the need to explicitly invert a matrix and reduces the computation time needed. These result in a fixed calculation time for each step of the R3LS method, which ideally can be completed within one sampling interval.
The R3LS method was tested with simulation data using the Monte-Carlo method to reveal its preferred statistical properties. Also, the R3LS method was applied to the field measurement data before a major blackout in western North America to show the potentials of the proposed algorithm in detecting an approaching small signal stability problem [16].

2.4.5 Least Mean Squares (LMS) Method

The gradient-based least mean squares (LMS) adaptive filtering algorithm is well suited for mode estimation in power systems because of its computational simplicity and reliance on the ambient noise that characterizes the data [19, 20]. The random swings associated with power system operation are the result of ambient noise inherent in the data. This ambient noise is assumed to be relatively statistically stationary for a block of data for the frequencies of interest. The LMS filter is used in a linear predictor to make a whitening filter as shown in Figure 2-7. The dominant modes of the whitening filter correspond to the dominant modes of the time-varying power system.

![Figure 2-7: Sampled time-varying power system data with additive noise analyzed by an adaptive whitening filter.](image)

The structure of the LMS filtering algorithm consists of a filtering stage and an adaptive stage as shown in the flow diagram in Figure 2-8.

![Figure 2-8: Flow diagram of LMS filtering algorithm [19].](image)

In the filtering stage, the LMS filter computes the output for the given input (power system data, in this case) and generates an estimation error by comparing the produced output with the desired response. The adaptive process involves automatic adjustment of filter parameters (tap weight vectors) in order to reduce the estimation error until the filter operation becomes optimized in the mean square error sense [21]. The FIR filter is in the main loop and the adaptive weight control mechanism is formed by the feedback loop. In [5] the weights of the FIR (Finite Impulse Response) filter are found by solving the Wiener-Hopf equations for linear prediction, which is equivalent to the YW AR approach where the least squares problem is not over-determined. The LMS algorithm is used to adapt the weights for tracking time variations in the power system.
The LMS flow diagram shown in Figure 8 is defined by the following three relationships [21]:

\[ y(n) = w^H(n)u(n) \]  
(17a)

\[ e(n) = u(n) - y(n) \]  
(17b)

\[ w(n + 1) = w(n) + \mu u(n - 1)e^*(n) \]  
(18)

where \( u(n) \) is the actual power system data in the form of tap input vectors, \( u(n-1) = [u(n-1) \ u(n-2) \ \ldots \ u(n-M)]^T \) is the data vector with \( M \) past values, \( w(n) = [w_1(n) \ w_2(n) \ \ldots \ w_M(n)]^T \) is a time-varying vector of filter tap weights, \( y(n) \) is the filter output, \( e(n) \) is the error output, \( \mu \) is the step size parameter, and \( M \) is the filter order. As the number of iterations reaches infinity, the value of the tap weight vector of the LMS filter may come close to \( w_0 \) (Wiener Solution), which shows that the filter’s solution is close to the optimal solution [21]. The tap weight vectors \( w(n) \) generated from the LMS process can be used to track the modes of the input data \( u(n) \). The modes are first tracked by finding the roots of the z-domain polynomial and are then converted to the s-plane. The z-domain polynomial is defined by

\[ \mu  - \mu  - \ldots - \mu  - M \]  
(19)

The function used to convert the roots from the z domain to the s-domain is as follows:

\[ \mu = \mu T \]  
(20)

where \( T \) is the sampling period. The total number of roots will be equal to the filter tap length \( M \). The information about the frequency and damping ratio of the dominant mode can be extracted from the roots and the results can be tracked over a finite length of data or over the full data length.

Two forms of convergence exist for the LMS algorithm. These include convergence in the mean and convergence in the mean square. For wide-sense stationary data and small \( \mu \), the LMS algorithm is guaranteed to converge [21] where the step-size \( \mu \) is set according to the interval

\[ < \mu < \]  
(21)

The total mean square value (MSV) equals \( \mu \) (number of filter weights) times the mean square value of the input data

\[ = \]  
(22)

\[ \approx - \sum \]
where $y$ is the noisy input data and $w$ is the window length of the input data. For higher values of $\mu$, the error is large, but the LMS algorithm is faster to converge. For lower values of $\mu$, the error is small, but the LMS algorithm is slower to converge [21].

Selection of the step-size parameter $\mu$ and an initial weight vector close to the actual weight vector is critical in using the LMS algorithm for modal estimation in power systems [20]. Results of implementing the LMS filtering algorithm using a Monte Carlo simulation for interarea modal analysis on model power system data generated using a fourth-order all-pole filter have shown that, in general, the method estimates the mode frequency (see Figure 2-9) with reasonable statistical error in comparison to the known modal frequencies. However, the estimates of the damping ratio (see Figure 2-10) for the known modal frequencies have a much higher statistical error than for the frequency estimates.

![Figure 2-9](image1.png)

**Figure 2-9:** One trial of the LMS estimated frequency (solid lines) and the true frequencies (dashed-dot lines) for both modes.

![Figure 2-10](image2.png)

**Figure 2-10:** One trial of the LMS estimated damping ration (solid lines) and the true damping ratios (dashed-dot lines) for both modes.

Implementation of the LMS algorithm using a zero initial weight vector (cold start) on actual ambient power system data for the real power flowing on a major transmission line in the
western North American grid shows that the LMS algorithm predicts frequency, but not damping with reasonable error. Therefore, LMS methods using a more accurate initial weight vector and using an adaptive step-size LMS (ASLMS) algorithm have been explored [19]. Figures 2-11 and 2-12 show the LMS estimates of mode frequency and damping ratio, respectively, for Malin-Round Mountain #1 June 2000 ambient data using a cold start, using initial estimates from AR block processing and using initial estimates from 10 minutes into a previous LMS run.

![Figure 2-11: Mode frequency vs. time by cold start, initial estimate from 10 minutes run into previous LMS and initial weight vector from AR block processing.](image)

![Figure 2-12: Damping ratio vs. time using cold start, initial estimate from 10 minutes into previous LMS run and initial weight vector from AR block processing.](image)

Using initial weight vector estimates from the AR block processing algorithm and initial estimates from 10 minutes into the previous LMS run have both reduced the variability in the frequency and damping ratio estimates of the LMS algorithm compared to the cold start case. The mode frequency estimates of the LMS algorithm with initial estimates from AR block processing and initial estimates from the previous LMS run are similar in terms of accuracy. However, the damping ratio estimates of the LMS algorithm with initial estimates from AR block processing are closer to the AR estimate than the damping ratio estimates of the LMS algorithm with initial estimates from the previous LMS run. This is because the AR estimate is used as the initial weight vector to start the LMS algorithm and also as the true mode estimate in this case.
Although the LMS technique offers computational simplicity and reasonable accuracy in modal frequency estimation for power system data, it suffers from limitations in accurately estimating damping associated with known modes.

2.4.6 Numerical Algorithm for Subspace State Space System Identification (N4SID)

Subspace methods have become popular because of their numerical simplicity, robustness of the techniques that are used in the algorithms, and their state-space form which is convenient for control purposes [22]. N4SID methods have been successfully applied to the power-system problem, including results in [8, 9, 37, 61]. However, subspace methods are suboptimal, as they are not based on the minimization of a criterion. Some of the main advantages of these methods over previously mentioned identification methods are:

• **Model order selection:** For PEM, different criteria have been proposed in order to select the best order of the model, such as Akaike Information Criterion (AIC) and Final Prediction Error (FPE) [14]. These techniques are defined based on prediction error variance. For instance, to choose the orders \( p \) and \( d \) of an ARMA(\( p,d \)) model, first a range is selected for \( p \) and \( d \), then, for each \( (p,d) \) pair, the parameters of the model are estimated; the pair obtaining the lowest value of AIC or FPE is selected as the best estimate of the model order. This approach results in high computational costs because all the possible models in the selected range have to be first estimated in order to obtain the prediction error variance. On the other hand, in subspace methods, there is the possibility to estimate the model order without having to estimate the parameters of a variety of different models. Order estimation in this case is one step of the algorithm, and only requires a singular value decomposition (SVD).

• **Handling large data:** When the amounts of data are large, order selection with PEM-based techniques is just not feasible [23]. This is the case of the application being considered here, which requires long “blocks” of data in order to truly estimate the power system mode(s).

The state-space matrices are identified in this case by using measured outputs and robust numerical techniques such as QR-factorization, least square and SVD without the need to calculate correlations, i.e. it is data driven instead of covariance driven. Therefore, these types of techniques are computationally efficient.

2.5 Mode Shape Estimation

Similar to the modal damping and frequency information, near-real-time operational knowledge of a power system’s mode-shape properties may provide critical information for control decisions. For example, modal shape may someday be used to optimally determine generator and/or load tripping schemes to improve the damping of a dangerously low-damped mode. The optimization involves minimizing load shedding and maximizing improved damping. This section describes how mode shape can be estimated from time-synchronized measurements.

Mode shape is defined to be components of the right eigenvector of a linearized system model. The first published result for estimating mode shape from time-synchronized ambient measurements is contained in [24]. Since then, several improvements have been published including the results in [27] thru [30].
2.5.1 Defining Mode Shape

Using the model in (1), the eigenvalues and eigenvectors for (1) are defined from the equations

\[ |\hat{\lambda}_i I - A| = 0, \quad A u_i = \hat{\lambda}_i u_i, \quad \psi_i A = \hat{\lambda}_i \psi_i \]

(23)

where \( \hat{\lambda}_i \) is the \( i \)th eigenvalue \((i = 1 \cdot \cdot \cdot n)\), \( u_i \) (order \( nx1 \)) is the \( i \)th right eigenvector, and \( \psi_i \) (order \( 1xn \)) is the \( i \)th left eigenvector, and \( I \) is the \( nxn \) identity matrix. As shown in [2] when considering the ambient case, each system state can be written as

\[ x(t) = \sum_{i=1}^{n} z_i(t) u_i, \]

(24)

where

\[ z_i(t) = \psi_i x(t), \]

(25)

\[ \dot{z}_i(t) = \hat{\lambda}_i z_i(t) + \psi_i B_k q(t), \]

(26)

for \( i = 1 \cdot \cdot \cdot n \).

The solution of (26) results in \( z_i(t) \), which is the \( i \)th mode’s response to \( q(t) \). Equation (24) provides information on how the modes are combined to create the system states. Examination of (24) reveals that element \( u_{ik} \) (the \( k \)th element of \( u_i \)) provides the critical information on the \( i \)th mode in the \( k \)th state. The amplitude of \( u_{ik} \) provides the information on the magnitude of mode \( z_i \) in state \( x_k \). The angle of \( u_{ik} \) provides the information on the phasing of \( z_i \) in state \( x_k \). By comparing the \( \angle u_{ik} \) for a common generator state (such as the speed), one can determine phasing of the oscillations for the \( i \)th mode. As such, \( u_{ik} \) has been termed the “mode shape” vector [2]. Knowledge of \( u_i \) provides all the required information to completely determine the pattern of generator swings for the \( i \)th mode [2].

As described above, the right eigenvector \( u_i \) completely describes the mode shape of mode \( z_i \). The question here is: how can the properties of \( u_i \) be estimated from direct power-system measurements without the dependence on the linear differential model (1)? The following subsections present two basic approaches.

2.5.2 Estimating Mode Shape from Spectral Analysis

The first published approach for estimating the mode shape from synchronized measurements is [24]. The following is a summary. Begin by defining two spectral functions.

\[ S_{kl}^{(\omega)}(\omega) = \lim_{T \to \infty} \frac{1}{T} E \{ \psi_k^* \psi_l (\omega) \} \]

(27a)

\[ S_{kl}^{(\omega)}(\omega) = \lim_{T \to \infty} \frac{1}{T} E \{ \psi_k^* \psi_l (\omega) \} \]

(27b)

where \( S_{kl}^{(\omega)}(\omega) \) is the cross-spectral density (CSD) function between general signals \( y_k(t) \) and \( y_l(t) \), \( S_{kl}^{(\omega)}(\omega) \) is the power-spectral density (PSD) of signal \( y_k(t) \), \( Y_k^{(\omega)}(\omega) \) is the discrete Fourier transform of signal \( y_k(t) \) at frequency \( \omega \), \( Y_k^*(\omega) \) is the complex conjugate of \( Y_k^{(\omega)} \), and \( E\{\cdot\} \) is the expectation operator. These definitions are found in many signal processing textbooks such as [26].

Now assume that \( \hat{\lambda}_i \) is a lightly damped mode with

\[ \hat{\lambda}_i = \alpha_i + j\omega_i \]

(28)

where \( \alpha_i \ll \omega_i \). As shown in [24], the following relationships result
Equations (29) and (30) are used to estimate the mode shape. Assume for the moment that all generator speed signals are time-synchronized sampled. Also assume that the frequency of the oscillation mode \( \omega_i \) is known. The PSD is calculated for each generator speed signal.

From (30), the PSD of each signal is scaled by \( |u_{i,k}|^2 \); therefore, the PSD is a direct measure of the observability of the mode at that generator.

The phasing of the mode among the generators is directly estimated from the angle of the CSD by (29). A reference generator with high mode observability is chosen as the reference generator \( k \). The angle of the CSD is calculated for all other generators at mode frequency \( \omega_i \). From (29), the angle of the CSD for generator \( l \) represents the phasing of the oscillation.

The squared-coherency function is defined as

\[
\gamma_{k,l}^2(\omega) = \frac{|S_{k,l}(\omega)|^2}{S_{k,k}(\omega)S_{l,l}(\omega)}
\]

(31)

It represents a measure of the correlation between two signals as a function of frequency [24]. As the two signals become uncorrelated, the coherency converges to zero. Similarly, as the signals become totally correlated, the coherency converges to unity. Basically, it is a measure of percent correlation.

The coherency function can be used to determine if a mode of oscillation is due to one mode or multiple modes at the same frequency. For example, given two signals \( y_1(t) \) and \( y_2(t) \), if both \( S_{1,1}(\omega_i) \) and \( S_{2,2}(\omega_i) \) have peaks at frequency \( \omega_i \), this indicates that the system contains one or more modes at this frequency. If \( \gamma_{1,2}^2(\omega_i) \) is near unity, then the same mode is contained in both \( y_1 \) and \( y_2 \). Alternatively, if \( \gamma_{1,2}^2(\omega_i) \) is near zero, then the system contains at least two different modes at frequency \( \omega_i \).

In [27], a method of using efficient weighted updates was applied to this mode shape estimation method to provide recursive estimates of mode shape and coherence in an online environment. It was able to successfully track mode shape and coherence through system changes in near real time.

2.5.3 Transfer Function (TF) Methods

A general relationship between mode shape and a transfer function estimated between system outputs was derived in [28]. Define the Laplace-domain transfer function between measured system outputs \( y_k(t) \) and \( y_l(t) \) as

\[
G_{kl}(s) = \frac{Y_l(s)}{Y_k(s)}
\]

(32)

It is shown that assuming measured output \( y_k(t) \) is a direct measurement of, an approximation of, or can be linearly transformed into, system state \( u(t) \), then the relative mode shape between states \( x_k(t) \) and \( x_l(t) \) at mode \( \lambda_i \) is found by evaluating the transfer function at \( \lambda_i \):
Any number of system identification techniques may be used to estimate the transfer function $G_{kl}(s)$. In [28], a parametric Autoregressive Exogenous (ARX) model was created to estimate the transfer function. The ARX model, in the $Z$-Domain is

$$Y_i(z) = \frac{B(z)}{A(z)} Y_k(z) + \frac{1}{A(z)} E(z)$$

(34)

where $E(z)$ is the $Z$-Transform of the prediction error $e(k)$ and $A(z)$ and $B(z)$ are polynomials of degrees $n$ and $m$, respectively, in $z$. The mode shape is then approximated as

$$G_{kl}(\lambda_i) \approx \frac{B(z)}{A(z)} \bigg|_{z = e^{j\lambda T}}$$

(35)

where $T$ is the sampling period.

In [29] and [30], mode shape was estimated from the frequency response of a channel matching filter. In [29] a narrow bandpass filter was applied to the measured data to isolate a single mode, resulting in the use of a channel matching filter with structure

$$Y_i(z) = B(z) Y_k(z) + E(z)$$

(36)

where $B(z) = b_0 + b_1 z^{-1}$ is only 1st order due to the use of the narrowband filter. In [30] this method was improved upon by removing the narrowband filter to accommodate multiple modes, extending the order of $B(z)$ to a general $m^{th}$ order, and including an $n^{th}$ order $A(z)$ filter, resulting in

$$A(z) Y_i(z) = B(z) Y_k(z) + E(z).$$

(37)

In both [29] and [30], the mode shapes are found by evaluating the channel matching filter frequency responses at the frequency of the mode of interest, i.e.

$$B(z) \bigg|_{z = e^{j\omega T}}$$

(38)

and

$$\frac{B(z)}{A(z)} \bigg|_{z = e^{j\omega T}}$$

(39)

respectively.

It is obvious by inspection that the channel matching filters of (36) and (37) are actually the structures to form transfer functions between the pair of system outputs. In fact, (37) is equivalent to (34). Thus, the channel matching method of [29] and [30] is a special case of the transfer function method, where a parametric estimate of the transfer function is evaluated at $s = j\omega_i$ instead of $s = \lambda_i$. 
It can also be shown that the Spectral Method of mode shape estimation detailed above in Section 2.5.2 is also a special case of the transfer function method. Referencing the derivations found in [24], one may generalize (29) and (30) to show that

\[
\frac{S_{kl}(\omega)}{S_{kk}(\omega)} \approx \frac{u_{i,l}}{u_{i,k}}. \tag{40}
\]

Using the well-known relationship between the transfer function between two signals and the spectra of those signals, (40) may be expressed as

\[
G_{ij}(\omega) = \frac{S_{kl}(\omega)}{S_{kk}(\omega)} \approx \frac{u_{i,l}}{u_{i,k}}. \tag{41}
\]

It is obvious from (41) that the Spectral Method of [24] is a special case of the transfer function method, where a nonparametric estimate of the transfer function is obtained and evaluated at \(s = j\omega_i\) instead of \(s = \lambda_i\).

The authors of [31] use the Frequency Domain Decomposition (FDD) method to estimate mode shape. FDD begins by constructing a spectral density matrix, with power spectral densities in the diagonal elements and cross spectral densities on the off diagonal:

\[
S(\omega) = \begin{bmatrix}
S_{11}(\omega) & \cdots & S_{1p}(\omega) \\
\vdots & \ddots & \vdots \\
S_{p1}(\omega) & \cdots & S_{pp}(\omega)
\end{bmatrix}. \tag{42}
\]

The Singular Value Decomposition (SVD) is performed on (42) and it is assumed that there is only a single dominant mode in the vicinity of frequency \(\omega_i\) such that rank reduction can be performed to represent \(S(\omega)\) as a single degree of freedom system:

\[
S(\omega) = \Sigma_i(\omega) \mathcal{N}_i(\omega) \mathcal{N}_i^H(\omega) = \begin{bmatrix}
\sigma_i(\omega) \mathcal{N}_{i,1}(\omega) \mathcal{N}_{i,1}^*(\omega) & \cdots & \sigma_i(\omega) \mathcal{N}_{i,1}(\omega) \mathcal{N}_{p,1}^*(\omega) \\
\vdots & \ddots & \vdots \\
\sigma_i(\omega) \mathcal{N}_{p,1}(\omega) \mathcal{N}_{i,1}^*(\omega) & \cdots & \sigma_i(\omega) \mathcal{N}_{p,1}(\omega) \mathcal{N}_{p,1}^*(\omega)
\end{bmatrix}. \tag{43}
\]

It is stated in [32] that FDD obtains mode shape estimates via the ratio of elements in the largest singular vector such that

\[
\frac{V_{i,1}(\omega)}{V_{k,1}(\omega)} \approx \frac{u_{i,l}}{u_{i,k}}. \tag{44}
\]

Using (44) along with the approximation in (43), the definition of the elements of \(S(\omega)\) in (42), and the relationship between transfer functions and spectral densities in (41), it is apparent that

\[
G_{kl}(\omega) = \frac{S_{kl}(\omega)}{S_{kk}(\omega)} \approx \frac{\sigma_i(\omega) \mathcal{N}_{i,1}(\omega) \mathcal{N}_{i,1}^*(\omega)}{\sigma_i(\omega) \mathcal{N}_{k,1}(\omega) \mathcal{N}_{k,1}^*(\omega)} = \frac{V_{i,1}(\omega)}{V_{k,1}(\omega)} \approx \frac{u_{i,l}}{u_{i,k}}. \tag{45}
\]
Thus, FDD is also another special case of the transfer function method where a nonparametric estimate is used $s = j\omega_i$ instead of $s = \lambda_i$. In fact, FDD arrives at the same relationship between spectral densities and mode shape as does The Spectral Method of [24], despite the use of a different approach to the problem.

For each of the special cases of the transfer function method discussed here that evaluate the transfer function at $s = j\omega_i$ instead of at the complex mode $s = \lambda_i$, there is danger in biasing the mode shape estimates if the true damping of mode $\lambda_i$ is high. If the damping is low enough, then the approximation that $\lambda_i \approx j\omega_i$ may be made and one may proceed with little worry about biasing the estimates. In fact, the authors of [31] recommend FDD be used only for modes with damping of less than 5%.

In [30], the coherence squared was obtained from the channel matching filters, and since this is a special case of the transfer function method, it shall be discussed here. The coherence was derived from the product of the magnitude of the frequency response of the transfer functions with both directions of signal flow taken into account, i.e. one transfer function where $y_k(t)$ is assumed as the input and $y_l(t)$ as the output, and vice versa:

$$\gamma^2_{k,l}(\omega) = |G_{kl}(\omega)||G_{lk}(\omega)|.$$  \hspace{1cm} (46)

With any of these applications of the transfer function method, a strong estimate of the mode $\lambda_i$ is obviously critical to the performance of the mode shape and coherence estimates. This estimate does not directly provide estimates of the modes so any one of the mode-estimation methods may be appropriate to use in conjunction with this method.

### 2.6 Performance Evaluation and Validation

#### 2.6.1 Needs and Approaches

While sections 2.4 and 2.5 focused on mode estimation and mode shape estimation, respectively, this section address the issue of performance evaluation and validation. Performance evaluation has to do with how well the algorithm is working in the sense of how good are the estimates of mode damping, frequency, and shape. Under ambient or probing conditions, the measured data is stochastic so estimates of modal parameters are functions of random data and therefore are random parameters themselves. Thus, an exact correct answer is not possible. The goal is that, on average, the estimates are close to the true mode parameter values, i.e. that the estimates have a small bias. Also, the goal is that, on average, the variability in the estimates is small, i.e. the estimates have a small variance. The bias and variance of the estimates depend on many factors including how well the assumed model matches with reality, the time aperture of the observed data, the observability of the modes in the particular signal or signals, the algorithm used, and if probing, the amplitude, shape, and duration of the probing signal. For simulated data, estimating the bias and variance can be accomplished through Monte Carlo trials. Monte Carlo simulations repeat a certain number of trials (experiments) under identical conditions where mode estimates are calculated for each trial and then mode statistics (such as the mean and variance of the damping ratio) can be computed. With actual power system data, Monte Carlo analysis is less realistic because it is usually not possible to repeat the experiment under identical conditions. In this case, only one data set is available for a given operating condition, so bootstrap and theoretical methods are frequently used to estimate the variance of the mode estimates. Related to estimating the bias and variance in mode estimates is the topic of estimating confidence intervals in the estimates. For example instead of the algorithm outputting a mode damping estimate of 5%, the algorithm output states “90% confident that the damping is between 4 and 6%.”
In addition to looking at performance, this section also addresses validation. Validation has to do with whether or not the results of the algorithm are reasonable. Do the estimates concur with the measured data? There are a number of approaches to address this question. A straight-forward approach is to compare a parametric model that has been estimated to a non-parametric method to check for consistency in the results. For example, if an ARMA model was estimated using RLS, then the parametric spectral estimate from that ARMA model could be compared to a non-parametric spectral estimate such as a Welch periodogram. If the estimated spectrums are not similar then the underlying assumptions, such as model order, may be invalid.

A common method of validating estimates is to use residual analysis. Residuals are the difference between the measured data and what the algorithm estimates the measured data to be. Many algorithms such as RLS or LMS produce these residuals as part of the computation process. These residuals correspond to what is leftover, i.e. what the model could not reproduce [14]. If the modeling estimation process has performed well, then all the useful information should have been pulled out of the data and the residuals should be white noise. Thus, it is common to perform whiteness testing on the residuals. If the residuals are not white, the model is not valid and there is likely a mismatch between the model and the actual system conditions. For example, if the model order chosen is too small, there is a mismatch between the model and reality so there are not enough degrees of freedom in modeling the measured data. Thus, the residuals will not be white noise. If there is a probing signal present, another common residual test is to see if the residuals and the past probing signal samples are uncorrelated. This can be tested by estimating the cross-correlation between the residuals and past probing signal samples. If these values are not small, it suggests that all the information from past inputs has not be utilized in creating the model and the model could be improved. This question of performance evaluation and validation can also depend on the end use of the estimates. For example, one would likely need fairly accurate estimates of mode damping when the mode damping is low, but when the actual damping is high, the estimate may not need to be as accurate.

The following subsections discuss various aspects of validation and performance. Subsection 2.6.2 looks at the use of Monte Carlo simulations and 2.6.3 gives an example of the variance and bias in mode estimates. Subsection 2.6.4 investigates error bounding while subsection 2.6.5 studies residual analysis. The final subsection, 2.6.6, looks at benchmarking methodology for actual power systems.

2.6.2 Monte Carlo Analysis

As discussed above, power-systems are stochastic. In developing, applying, and evaluating measurement-based modal analysis algorithms, it is imperative that one consider the stochastic nature of the system. Power systems are continually excited by random inputs with high-order independence. In equation (1), this is modeled by \( q(t) \). Because of this stochastic nature, no algorithm can exactly estimate the modal properties of the system from finite-time measurements. There will always be an error associated with the estimate. When evaluating estimation algorithms, one must address these error properties. This includes the bias error as well as the variance of the estimate.

Because of the stochastic nature of the system, there are limits to the accuracy of any estimator. To fully evaluate a problem, estimates need to be validated and the accuracy of the estimates needs to be assessed. The stochastic nature of the problem cannot be overemphasized. This stochastic nature of the measured data results in statistical variability of the estimates. The validity of the estimates needs to be tested with as many techniques as possible and the accuracy of the estimates needs to be determined.
Monte Carlo simulations are an accepted and often utilized approach for evaluating modal-analysis algorithms. With such an approach, a transient simulation is created with a small portion of the system loads represented as random and each load is independent. With each iteration, the seed of the random generator for the loads is randomly changed. Modal analysis is applied to each iteration and the statistics are calculated from the set of simulations.

2.6.3 Variance and Bias Mode Estimates

2.6.3.1 Ambient Data

An ARMA model is more likely to adequately represent the true power system than an AR model; in this case, the estimated parameters would be asymptotically unbiased if the model order of the true system matches the ARMA model order. If an AR model is used, a high order may be used to reduce the bias in the estimated modes.

Assuming that the underlying load switching in the power system throughout the day is white noise, the subspace methods, prediction error methods, Modified Yule-Walker method and Least Square Modified Yule-Walker method are asymptotically equivalent, i.e. these methods behave the same for long enough data blocks. From the computational burden point of view, iterative algorithms take longer to compute than non-iterative algorithms. Some of these points are illustrated in the example below.

2.6.3.2 Example

In this example, first, the bias problem one may encounter due to the wrong model selection (choosing an AR model instead of an ARMA model) is illustrated. Then, the effect of size of data blocks and asymptotic behavior of different models are studied. For the 2-area benchmark system in Figure 2-4, a Monte-Carlo type of simulation with 150 independent simulations is performed. For these trials, 1% of the loads are represented as Gaussian noise; 4-minutes data blocks of a generator output power are recorded in each simulation and white Gaussian noise is added to the output signals as measurement noise, so that the SNR is 20 db. The signals are then passed through a Chebyshev low-pass filter with a cut-off frequency of 2 Hz, and resampled at 10 Hz rate. The pre-processed data blocks along with a prediction error method (PEM) are employed to estimate the parameters of an ARMA($p,d$) model representing the power system transfer function.

An ARMA($p,d$) model with different $p$'s and $d$'s is employed to model the measured signal in every simulation. The mean of the estimated electromechanical mode corresponding to each model for 150 trials is depicted in Figure 2-13, together with the “true” mode, $-0.1228 \pm j4.7824$, obtained from a linearized model (LM) of the power system. Observe that when a pure AR(15) model, i.e. ARMA(15,0), is selected, the result is biased and is not as close to the LM mode as ARMA($p,d$) with $d \neq 0$; this was also noticed in [6]. From the system identification point of view, an ARMA model set is more likely to adequately represent the true system than an AR model; in this case, the estimated parameters would be asymptotically unbiased [14] if the model order of the true system matches the ARMA model order.
Figure 2-13: Mean of identified inter-area mode \(-0.1228 \pm j4.7824\) for the 2-area benchmark system using a Monte-Carlo approach with 150 independent simulations.

The effect of using different sizes of data blocks on the accuracy of the method is discussed next for the 2-area benchmark system, and the asymptotic behavior of the CVA subspace method is compared to an ARMA model and a state-space model whose parameters were estimated using the prediction error method (PEM). For this purpose, the following built-in functions of MATLAB were used: \texttt{n4sid()} for the subspace method; \texttt{armax()} for the ARMA model; and \texttt{pem()} for the state-space model. Note, \texttt{n4sid()} is non-iterative, while \texttt{pem()} and \texttt{armax()} are iterative algorithms. The data was generated for an inter-area mode \((\alpha \pm j \beta = -0.1228 \pm j4.7824\), damping 2.57\% and frequency 0.7611 Hz). The standard deviation of the estimated real part and frequency of the mode for different size of data blocks are depicted in Figures 2-14 and 2-15, respectively. Notice that for short data blocks, the ARMA and state-space models yield better performance. On the other hand, for long data blocks, as explained before, the subspace method and ARMA and state-space models behave similarly.

From the computational burden point of view, the previously mentioned MATLAB built-in functions were used to estimate the parameters of the corresponding models for the same 4-minute block data. Table 2-1 shows the CPU times obtained for each function as the system order \(n\) changes. Observe that the subspace method is faster than the others. That is because the other two methods are iterative. Non-iterative algorithms such as the YW and its modifications would also be fast compared to the iterative methods. Also recursive algorithms, such as R3LS, are fast because they are continuously updating the estimates.
2.6.4 Estimating Error Bounds

In parameter estimation, second order statistical properties of the estimates, such as confidence intervals, are important when discussing the quality of the estimates, i.e. the tighter the confidence intervals (error bounds), the better are the estimates. The mean value and the confidence interval (standard deviation of the estimates) are usually calculated to ensure that the estimates are both accurate and dependable. The confidence intervals, however, are typically obtained by means of a Monte-Carlo type of simulation, which requires repeating the simulation for several realizations of the noise, thus making them
computationally expensive and in some cases impractical. For example, in a real power system, the system operating conditions as well as its dynamic characteristics may change over a long time window due to load change, rescheduling, tap changer operation, etc. Therefore, it is important to quickly conduct the experiment to ensure that the system does not experience significant changes, and that the stationarity assumption is not violated over the measurement time.

This section starts with a background on estimating the covariance of parameters of a linear parametric model such as ARMA. This information is useful in determining the error bounds of the identified electromechanical modes. These modes, however, have a nonlinear relationship with the parameters of the model, and so do their variances; this nonlinear relationship is due to the fact that the electromechanical modes are basically the roots of the characteristic equation associated with the model. A technique that has been used in civil engineering for identification of civil structures to establish a connection between the variance of parameters and the variance of modal parameters is used here [33], [34]; the corresponding equations are explained in detail next.

A linear parametric model such as AR or ARMA used for representing a power system presents the following typical form:

\[ y(k) = H(q) e(k) = \frac{B(q)}{A(q)} e(k) \]  \hspace{1cm} (47)

where \( H(q) \) is a rational transfer function with unknown parameters \( \theta \). One-step-ahead prediction \( \hat{y}(k|\theta) = \hat{y}(k|\theta) \) uses the observations available up to time \( k-1 \) to predict \( y(k) \), i.e.

\[ \hat{y}(k|\theta) = [1 - H^{-1}(q)] y(k) \]  \hspace{1cm} (48)

This notation is adopted from [14] and is used here to emphasize the dependence on the parameter vector \( \hat{\theta} \). Within the context of prediction error methods (PEMs), for an ARMA\( (p,d) \) model, vector \( \hat{\theta} \) may be computed as follows:

\[
\hat{\theta} = \arg \min_{\theta} V_N(\theta) \\
V_N(\theta) = \frac{1}{N} \sum_{k=1}^{N} \frac{1}{2} \varepsilon^2(k, \theta) \\n\varepsilon(k, \theta) = y(k) - \hat{y}(k|\theta) 
\]  \hspace{1cm} (49)

where “arg min” stands for the minimization argument of the loss function \( V_N(\theta) \); \( N \) is the number of samples; and \( \varepsilon(k, \theta) = \varepsilon(k, \theta) \) represents the residuals. This requires an iterative search for \( \theta \) that yields the minimum of the loss function \( V_N(\theta) \), which may be a local minima of the nonlinear optimization problem (49). As previously mentioned, only the coefficients of polynomial \( \hat{A}(q) \) in (47) are of interest, since it is aimed at extracting the modal content of the signal, which are the roots of \( \hat{A}(q) \); hence, one may consider applying techniques such as the Yule-Walker method that only estimates the parameters of \( \hat{A}(q) \), thus employing more simplified and robust numerical techniques.

The theory of variance of identified model parameters is well-understood and developed in the context of PEMs [14]. Therefore, this theory, as explained next, is used here to estimate the variance of identified modes. Thus, at the solution point \( \hat{\theta} \), the differentiation of \( V_N(\theta) \) with respect to \( \theta \) has to be zero, i.e.

\[ V'_N(\hat{\theta}) = 0 \]  \hspace{1cm} (50)
Thus, an iterative algorithm, such as Newton, can be used to solve for \( \hat{\theta} \) by means of the Taylor series expansion of (50) around a given point \( \theta^* \) close to \( \hat{\theta} \) [14]

\[
0 \approx V_N'(\theta^*) + V_N''(\theta^*) (\hat{\theta} - \theta^*)
\]

or

\[
(\hat{\theta} - \theta^*) = -[V_N''(\theta^*)]^{-1} V_N'(\theta^*)
\]

This requires the first derivative (gradient) and second derivative (Hessian); thus:

\[
V_N'(\theta^*) = -\frac{1}{N} \sum_{k=1}^{N} \psi(k, \theta^*) \varepsilon(k, \theta^*)
\]

\[
V_N''(\theta^*) = \frac{1}{N} \sum_{k=1}^{N} \psi(k, \theta^*) \psi^T(k, \theta^*) + \frac{1}{N} \sum_{k=1}^{N} \psi'(k, \theta^*) \varepsilon(k, \theta^*)
\]

where

\[
\psi(k, \theta^*) = -\frac{d}{d\theta} \varepsilon(k, \theta)|_{\theta^*} = \frac{d}{d\theta} \hat{\gamma}(k|\theta)|_{\theta^*}
\]

Close to the solution \( \hat{\theta} \), the predicted errors \( \varepsilon(k, \theta) \) are independent; thus,

\[
V_N''(\theta^*) \approx \frac{1}{N} \sum_{k=1}^{N} \psi(k, \theta^*) \psi^T(k, \theta^*)
\]

2.6.4.1 Covariance of Parameters

It is known that \( \sqrt{N}(\hat{\theta} - \theta^*) \) is asymptotically Gaussian distributed with zero mean and a covariance matrix \( \hat{P} \), i.e. \( \mathcal{N}(0, \hat{P}) \) [14]. Therefore, an estimate of \( \hat{P} \) from available data can be obtained as follows:

\[
\hat{P} = \hat{\lambda}_0 \left( V_N''(\theta^*) \right)^{-1}
\]

\[
\hat{\lambda}_0 = \frac{1}{N} \sum_{k=1}^{N} \varepsilon^2(k, \theta^*)
\]

where \( \hat{\lambda}_0 \) is an estimate of the variance of the errors. Then, the covariance of parameter estimates, i.e. \( P_{\hat{\theta}} = E[(\hat{\theta} - \theta^*)(\hat{\theta} - \theta^*)^T] \), can be approximated as:

\[
P_{\hat{\theta}} \approx \frac{1}{N} \hat{P}
\]

The modes of a system are the roots of the characteristic equation, and hence are only dependent on the AR part of an ARMA(\( p, d \)). Thus, the covariance matrix \( P_{\hat{\theta}} \) is partitioned so that the rows and columns corresponding to the AR and the MA parts are separate as follows:
A relationship between $P_{\theta_{AR}}$ and the covariance of modes (error bounds) is established next.

### 2.6.4.2 Relationship Between Error Bounds and Covariance of Parameters

System modes can be related to $\theta_{AR}$, which are the coefficients of the characteristic equation, as follows \[34\]:

$$
\Phi = \gamma(\theta_{AR})
$$

(60)

where $\gamma(\theta_{AR})$ is a nonlinear function, and $\Phi$ denotes a vector containing the modal parameters. For instance, the real part $\alpha$ and the frequency $f$ of the modes can be used to define:

$$
\Phi = [\alpha_1, f_1, \alpha_2, f_2, \ldots, \alpha_p, f_p]^T \in \mathbb{R}^{2p}
$$

(61)

In order to obtain the mean and variance of the modes, the expected value operator may be applied to a Taylor series expansion of the function $\gamma$ about an operating point $(\Phi, \theta_{AR})$; thus:

$$
\Phi \approx \Phi + J(\theta_{AR})(\theta_{AR} - \tilde{\theta}_{AR})
$$

(62)

where

$$
J(\theta_{AR}) = \left. \frac{\partial \gamma(\theta_{AR})}{\partial \theta_{AR}} \right|_{\tilde{\theta}_{AR}} \in \mathbb{R}^{2p \times p}
$$

(63)

Rearranging (5.4.15) and applying the second moment operator (covariance) yields:

$$
\text{Cov} \ \Phi = E[(\Phi - \tilde{\Phi})(\Phi - \tilde{\Phi})^T] = J(\tilde{\theta}_{AR}) \ P_{\theta_{AR}} \ J^T(\tilde{\theta}_{AR})
$$

(64)

This clearly shows the connection between the covariance of estimates $P(\tilde{\theta}_{AR})$ and the covariance of modes (error bounds) $\text{Cov} \ \Phi$. Therefore, in (64), $P_{\theta_{AR}}$ can be estimated using (58), and a numeric Jacobian $J(\tilde{\theta}_{AR})$ can be approximated as follows:

$$
J_{ij}(\tilde{\theta}_{AR}) \approx \frac{\gamma_i(\tilde{\theta}_{AR} + \Delta \theta_j) - \gamma_i(\tilde{\theta}_{AR} - \Delta \theta_j)}{2\Delta}
$$

(65)

where $\Delta \theta_j = [0 \ \cdots \ 0 \ \frac{\Delta}{\theta_j} \ 0 \ \cdots \ 0]$, with $\Delta$ being a small number.

### 2.6.4.3 Calculating the Error Bounds of Estimated Modes

Regardless of the approach used to estimate an electromechanical mode and the corresponding uncertainty (Monte-Carlo or the abovementioned method), selecting proper orders is essential for obtaining accurate results. For power system monitoring applications, model order selection has been mostly done by means of trial and error. However, analytical approaches have been employed as well [35] and [36]. For PEM-based techniques, different criteria have been proposed to select the best order of the model, such as Akaike information...
criterion (AIC) and final prediction error (FPE) [14]. These techniques are defined based on prediction error variance. For instance, to choose the orders of an ARMA \((p,d)\) model, first a range is selected for \(p\) and \(d\), and then, for each pair, the parameters of the model are estimated; the pair obtaining the lowest value of AIC or FPE criterion is selected as the best estimate of the model order.

For a given ARMA\((p,d)\) model, the following steps can be followed to obtain the covariance of estimated modes:

1. Compute \(\hat{\theta}\) and \(\tilde{\theta}\), which also leads to \(P_{\hat{\theta}}\) when partitioned as (59), by means of the PEM (49).
2. Define \(\Phi\) for the mode or modes of interest and compute the corresponding numeric Jacobian \(J(\hat{\theta}_{AR})\) as per (61) and (65), respectively.
3. Calculate the covariance of modes (error bounds) as per (64).

2.6.4.4 Example

The estimated standard deviation of both the real part and frequency of the inter-area mode for the 2-area benchmark system obtained using (64) is depicted in Figure 2-16. Observe that the estimates track the results corresponding to the Monte-Carlo simulation, and the mean of the estimates can provide reasonably good accuracy with a significantly reduced number of trials. For instance, in Figure 16, the convergence speed of the standard deviation of estimates is nearly three to four times faster than the Monte-Carlo method, thus yielding significant reduction in the monitoring time. Notice as well that the uncertainty associated with the real part of the mode is relatively large when compared to that of the frequency (e.g. the standard deviation of the real part of the mode depicted in Figure 2-16 is about 25% of the actual real part, whereas it is only about 0.6% for the frequency). This is due to the fact that obtaining accurate estimates of mode damping in power systems using system identification is more difficult [6], [36].
Figure 2-16: Standard deviation of the real part and the frequency of the identified inter-area mode $-0.1228 \pm j4.7824$ for the 2-area benchmark system using Monte-Carlo and equation (64) for an ARMA(10,10).

2.6.5 Residual Analysis

Residuals, also known as prediction errors, can be defined as the differences between the measured outputs from a system and the predicted outputs for an identified model. The residuals account for the portion of measurement data that cannot be explained by the model. A good model usually can predict system responses well and thus has small residuals. As such, residuals are often used as an indicator of model quality. For example, [37] uses a subspace method to build a state space model from probing responses. To check the quality of the identified model, the residuals from the model are compared with original measurements as in Figure 2-17. The relatively small residuals are one indicator of a good model.
Most mode meter algorithms use linear models to describe power system responses. After a model is built using measurement data, modes are calculated as the eigenvalues of the linear model. Thus, the quality of the identified model determines the accuracy of the estimated modes.

To identify a model and evaluate the quality of a model, data can usually be divided into two groups, i.e. estimation data and validation data [14]. The estimation data are used to build the model. The validation data are reserved and not used in model identification procedure so that they can be used later to independently verify the performance of the model. A good model can predict the system responses, and thus shall produce small residuals for both estimation data and validation data. Independent performance validation using reserved validation data can help reduce the chance of over-valuing a model that over-fits the estimation data.

Ambient data are generated assuming that a power system is stimulated with white noises [5]. The residuals are the estimates of system stimulations. Thus, to evaluate the model quality, it is a good practice to check the whiteness of the residuals to verify the assumption. The whiteness of residue can be checked through the χ² tests of its auto-correlation function [14]. The normalized auto-correlation function of an ideal white noise is an δ(τ) function, which takes 0 value for all the τ’s except at τ=0. A good model should produce residuals close to white noises, which indicates that all the correlations in the measurement data have been captured by the model. Figure 2-18 shows the auto-correlation function of both measurement data and model residuals. Observe that the auto-correlation function of the residuals is relatively small at τ>0. This indicates that the residuals are close to white noises.
Even though a good model usually produces small residuals, those residuals are not sufficient to guarantee a good model. Small residuals only mean that the model can reproduce the particular system responses for either the estimation or validation data. When these data sets only represent partial dynamic features of a system, the small residuals only mean that the corresponding model captures the partial features shown in the data. As such, residuals shall be used as an indicator a good quality model.

2.6.6 Benchmark Testing Methodology

The benchmarking objectives and assessment procedures are specified from the point of view of the system operator [59]. In particular, due to the proprietary nature of some modal estimators, the system operator is unlikely to have detailed knowledge of the estimation algorithms employed in vendors’ estimation systems. Therefore, the modal estimator must be treated as a black box when designing and assessing the benchmarking tests. Typically, the system operator requires benchmark tests to be conducted using realistic test-signals generated from detailed models of the operator’s own power system. Benchmarking tests as outlined below have been successfully conducted in practice by system operators in Australia (2004) and New Zealand (2008).

In general the objectives of the benchmark tests are (i) to assess the bias and variability of the modal estimates and the limitations of the algorithms, (ii) to determine whether system operating conditions affect their performance, (iii) to establish the ability of the algorithms to track changes in system damping, (iv) to assess the performance of the modal estimators for staged system tests, (v) to determine if turbine/governor modes affect estimation of lightly-damped low-frequency electromechanical modes, and (vi) to assess the quality of damping information for use by both system operators and system planners. Additionally, the benchmarking tests are employed to assess the practical performance of modal-estimators in (ii) triggering of alarms due to the degradation of modal damping below a specified threshold, and (iii) non-invasive, special purpose or staged system damping tests (such as determining the improvement in damping that results from the introduction of a stabilizer).
It is typically assumed that ambient fluctuations actually observed in line power flows or bus voltage-angles are attributable to the random switching of loads at all levels in the power system. For the purposes of analysis, it is assumed that (i) small load perturbations excite the electromechanical modes of the system, (ii) the random fluctuation in the $i^{th}$ load is driven by the integral of white-noise [38], and (iii) white-noise driving the $i^{th}$ load is uncorrelated with the white-noise driving the random fluctuation in the $j^{th}$ load.

For benchmarking purposes, it is essential that the test signals are derived from a system model for which the dominant electromechanical modes of the system can be accurately calculated. Because in practice, the ambient noise fluctuations are small perturbations, linearized models of the actual power system are used as the basis for generating the test-signals. It is desirable to use detailed system models so that the test signals accurately reflect the full modal content that occurs in the measured signals in practice.

Test signals for the selected responses (e.g., node voltage-angle) are typically generated with a step size of 20 minutes and a duration of 48 hours. The long duration is required to ensure that reliable measures of the bias and variance of the modal estimates are obtained.

1) Stationary test signals
Stationary test signals for a range of system operating conditions are generated in order to establish the base-line accuracy of the modal-estimator under ideal (i.e. stationary) conditions. To assess the performance of the modal-estimator across a range of system operating conditions, a number of cases are generated that cover light, medium and high system load conditions, with a variety of power transfer levels on key interconnectors. In addition, a set of cases is devised in which the damping of the dominant system modes is varied from very well to very poorly damped.

2) Non-stationary test signals
The non-stationary tests are designed to assess the ability of the modal-estimators to track changes in the modal parameters. Three types of non-stationary signals are generated. a) Relatively infrequent step-changes in the damping constant and/or frequency of selected modes. This is achieved by applying step-changes in selected system parameters (e.g., gains of selected power system stabilizers). The step changes are of varying amplitude and spacing. b) Continuous changes in the damping constant of selected modes. Typically, the damping is changed at a constant rate defined by the user. A test signal will typically be composed of a number of ramps of various rates resulting in an irregular saw-tooth wave. The damping constant is changed by modifying the parameters representing the damping torque coefficients of selected machines at the required rate. c) Step changes in damping constants of the least-damped inter-area mode every 3 hours in the range from ±0.01 to ±0.1 Np/s. These latter conditions represent non-invasive, staged system tests; in benchmark testing the times at which the damping changes occur are given as would be the case in practice.

1) Assessment of Benchmarking Results: Stationary cases (See Figure 2-19)
The objective of the assessment of the results for the stationary cases is to determine (i) the error between the estimated modal parameters and their actual values, and (ii) the variability of the estimates, over the period of the test sequence. The modal-estimator generates a time-series of estimates of the damping constant (Np/s) and modal frequency (rad/s) for each mode of those identified. The duration of the test signal should be long enough to obtain an accurate measure of the mean and standard deviation – 48 hours or more is typical.

2) Assessment of Benchmarking Results: Non-stationary cases (See Figure 2-20)
In practice, the power system is non-stationary and it is essential that the modal estimates closely track the actual values. For benchmarking purposes, the objective in the assessment of the performance is (i) to determine, qualitatively and quantitatively, how closely the estimates track the actual trajectories of the modal parameters; (ii) to determine the rate of missed and
false damping alarms; and (iii) to determine the accuracy with which changes in damping can be determined during staged tests.

The above benchmarking approach should be extended to include the following aspects:

1. The accuracy of the modal estimates in the event of significant transient events including faults and network topological changes.
2. The performance of the signal measurement system.

![Figure 2-19: Comparison of the estimated and actual modal parameters, damping-constant and frequency, over a 48-hour period for a stationary signal.](image)

![Figure 2-20: Response of the estimated damping-constant trajectory of a mode when the actual trajectory of the damping-constant is a series of ramps. (For purposes of comparison the time scale of the estimates has been advanced by half the window length).](image)
2.7 Automated Implementation Issues

2.7.1 Data Processing

The primary source of data for estimating modal information is derived from time-synchronized Phasor Measurement Unit (PMU) signals. Typically, raw PMU output phasor voltages and currents are sampled at 30 samples per second (sps) to 60 sps and have a bandwidth on the order of 5 Hz to 15 Hz. For example, a common signal to use is the derivative of the phase angle of the positive-sequence phasor voltage. Other common signals include real power flows and voltage magnitudes.

Before applying modal analysis algorithms, one typically pre-processes the PMU data by:

1. Low-pass filtering the data to prevent aliasing and down-sampling data to a lower sample rate.
2. Filtering data to remove unwanted effects.

Down sampling is often employed to reduce the sensitivity of the algorithms. The discrete-time poles become more spread out in the Z domain. Research [9] has supported that many mode estimation algorithms work best with preprocessed data down sampled to approximately 5 sps. The order of the above two filtering steps is interchangeable and is still an open argument.

Many standard low-pass filters will provide the necessary anti-aliasing effect. Many researchers prefer FIR filters to avoid any added modes to the data due to the filter poles. Other researchers employ IIR filters. No comparison of the different low-pass filtering options has been published to date.

2.7.2 Mode Selecting

Although power systems are very high order, any one measured signal is typically dominated by a few modes or less. Therefore, the “dominant order” of any signal is typically six or less. For improved numerical conditioning, the model order in the previously described algorithms is often assumed large; a typical value is 20 to 25. This results in several estimated modes. The problem then becomes determining which mode is the dominant electromechanical mode and which are numerical artifacts and weaker modes. This is termed the mode selection problem.

When selecting signals for analysis in the algorithms, the goal is to use signals that have high observability of the desired mode and low observability for other modes. The observability is measured in terms of the peak in the power spectrum or energy in the autocorrelation. Because the signals are random, one cannot directly calculate the energy of a given mode within the time domain. But, one can estimate the “pseudo energy” of a given mode within the autocorrelation function [9]. The modes are then sorted according to the pseudo energy and the largest one is assumed to be the dominant system mode.

2.7.3 Missing Data and Outliers

To automatically monitor power system modes, a mode-meter algorithm needs to handle non-typical measurement data to avoid large estimation errors. Non-typical data are data that do not carry information about system modes and cannot be described by the linear models used by mode-meter algorithms. Commonly encountered non-typical data include 1) missing data and 2) outliers. Missing data are dropped data points, which may result from temporary communication and measurement device failure. Outliers are values that significantly deviate
from normal expected values, and could result from a high-level disturbance and/or sensor failure. Outliers and missing data are not uncommon in the PMU measurement data. If they are not addressed, they may have significant negative influence on the mode estimation results.

2.7.3.1 Missing Data

Missing data, which may be generated by temporary communication failure, are usually marked by a flag and thus can be readily identified. Depending on the duration of missing data, different methods can be applied for processing:

- When the duration of the missing data is short, the lost data can be reconstructed through interpolation or prediction during the data preprocessing procedure. Because of the short duration, interpolation can usually patch the data without introducing a significant amount of noise and, in turn, help a mode-meter algorithm ride through the missing data.
- When the duration of the missing data account for only small portion of analysis window size, the lost data can be simply cut out through concatenating the good data. As shown in [9], a mode-meter algorithm does not degrade very much when 6% missing data are removed through concatenation.
- When sizes of missing data are large, removing data through concatenation may lead to a fictitious transient; interpolating data may introduce significant noises, all of which may degrade performance of mode estimation. A common strategy for dealing with a large amount of missing data is to consider the good data as several segments separated by missing data [14], [17] and then formulating and lumping together the objective functions for all segments of good data. For example, considering data consisting of several segments of $T_1, T_2, \cdots, T_n$, a least square objective function can be formulated as

$$\hat{\theta}(t) = \arg\min_{\theta} \left\{ \sum_{i=1}^{n} \sum_{k \in T_i} \epsilon^2(k | \theta) \right\},$$

where the $\epsilon$ is the prediction errors, which is the function of model parameter $\theta$. Note that the prediction errors involving missing data are removed from the objective function.

2.7.3.2 Outliers

Outliers, which may be generated by temporary sensor failure or high-level interference, are data that have significantly deviated from normal measurements. Unlike missing data, outliers are usually not marked out in the measurement data. Thus, the first step is usually trying to detect the outliers during the data preprocess. Once an outlier is detected, it can be treated in the same way as a missing data point. An outlier is usually detected through inspecting the model prediction errors or residuals [14, 21]. Measurement data with large prediction errors are usually considered outliers. For example in [13], a data point is identified to be an outlier when its corresponding prediction error exceeds five times the standard deviation.

It is possible that some outliers are not captured during the data preprocessing and show up in the mode-estimation algorithm. In that case, it may be beneficial for a mode-estimation algorithm to have some robustness against outliers. A common strategy is to use a robust objective function. For example, the commonly used least square objective function is very sensitive to large prediction errors. To improve the robustness, some robust objective functions (e.g., ‘Hard’ rejection function, Hampel nonlinearity function [18]) may be used.
These robust functions reduce and/or confine the influence of large residuals on the objective functions. Thus, the negative influence from some outliers can be reduced. Note that the influences of some normal measurement data with large prediction errors are also reduced. Thus, for a data set with no outliers, the efficiency of the algorithm may be reduced by the robust function. Due to the statistical nature of the measurement, there is almost always a tradeoff between robustness and efficiency for mode estimation in processing outliers.

In summary, the non-typical data (such as missing data and outliers) must be processed properly to reduce the negative influence on the mode estimation. It is worth noting that in addition to missing data and outliers, there may be some other non-typical data that can degrade performance of a mode-meter algorithm. For example, the initial transient responses right before a ringdown can be harmful to the normal recursive least square (RLS) algorithm as shown in [17]. In this case, the initial transient responses are true responses that reveal the system dynamic features. The limitation is with the AutoRegressive (AR) model structure used by the RLS algorithm, which cannot describe the initial transient responses. Thus, the corresponding data, if detected, shall be treated as non-typical data in a RLS algorithm with an AR model. Another example is the system responses from probing injection. In [13], it is shown that if the injection is known and used in an ARMAX model, the probing responses can help mode estimation. In contrast, if the injection is unknown and therefore not used, using probing responses as normal ambient data leads to biased mode estimation. As such, whether the data is non-typical or not depends on whether the data can be properly assimilated into the model used. The non-typical data should be treated with care to reduce their negative influences on the mode estimation.

2.8 Examples

2.8.1 US WECC

The western North American power system (wNAPS) shown in Figure 2-3 has a long history of testing and examining power-system dynamics. As discussed in section 2.3, testing typically includes pulsing the 1400-MW Chief Joseph dynamic brake and/or modulation of the Pacific DC intertie (PDCI) to inject known probing signals into the system. Referring to Figure 2-3, the PDCI is the DC line flowing from Oregon to southern California. The PDCI has been modulated with a number of different signals including short duration mid-level probing resulting in transient responses and long-duration low-level probing that result in measured signals only slightly above the system ambient noise floor.

The wNAPS contains several electromechanical modes; but, the two are very wide-spread. They are the “NS” mode typically near 0.25 Hz and the “Alberta” mode typically near 0.4 Hz. Figure 2-21a and Figure 2-21b summarize the typical shapes of these wide-spread modes. The NS mode has the northern part of the system oscillating against the southern half. The Alberta mode has the Alberta area oscillating against the northern portion of the system, which in turn oscillates against the southern half of the system.

As an example, consider a test conducted in 2011. The test consisted of four 0.5-second brake insertions and two 20-minute PDCI low-level probes. Figure 2-22 shows the PDCI probing signal normalized between +1 and the discrete Fourier transform of the signal. As seen in the plot, the signal is 100 seconds long and has frequency content from 0.02 to 5 Hz. Specifically, the frequency content rolls off from 0.1 to 0.02 Hz at 20 dB/decade; rolls off at 20 dB/decade after 1 Hz; and every other bin outside 0.2 Hz to 0.5 Hz is removed. Considerable testing was conducted to obtain this signal. The goal is to sufficiently excite the dynamics in the electromechanical frequency range. The probing signal has also been used to
estimate transfer functions for PDCI damping control studies. For a given probe test, the signal is scaled to \( \pm 20 \text{ MW} \) and applied to the PDCI DC power-flow controller and repeated 12 times for a total of 20 minutes.

The response of the system for the considered test day is shown in Figure 2-23 and Figure 2-24. Figure 2-23 shows the detrended PDCI real-power over 500 min. Figure 2-24 shows the derivative of the phase angle between two key buses (buses A and B) in the system. The derivative is scaled to Hz; we term this a “relative frequency” signal. The PDCI flow is detrended and the names of the buses are protected for data confidentiality. Using the notation of MIN:SEC, key points in the test are

- Brake insertion 1 = 89:05.4
- Brake insertion 2 = 94:15.5
- PDCI probe 1 = 208:22 thru 228:22
- PDCI probe 2 = 325:11 thru 345:11
- Brake insertion 3 = 368:45.0
- Brake insertion 4 = 374:25.0

Figure 2-24 shows the relative frequency signals for the full 500 minutes on the top plot and zoomed into brake insertion 1 on the bottom plot.

Figure 2-25 shows the waterfall spectrum of the A-B relative frequency signal estimated using Welch’s periodogram averaging. Note the increased energy in the spectrum during the brake insertions and the PDCI probes. The PDCI probe typically excites the Alberta mode with more energy than the NS mode.

Figure 2-26 and Figure 2-27 show the mode estimates from modal analysis of the A-B signal in Figure 2-24 using a modified extended Yule Walker algorithm and Prony analysis on the brake responses. A 30-minute window of data was used to estimate the modes using the LSMYW and repeated every 1 minute. Green markers indicate the brake insertions and PDCI probe times. Note the reduced variance of the LSMYW estimates for the Alberta mode damping immediately after the brake insertions. This is because there is more energy in the system due to the brake insertion. The slight bias error from the Prony damping estimates is typical and is related to the basic assumption of the system input for the YW formulation. Also note that the variance of the damping estimates for the Alberta mode are reduced after insertion of the probing signal. This is also due to increased energy in the system.
Figure 2-21a: WECC NS mode shape. Generators within blue oscillate against generators within red.

Figure 2-21b: Alberta WECC NS mode shape. Generators within blue oscillate against generators within red.

Figure 2-21: The typical shapes of these wide-spread modes.

Figure 2-22: Probing signal normalized to unity and its transform.
Figure 2-23: PDCI detrended DC power flow.

Figure 2-24: Relative frequency between buses A and B. Top plot is the full test; bottom plot zooms to brake insertion 1.
Figure 2-25: Waterfall spectrum of relative frequency between buses A and B.

Figure 2-26: NS mode estimation from the relative frequency between buses A and B using modified extended Yule Walker algorithm and Prony analysis of brake insertions.
2.8.2 US Western Electricity Coordinating Council (WECC) Mode Shape

Now consider estimating mode shape from wNAPS data. For demonstration purposes, six buses near major generation sites spread across the system are selected. The actual locations of the sites are not revealed in order to protect data confidentiality. The buses are termed bus A through bus F. Nearly 20 minutes of phasor data was collected during a recent probing test from a PMU at each bus. The frequency error at each bus is estimated from the phasor angle using a forward differencing calculation.

Results from the spectral analysis are shown in Figure 2-28 through Figure 2-30. The relatively large peaks at 0.25-Hz for buses B through F in the PSD and coherency indicates that these buses participate in a single mode at this frequency. The relatively small peak for bus A indicates that this bus does not significantly observe this mode. Bus D is selected as the reference bus for the analysis as it has a large peak in the PSD. It should be noted that any signal with a large peak may be selected as the reference bus in the analysis. The CSD in Figure 30 shows the mode phasing. The plot indicates that buses D and C swing together against buses B, E, and F.
Figure 2-28: Estimated PSD for wNAPS mode-shape example.

Figure 2-29: Estimated coherency for wNAPS example. Bus D is the reference signal. 
* indicates the 0.25-Hz mode.

Figure 2-30: Estimated CSD angle for wNAPS example. Bus D is the reference signal. 
* indicates the 0.25-Hz mode.
2.8.3 US Eastern Interconnect

While this chapter focuses on modal identification of electromechanical modes, this section looks at another cause of oscillations in power systems. Unwanted oscillations within a system can occur for various reasons, such as an inter-area modal oscillation or a forced response of the system due to a rogue controller on a generator. Inter-area modes are characteristics of the system and oscillations associated with these modes are part of the natural response. On the other hand, forced oscillations are not a characteristic of the system but are a response by the system due to a driving function and are thus part of the forced response of the system. These two distinct types of oscillations show up in synchrophasor measurements. It is desirable to be able to accurately detect forced oscillations in order to distinguish them from system inter-area modal activity. Also, these forced oscillations are not to be confused with transients caused by line-switching and load shedding within the system.

Forced oscillations can be small to large in amplitude and can occur at a wide range of frequencies. If the frequency of the forced oscillation is near that of an inter-area electromechanical mode, the forced oscillation is likely to be observed over a wide geographical region because of the high system gain near modal inter-area frequencies. Forced oscillations can occur in any power system.

In this subsection, examples from the Eastern United States Interconnect (EI) of forced oscillations, which are in the same frequency range as inter-area modes, are investigated. System-wide forced oscillations are of particular interest. These forced oscillations can be estimated from Phasor Measurement Unit (PMU) data using a variety of methods. For this example, both real power (MW) and frequency error are calculated from individual PMU channels, and then analyzed. Channel locations and names are confidential and not shown. From the many PMU channels within the EI, six that are geographically separated have been chosen for this study.

The ability to detect a forced oscillation depends on the magnitude and time duration of the oscillation. If the oscillation is very large in amplitude then it can readily be observable in the time domain. In these particular examples the amplitude of the oscillation is not large enough to readily be seen in the time domain as apparent in Figure 2-31. Thus, frequency domain techniques need to be used for identifying forced oscillations. Constructing a Welch Periodogram is a useful approach that averages together the squared magnitudes of many shorter FFTs. Figure 2-32 compares spectrums of one hour of MW data from two channels during 2008. Periodograms are calculated with von Hann windows of 90-second length. In Figure 2-32, a narrow peak is observed at 0.45 Hz. The question immediately becomes, is this peak caused by a lightly damped electromechanical mode or is it caused by a forced oscillation? A direct approach to distinguish them is by simple FFT analysis of long record lengths, in this case the full hour of data. The frequency resolution is inversely proportional to the data record length used in the FFT. The absolute value squared of the FFT is also shown in Figure 2-32. Here the sinusoidal forced oscillation can be observed as the extremely narrow peak at 0.45 Hz. Since the peak has no width there is no damping and this oscillation is indeed a forced oscillation. These oscillations are generally visible in both real power flow as well as electrical frequency data, although with varied comparative magnitude. Only the MW results are shown here.
Figure 2-31: Time-domain real power flow on Ch.6 during 2009.

Figure 2-32: Comparison of two channels of MW data, channels two and four. The raw FFT shows a distinct sinusoidal component.

It is possible to observe a forced oscillation throughout time, rather than from taking an entire hour of data, by using an averaged spectrogram. Figure 2-33 shows the varying frequency content throughout time. The forced oscillation is clearly visible in Figure 2-33 throughout the entire time period as the narrow red ridge at 0.45 Hz.
Figures 2-31 through 2-33 use data collected in 2008. Forced oscillations of nearly the same frequency have been found in data sets from various other years. An interesting note is that the forced oscillations throughout each different dataset from 2007-2009 all contain nearly the same oscillatory frequencies, which are all within the same frequency range as inter-area modes. Table 2-II shows the center frequencies of the oscillation in the various datasets.

### Table 2-II: Comparison of sinusoidal oscillatory frequencies over various datasets collected from the EI

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Freq. (Hz)</td>
<td>0.4556, 0.4167</td>
<td>0.4539</td>
<td>0.4533</td>
</tr>
</tbody>
</table>

Forced oscillations may not always be of magnitude large enough to “jump out” of the signal spectrum. Yet another dataset shows a system-wide forced oscillation, but of shifting frequency and including a visible 2nd harmonic, occurring in 2007. Figure 2-34 shows a spectrogram encompassing the oscillation. This oscillation first appears during the second hour and is small in magnitude. During the sixth hour the magnitude becomes larger after shifting down in frequency. Then, at the end of the seventh hour, the forced oscillation ends.

---

Figure 2-33: Averaged spectrogram from ambient MW data Ch.6 showing the presence of a 0.45Hz forced oscillation.

Figure 2-34: Real power flow from 2007 showing a low magnitude forced oscillation shift in frequency.
It can be insightful to estimate other characteristics of the sinusoidal disturbance such as the amplitude and phase. Using further Fourier analysis to estimate sinusoidal parameters of the oscillation, the output can be compared at various geographical locations. Figure 2-35 shows a comparison of the estimated sinusoidal disturbance across six PMU channels, which are located hundreds to thousands of miles apart. Estimated parameters are amplitude, frequency, and phase. The oscillations observed in these examples are not large in magnitude, often less than a MW in amplitude. Perhaps further analysis could narrow down the origin of the oscillation by observing the phase of the oscillation on various channels.

![Figure 2-35: Estimating a system-wide forced oscillation as a pure sinusoidal component of the same frequency, data from 2008 real power flow.](image)

### 2.8.4 Mexico

In this section we discuss how block-processing algorithms using parametric and non-parametric frequency and damping estimation can be used to track changing dynamics of power systems. To this aim, we analyze phasor measurement data emerging from a planned interconnection test between the Mexican and Guatemalan power systems. Such analysis shows that spectrograms reveal important features of the dynamics of power systems, which are otherwise difficult to capture, and that continuous monitoring of inter-area modes is most relevant in weak interconnections. Analysis of these planned tests provides important information on how spectrograms behave under the connection of dominant transfer paths.

#### 2.8.4.1 Interconnection Characteristics, Measurements, and Staged Test Data

The interconnection between the Mexican and Guatemalan power system are illustrated in Figure 2-36. The interconnection consists of a double-circuit 400 kV reactive-shunt-compensated transmission line, which interconnects the TPH-400 substation on the Mexican side to the LBR-400 substation in the Guatemalan side. The interconnection includes a 400 kV system infeed from the ANG-400 substation and an outfeed to the local 115 kV network at TPP-115 in the Mexican side, while at the Guatemalan side, the interconnection joins the main 230 kV grid. PMUs are installed throughout the interconnection at all buses as indicated by the red diamonds in Figure 36. The only PMU data used for ambient data analysis comes from PMUs THP-LBR at the Mexican side and LBR-THP from the Guatemalan side; however, all data is available as it can be observed in Figure 37, which shows the frequencies before the synchronization, during it, and after the systems have been separated. During this test the transmission line from THP-LBR to LBR-THP carried about 20 MW of transfer maximum as shown in Figure 2-37.
2.8.4.2 Time-Frequency Analysis (Spectrograms) and Damping Estimates

The Welch and LSMYW algorithms were applied to the measurement data available from the THP-LBR and LBR-THP PMUs. For the estimation of each periodogram, the data was divided in 5-minute parcels, sliding every 2 minutes. Linear trends were removed, and a low pass filter with 2 Hz cut-off was applied to the data to limit it to the electromechanical dynamics. The data were then down-sampled from 20 samples per second to 5 samples per second and filtered with a high pass filter with 0.1 Hz cut-off to remove load-governor dynamics. For Welch’s method, a block size of 1500 samples was used, and a segment size of 500 samples was used (equally for the number of points used for the FFT). A Hamming window with 50% of overlap was applied to the data. The estimated periodograms from Welch’s method were used to refine the AR model order of the LSMYW method by obtaining good agreement between the PSDs of both methods, while trying to keep the model order as low as possible (in this case the order model is 20). As a result, an excellent agreement was obtained between the PSDs estimated from each method.

The spectrograms in Figure 2-38 are constructed from all the computed periodograms, showing the estimated mode frequencies from the power and bus frequencies. Note that the estimates from the active power can only give a mode estimate when there is a transfer in the tie-line, while the bus frequencies can always provide an estimate of each system. Before synchronization, the modes in the Mexican power system are: 0.15, 0.4, 0.65, and 0.9 Hz, where 0.15 and 0.9 Hz are the most active mode as can be observed in Figure 2-39b. When the synchronization occurs, the 0.9 Hz mode vanishes and the 0.62 Hz dominates – this is the inter-area mode between both systems, as can be observed in Figure 2-38a. It has the highest content in the active power flow mode estimates. This observation is relevant, as it shows that
different power system signals have a diverse mode observability which becomes crucial when choosing the proper signal to monitor inter-area modes.

![Time-frequency plots for the active power flow and bus frequency at THP-LBR during the staggered test.](image)

**Figure 2-38:** Time-frequency plots for the active power flow and bus frequency at THP-LBR during the staggered test.

In addition, it is important to highlight that the estimated modes at THP-LBR and at LBR-THP from the bus frequency signal show the explicit mode frequencies of each network before synchronization, and contained the combined dynamics of the two networks when they are synchronized. Table 2-III shows that for the Mexican power system, the main mode is 0.97 Hz before the synchronization, while the other power system is 0.42 Hz; when both power systems are synchronized, the 0.97 Hz mode vanishes leaving the 0.62 Hz mode from the Mexican power system to dominate the interconnection. This becomes apparent in Figure 2-39 (a) and (b) at t=60 min when the synchronization begins, and t=110 when the systems are separated.

**Table 2-III: Mode Frequency Estimates (Average) During Each Stage of the Test**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Before Synch. (Hz)</th>
<th>During Synch. (Hz)</th>
<th>After Synch. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (THP-LBR)</td>
<td>0.15, 0.4, <strong>0.62, 0.9</strong></td>
<td>0.15, 0.4, <strong>0.62, 0.97</strong></td>
<td>(Not available, data quality issues)</td>
</tr>
<tr>
<td>P (THP-LBR)</td>
<td>---</td>
<td>0.2, 0.42, <strong>0.62, 0.97</strong></td>
<td>---</td>
</tr>
<tr>
<td>f (LBR-THP)</td>
<td>0.13, 0.2, <strong>0.4, 0.7</strong></td>
<td>0.13, 0.2, <strong>0.42, 0.62, 0.97, 1.07</strong></td>
<td>0.13, 0.2, <strong>0.4, 0.63, 1, 1.18</strong></td>
</tr>
</tbody>
</table>

![Mode frequency estimates from the bus frequency signal at THP-LBR and LBR-THP](image)

**Figure 2-39:** Mode frequency estimates from the bus frequency signal at THP-LBR and LBR-THP
2.8.4.3 Insight on Power System Dynamics: Damping Estimates

Due to the fact that the Mexican power network is electrically stronger than the network in Guatemala, it is possible to observe, by applying ambient data analysis, that the impact of the interconnection of a weakly-coupled system to a larger grid results in low damping of the inter-area mode traveling through the interconnection and the manifestation of common modes from the stronger grid in the weaker network. This fact can be observed in Figure 2-40(a) and (b), which provide damping estimates from the frequencies at THP-LBR and LBR-THP. Note that before the synchronization at $t=60$ min, the 0.62 mode is hard to estimate. When it is estimated at THP-LBR, it shows damping between 5-8%. However, when the synchronization occurs, not only the mode becomes dominant, but it also does so with low damping. Another interesting fact is that when the system is more stressed, the damping estimates are lower, as expected in a weak link. Although this correlation between the intensity of the activity of an excited mode, and mode damping is known, it is rarely possible to pin-point the causes for the stress of the system, as it is in this case: power loading in the interface. Analysis of this kind of planned tests provides important information on how spectrograms behave under the connection of dominant transfer paths.

![Figure 2-40: Damping estimates for the inter-area mode between both networks during the synchronization.](image)

2.8.5 Australia (By G. Ledwich)

2.8.5.1 Historical Development in Australia of Modal Analysis

The first application of phasor data in Australia was across three nodes and 1200 km of the Queensland network [38]. The live transmission of this 8-bit data enabled evaluation of the travel time issues. But even this low resolution data gave the spectral response with modal peaks and a magnitude which rose as frequency drops as seen in Figure 2-41. The same shape was also seen the US data in [43]. This translated into a representation of the load changes as white noise, the model was later refined as non-Gaussian from the combination of major industrial loads with many smaller domestic loads.
On the basis of these initial results, a new PMU system was developed based on commercial GPS and A/D cards and a LabView analysis environment. This was deployed in the year 2001 at the major nodes of the interconnected Australian network. As seen in the system connections, the network is mainly a single line between major nodes which has longitudinal oscillations.

Because of the historical development of the systems as separate states, which later interconnected, the weak points tend to be on the links between states. There are three major modes visible in the data from these four measurement points and very little other modes visible apart from Queensland. The Brisbane measurement showed evidence of one other mode which was confirmed as Southern vs. Central Queensland mode when a mobile monitor was placed near Rockhampton 800km north of Brisbane.
2.8.5.2 Modal Analysis Development

Because of the simple modal structure in Australia and measurements, which essentially operated as single machine equivalents, the process of validation proceeded from confirmations of frequencies and damping ratios to oscillation mode shapes [44]. The linear combination of measurements to extract a desired mode before extraction of the damping estimate was added by Don Geddey of partner company Transgrid [45]. This proved to be beneficial for modes that were masked by others and the concept was extended in [46] to an automatic search process.

The estimates of angle difference spectra from the measurements in Figure 44 clearly show modes at 2 and 3 rad/s while there is evidence of another mode. Forming linear combinations of existing measurements meant that the third mode at 3.5 rad/sec was more able to be extracted and identified in Figure 2-44 using the decoupling approach [46].

2.8.5.3 Processes Which Aid Accuracy of Modal Estimates

One process developed by Don Geddey was to prefilter autocorrelation data with an exponential filter with a time constant matched to the expected damping ratio of the modes of interest. This added damping was then corrected numerically once the modal parameters were extracted. This constitutes a form of matched filtering and as seen in [47] shows a substantial benefit for short data length estimation of modal parameters.
2.8.5.4 Development of Alarming

Much of the development of algorithms for modal measurement has centered on frequency and damping estimates from ambient system variations. The difficulty is that accurate determination of damping data requires long data records and precise operator intervention as a loss of damping is difficult to trigger based on damping estimates. The work centered on techniques to characterize the whitening filter for a stationary set of measurements and then establish the probability of different degrees of confidence for a precise alarming system with defined levels of false positives. [48, 49, 50, 51, 52].

For an alarm threshold of 0.1% false positives, a loss-of-damping event in Victoria is seen in Figure 2-45. The relevant measurements are processed to give white noise for the stationary data. A sudden change of damping has the output of the whitening filter rising around 0.58 Hz. Fortunately, this was an incorrectly connected stabilizer on one generator, which was not exciting a major inter-area mode.

2.8.5

The I in Ta: s at these node shape.
2.8.6 Nordic

In this section we focus on the application of block processing techniques for mode frequency estimation in the Nordic region. Damping estimation results are shown to highlight some current challenges for mode damping estimation under forced oscillations.

2.8.6.1 Block Processing Algorithms: Pre-Processing and Parameters for Mode Frequency Estimation

In this study, we used a non-parametric method, the Welch method, and a parametric method, the LSMYW method. PMU was obtained from two different substations of the Nordic power system located in Eastern Denmark; their locations are shown in Figure 2-47. Data from the Radsted substation (RAD132, in Figure 2-47) were obtained on 03-20-2008 and 03-21-2008. Data for the Hovegaård substation (HVE400, in Figure 47) were obtained on 04-15-2007 and 04-16-2007.

The data is segmented in blocks of 10 minutes, and pre-processed for analysis. First, linear trends are removed from these signals using the detrend algorithm. Because these analyses focuses on the inter-area mode range, a low-pass finite impulse response (FIR) filter with 2 Hz cut-off frequency is used to remove high frequency components beyond this cut-off frequency. The data is down sampled to 5 Hz and a FIR high-pass filter with cut-off frequency of 0.1 Hz is used to remove low frequency components related to governing control. To each 10 minute block of pre-processed data, the Welch and LSMYW methods are applied.

For Welch’s method, we use 100 seconds for both the block segments and the number of FFT points used to calculate the PSD. To these segments, a Hanning window with 50% overlap is applied. The estimated periodograms from Welch's method are used to refine the AR model.
order of the LSMYW method by obtaining good agreement between the PSDs of both methods, while trying to maintain the model order as low as possible (in this case the order model is 40). As a result, an excellent agreement was obtained between the PSDs estimated from each method.

Both methods were applied to all the signals from Radstead and Hovegård for 10 minute blocks. Note that the estimates for Hovegård were obtained for a different date than those of Radsted. Common to all of the data sets were three dominant modes at approximately 0.36, 0.54, and 0.83 Hz. The presence of a low-frequency oscillatory component at approximately 0.28 Hz is discussed below.

![PMU Locations in Denmark](image)

**Figure 2-47: PMU Locations in Denmark.**

### 2.8.6.2 Time-Frequency Analysis (Spectrograms) for 48-hour Data Sets

Next, a time-frequency analysis for the whole 48 hour data set is performed and the spectrograms in Figure 48 were obtained for the $P_1$ signal. For the $P_1$ signal of Radsted, its Welch spectrogram is shown in Figure 2-48(a), and its corresponding AR spectrogram in Figure 2-48(b). Observe the close agreement between both spectrograms confirming the existence of the modes discussed above. The changing dynamics of the power system are revealed by the 48-hour spectrograms; 03-20-2008 was a “typical day” (in terms of the power system loading). It is important to note that the frequency and damping ratio of the electromechanical modes are influenced by the system loading and configuration of the power grid. For example, Mode 2 (around 0.54 Hz) is present throughout 03-20-2008, however it is not visible during hours 32-42.
The reader might be misled by the distortions appearing around 24 hours in Figure 2-48, which is a result of the selected range of the temperature bar giving the coloring to the spectrogram. To clarify, in Figure 2-49, we show a zoom of the spectrograms for signal \( P_4 \) of Radsted for \( t = [16 - 32] \), centered at 24 hours and with a different setting for the temperature bar. Note that with this new range the "blur" is no longer present and the modes discussed above can be clearly appreciated.

Close inspection of the Welch spectrograms for \( P_1 \) and \( P_4 \) (Figures 2-48 and 2-49) reveals an important feature of this particular data set. As mentioned earlier, an oscillatory component is present at about 0.28 Hz. This component must be critically analyzed, and the Welch spectrograms serve to this purpose. The Welch spectrograms show that the 0.28 Hz component appears almost persistently at a very narrow band, centered at approximately 0.28 Hz, from hours 3-32 and 42-48. To further discuss the nature of this component, an enlargement of Figure 2-49 is shown in Figure 2-50.

Note that from Figure 2-50, the behavior of the 0.28 Hz component is much different from the one observed of Mode 1 where the mode has a broader variation frequency range and change
of intensity. By inspecting Figure 2-50, it becomes apparent that the 0.28 Hz component has a narrower and better-defined frequency band.

The behavior shown by this 0.28 Hz oscillatory component corresponds to what is expected of a sinusoid or forced oscillation. A more careful inspection of the Welch spectrograms reveals that presumably another sinusoid is present at about 0.18 Hz for the time frames of 0-3, 32-37, and 40-43 hours. At this point, it should be realized that it is very likely that both of these components are harmonics of a fundamental sinusoid of 0.09 Hz. It is important to note that some of the sinusoid harmonics are superimposed over the “true” system modes.

2.8.6.3 Damping Estimation Issues with Forced Oscillations

Regardless of their origin, the harmonics discussed above will create difficulties to obtain accurate damping estimates of the true system modes. Mode meter algorithms will have difficulties resolving the portion of the frequency spectrum caused by ambient load variation from the portion due to the forced oscillations. To illustrate the most relevant issue regarding the presence of forced oscillations, the damping corresponding to the 0.18 and 0.27 Hz components was computed as shown in Figure 2-51.

The computed average frequency for the component in Figure 2-51 (a) was of 0.184 Hz with a standard deviation of 0 Hz. Although the average estimate for the damping during 48 hours was 3.37%, this average comes with a standard deviation of 9.82%. Close inspection of Figure 2-51 (a) reveals that the damping estimates are about 0%, hence supporting the presence of this component as a forced oscillation. For the component in Figure 2-51 (b), the average frequency computed over 48 hours is of 0.276 Hz (with standard deviation of 0 Hz), and an average damping of 0.59% (with standard deviation of 1.08%). For this component, the estimation results (a much lower standard deviation) give confidence that a forced sinusoid does exist, and furthermore, that the estimated damping of these components is zero.

The main concern is not necessarily that these forced sinusoids exist, but more importantly that some of the sinusoid harmonics are superimposed over the “true” system modes. Similar to the results shown in Figure 2-51, the average frequency and damping for each of the power system modes were computed. Table 2-IV shows the computed averages along with their respective standard deviation. While it is possible to have confidence on the estimates for the frequency, it is not possible to do so for the damping estimates for the “true” system modes (Mode 1 through Mode 3). Therefore, it can be concluded that the presence of the superimposed harmonic components of the forced oscillation can deteriorate the damping
estimates for each of these frequencies. As a consequence, the analyst should be skeptical of the resulting damping estimates in the presence of such forced oscillations.

Figure 2-51: Damping and frequency estimation of the 0.18 and 0.27 Hz components.

### Table 2-IV: Mode Meter Estimates for Radsted

<table>
<thead>
<tr>
<th>Mode</th>
<th>Signal</th>
<th>$\bar{f}$ (Hz)</th>
<th>$\sigma_f$</th>
<th>$\bar{d}$ (%)</th>
<th>$\sigma_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$P_1$</td>
<td>0.3615</td>
<td>0.0044985</td>
<td>4.384</td>
<td>7.3948</td>
</tr>
<tr>
<td></td>
<td>$P_3$</td>
<td>0.36038</td>
<td>0.0063084</td>
<td>8.0646</td>
<td>9.1122</td>
</tr>
<tr>
<td></td>
<td>$P_4$</td>
<td>0.36064</td>
<td>0.0048168</td>
<td>5.6049</td>
<td>7.7743</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$P_1$</td>
<td>0.55561</td>
<td>0.0066281</td>
<td>4.9269</td>
<td>9.1952</td>
</tr>
<tr>
<td></td>
<td>$P_3$</td>
<td>0.55556</td>
<td>0.0071702</td>
<td>6.4337</td>
<td>8.7984</td>
</tr>
<tr>
<td></td>
<td>$P_4$</td>
<td>0.55532</td>
<td>0.0062926</td>
<td>5.1723</td>
<td>6.4484</td>
</tr>
<tr>
<td>Mode 3</td>
<td>$P_1$</td>
<td>0.82916</td>
<td>0.0049495</td>
<td>2.5384</td>
<td>5.4367</td>
</tr>
<tr>
<td></td>
<td>$P_3$</td>
<td>0.83052</td>
<td>0.0051476</td>
<td>3.8604</td>
<td>6.8673</td>
</tr>
<tr>
<td></td>
<td>$P_4$</td>
<td>0.82996</td>
<td>0.0049805</td>
<td>3.2916</td>
<td>5.5067</td>
</tr>
</tbody>
</table>
2.9 Conclusions

This chapter has provided an overview of methods and examples in estimating power-system modal properties primarily from ambient data. An overview of algorithms and approaches for estimating modal frequency properties (both parametric and non-parametric), damping, and mode shape have been discussed. Several examples from around the world have also been provided. It is the hope of the authors that this chapter provides an introduction to the state-of-the-art in this area.

2.10 Chapter 2 References


G. Ledwich “Decoupling for Improved Modal Estimates” IEEE PES 24-28, pp. 1-6, June 2007


APPENDIX 2


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A2.1 Introduction

An early application of non-parametric modal identification of power system signal responses was reported in 1988 [1]. Prior to that, system identification techniques were investigated primarily for model validation and control tuning [2] but the interest for inter-area oscillations, which would become the main driver for power system small-signal analysis, only picked up in the late eighties [3]. While nonparametric FFT-based modal screening in the spirit of [1] has been in use on a heuristic basis in all organizations responsible for system dynamics, a radical methodological change occurred when Hauer [4] proposed a parametric approach, the so-called Prony analysis, to tackle this problem. This method, which improved the resolution of closely spaced modes and facilitated the estimation of damping and phase, is a crossover from the statistical signal modeling [5, 8] theory. It has emerged as the horsepower for identification of electromechanical methods and is routinely used to benchmark newly developed methods. Many variations in implementing Prony analysis exist but they all consist in estimating a transfer function whose poles are the natural modes of the response signals while their zeros are determined to achieve a least-squares fit of the signal to a sum of damped sinusoids.

Interestingly, state-space approaches for electromechanical-mode identification as discussed in this paper emerged shortly after pole-zeros based Prony analysis. As early as 1993, the ERA method previously developed in experimental modal testing of mechanical and civil engineering structures [6-7] was applied effectively to the analysis of ringdown responses arising in the process of system planning studies [9]. Later on, the ERA scheme was extended to pulse-response based MIMO model identification for small-signal analysis and damping control design [10]. During the nineties, the boom in N4SID algorithms [11-14] offered power system researchers yet another powerful state-space-based tool for electromechanical-mode identification. An advantage of subspace methods is that we do not need (nonlinear) optimization techniques, nor do we need to impose on the system a canonical form, so that subspace methods do not suffer from the inconveniences encountered in applying prediction error methods (PEMs) to MIMO system identification [13].

N4SID was applied initially to black-box MIMO model identification from simulations of power system small-signal responses to random forced excitations [15]. Recently, the idea was extended to probing excitations applied on actual systems [16]. Also well-known in mechanical vibration testing, a variant of the N4SID, called stochastic state-space subspace identification (SSSID), can deal with ambient power system responses [17-18]. This implies the identification of output data with un-forced excitation, available in a real-time tracking mode or in the form of long term time-series records. In the comparative study of three modal meter algorithms presented in [19], the SSSID (or a variant) is included.
Recently, it was shown in [20] that appropriate filter-bank pre-filtering in association with a TKEO-based mechanism, allowing us to select just the dominant channels for further modal analysis, considerably relaxed the robustness requirements on identification methods used in empirical analysis of electromechanical modes: both ERA and SSSID seemed well suited for this task, although the latter appears easier to tune for multi-output measurements.

The objective of this paper is to consolidate the different viewpoints on state-space model electromechanical-mode identification, until now scattered among various papers, into a single general framework, available at one stop. In our opinion, the relative complexity of state-space approaches and especially the level of statistical mystery surrounding N4SID methods have to a degree prevented power system engineers from reaping the full benefit of empirical state-space based modal analysis.

It is our hope that, by revisiting the theoretical background of ERA and N4SID in combination, and illustrating each target problem with an application developed on the same nine-bus test system [21, 22], our initiative will convincingly highlight the comprehensive benefits of this class of methods. One of the many advantages we see in them is the possibility of a seamless integration of stochastic and deterministic identification together with an indiscriminate treatment of MIMO, SIMO and SISO systems and signals, all this using non-iterative, linear analysis-based matrix solutions. Currently, the only way to move beyond N4SID and ERA accuracy is to opt for a nonlinear prediction error method (using N4SID for initialization as in Matlab’s System Identification Toolbox [41]), which definitely represents a big jump in software complexity and computation time.

**A2.2 Combined Simulation-Measurement Approach to Modal Analysis of Systems and Signals**

**A2.2.1 The Why and the How**

Motivated by the need to convert WAMS data streams into more useful and actionable information for system operators, recent papers tend to cast their focus on the modal analysis of actual measured system responses [23-24]. Although necessary, this evolution should not overshadow the original planning need to analyze simulations [1,9], understanding power system modal characteristics [9], and building models allowing damping controller tuning [10,14, 25-27]. Efficient power system operation requires a lot of simulation in fact, at both the design and the operations planning stages. These massive simulations need to be analysed as well, and, if a problem is found, system controllers have to be designed, tuned and coordinated off-line, before field validation through measurements [16]. Although imperfect, simulation models are necessary in day-to-day life operation of complex power systems and the trend towards near real-time decision making for economic reasons, increase the pressure for rapid assessment of simulated responses.

In this context, it clear to us that a convergence between modal analysis of simulated and measured data is suitable. We can broadly state this generalized problem in terms of identifying state-space models of systems and signals with forced or random excitations, followed by eigenanalysis of the resulting model (Figure A2-1). In the proposed framework, we can assume that the data come indistinctly from simulations or measurements. The only point to note is that in the latter case, stochastic identification methods will somehow be mandatory. Once the data source is determined, we face two options: MIMO black-box models or SIMO analysis of
multiple signals analysis. At this point, the specific method will depend on the assumptions on noise and excitation. When the free or pulse response assumptions hold reasonably well, and noise is not an issue (as in a simulation setup), ERA’s method seems a natural choice. In all other situations, stochastic methods apply: when there is no forced input, the stochastic subspace state-space algorithm (SSSID) is the natural choice [17-18], while the N4SID [12-14] algorithm lends itself to configurations with forced excitations and noisy measurements.

Figure A2-1: Framework for a combined simulation-measurement approach to electromechanical modes identification.

The result of the identification is a MIMO model (A,B,C,D) when dealing with input-output data, and a SIMO model (A,C) when dealing with output data only. In all cases, an eigenvalue analysis of the system matrix A provides the natural modes. Giving the state-space representation of the underlying phenomenon, additional information regarding structural modal properties is readily available, including the observability and controllability mode shapes [10, 28].

**A2.2.2 Test system: the Anderson and Farmer 9-bus system**

This test system was fully developed in [21] to mimic the dynamic behavior of a typical North-American interconnected grid with an installed generation capacity of 135,700 MW. It is a three-area network (Figure A2-2), with area 1, a medium-size control area totalling 5700-MVA generation for a 5000-MW peak load. The major share of the generation, 4400 MVA, is provided by a plant at bus 3, relatively close to the load. An additional 1300 MVA from a remote nuclear plant is connected radially to area 1, through a long 500-kV line C. Area 2 represents a large control area with 60,000-MVA generation for a 40,000-MW embedded load. It is connected to area 1 through two long lines A and B. The area 3 is the largest control area with 70 000 MVA of
generation to match a 50 000 MW peak load. It is also connected to area 1 by means of two 500-kV lines labelled F and G in Figure A2-2. The points “M” on the tie-lines mark the locations where inter-change power is measured for billing purposes.

In the present work, we have closely followed the data set described in Appendix B of [21]. Basically, the power plants have steam turbine generators with six-order synchronous machines and fast exciters modeled in detail. There is no stabilizer present in the original system and lines A, B and C are series-compensated at a 25% level. All the remaining lines are uncompensated. The model has been carefully built in the SimPowerSystems software from MathWorks/Matlab [41], and its results have been successfully validated against the sample time-domain plots shown in [21], although all loads are represented here by constant impedances. It is important to mention a 1000-Mvar capacitor at bus 7, which is mandatory to have the specified voltage profile. Notice that in contrast to [22], there is no SVC in dynamic studies, only the fixed capacitor is present. Also, in contrast to [22] which used phasor simulation mode, the present work uses discrete-time simulation mode with electromagnetic transients of network represented in full.
A2.2.3 Typical Applications of Electromechanical Modes identification

The first application is MIMO identification of a black-box open-loop model for investigating modal interactions and designing damping controllers [15, 22, 28, 32]. In the simplest scenario, the voltage references of the four generators in the system can be selected as inputs and the four rotor speed deviations as outputs. Applying to each input sequentially a short pulse 1% in magnitude and 0.4-s pulse excitation (Figure A2-3), we obtain the typical responses shown in Figure A2-4. These responses will later be used to develop a 4-input-4-output state-space model using the ERA method.
Another configuration of interest consists in exciting the four generators at their voltage reference, using band-limited random noise, and monitoring the resulting speed deviation responses [15, 16]. Sample input-output data simulated on the test network without PSSs are illustrated in Figure A2-5. Even if the random excitations are small and symmetrical around zero, the unstable system is unable to contain the slowly growing oscillations due to the impact of the initial step at t=1s. The data in Figure A2-5 will be used in section A2.8.2 to demonstrate the N4SID method and cross-validate its performance by comparison with the corresponding ERA-based model.

Next, three IEEE4B PSSs [29] were installed at generators 1, 2 and 4 to stabilize the system. Although the PSSs should be installed everywhere in real life, it was decided to leave generator 3 without a PSS to stress the system even more. The PSS tuning procedure and parameters are summarized in an appendix (Section A2.13) to make the paper as self-contained as possible. Under these conditions, a 0.5-s three-phase fault at bus 7, cleared by the outage of line C, resulted in the ringdown responses of Figure A2-6. This system is now clearly stable, thanks to the PSS. The modal characteristics of the ringdown of the four generators speed responses, including their mode shapes, will be analyzed in Section A2.8.3 using an ERA-based SIMO identification method.
In the last scenario, we considered the ambient power system response. To simulate this behavior, we decided to install two additional loads at buses 5 and 9. These loads were modeled as controllable three-phase current sources with constant amplitude and random phase. The amplitude is 1 kA (at 500 kV) while the phase modulation signal is a sequence of uniformly distributed random numbers filtered through a Butterworth low-pass filter with its 3-dB cut-off set at 1 Hz. PSSs are still present at generators 1, 2 and 4 but the gain of the generator 1 PSS is reduced by half to further stress the inter-area mode. Typical unfiltered ambient responses of this test system are shown in Figure A2-7 assuming that no excitation is applied to the grid other than the random load fluctuations. At the top of the figure, the speed deviation of generator 2 and 3 are illustrated while the spectrogram of generator 3 speed deviation is seen at the bottom. The latter is evaluated over a non-overlapping 15-s data block. The high variability of the spectrogram confirms that the amplitude and damping are changing quickly. The modal characteristic of this signal will be determined in section A2.8.4 using the TKEO-based MBMA approach of [20], with the SSSID [17-18] as the identification engine.
A2.3 Linear MIMO System Identification

The aim of state-space identification methods is to estimate a black-box model of the system without using any particular canonical parameterization. In general, such a model takes the form

\[
\begin{align*}
  x_{k+1} &= Ax_k + Bu_k + w_k \\
  y_k &= Cx_k + Du_k + v_k
\end{align*}
\]  

(1a)

The input vectors \(u_k \in \mathbb{R}^{n_u \times 1}\) and output \(y_k \in \mathbb{R}^{n_y \times 1}\) vectors are measured, but \(v_k \in \mathbb{R}^{n_v}\) and \(w_k \in \mathbb{R}^n\) are non-measurable observation and process noise vector sequences with a zero mean covariance matrix:

\[
E\left(\begin{bmatrix}v_p & y_q & v_q \end{bmatrix}^T \begin{bmatrix}v_p & y_q & v_q \end{bmatrix}\right) = Q S \delta_{pq}
\]  

(1b)

The identification problem can be stated as follows: given \(N\) input and output measurements \(\{u_1, \ldots, u_N\}\) and \(\{y_1, y_2, \ldots, y_N\}\) \((N \to \infty)\) and the fact that these two sequences are generated by an unknown linear time-invariant system of the form above, find the system order \(n\), the system matrices \(A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n_u \times n} \), \(C \in \mathbb{R}^{n_y \times n}, D \in \mathbb{R}^{n_y \times n_u}\) up to within a similarity.
transformation and the covariance matrices \( Q \in \mathbb{R}^{n \times n}, S \in \mathbb{R}^{n \times n}, \) and \( R \in \mathbb{R}^{n \times n} \). After the discrete-time system \( \{ ABCD \} \) is identified, its continuous-time counterpart \( \{ ABCD \} \) is reconstructed using discrete-to-continuous mapping, e.g. the inverse bilinear transform (c.f. the functions bilin.m in the MATLAB robust control toolbox or d2c.m in the control toolbox [41]).

The eigensystem realization algorithm and N4SID have their origins in the state-space realization theory [12-13]. This approach generally starts by organizing the input-output data in Hankel matrices (see equation 2 for a picture of a matrix having a so-called Hankel structure.) For instance, noting that the impulse response of (1) is given by

\[
H_0 = DH^{-1} = CA^{k}(Bk = 1, 2, \ldots, N),
\]

the associated Hankel matrix is defined at the reference time \( k>0 \) as follows:

\[
HA_{(i,j)} = \begin{bmatrix}
H_k & H_{k+1} & \cdots & H_{k+j} \\
H_{i+k} & H_{i+k+1} & \cdots & H_{i+k+j} \\
\vdots & \ddots & \ddots & \vdots \\
H_{j+k} & H_{j+k+1} & \cdots & H_{j+k+j}
\end{bmatrix} = \vartheta_{ij} \kappa^{-j} \varphi
\]

(2a)

where \( i \) and \( j \) are fixed integers, and

\[
\vartheta_{ij} = \begin{bmatrix}
C \\
CA \\
\vdots \\
CA^{i-1}
\end{bmatrix}, \varphi = \begin{bmatrix}
B & AB & A^{i-1}B
\end{bmatrix}
\]

(2b)

are the extended observability and controllability matrices, respectively. Similarly, any arbitrary input and output data sequences \( \{ u_k \} \) and \( \{ y_k \} \) can be collected into Hankel matrices \( U_{k(i,j)} \) and \( Y_{k(i,j)} \), with for instance:

\[
U_{k(i,j)} = \begin{bmatrix}
u_k & u_{k+1} & \cdots & u_{k+j} \\
u_{i+k} & u_{i+k+1} & \cdots & u_{i+k+j} \\
u_{j+k} & u_{j+k+1} & \cdots & u_{j+k+j}
\end{bmatrix}
\]

(2c)

Note that \( H_{k(i,j)} \) contains block matrices while \( U_{k(i,j)} \) and \( Y_{k(i,j)} \) contain block vectors.

### A2.3.1 Minimal Realization of Pulse Data

Starting from the sequence of impulse block matrices, \( \{ H_k \}, k=0, 1, \ldots, N \), the objective here is to construct a state-space representation in the form (1), whose impulse response matches the sequence \( \{ H_k \} \). To solve this problem, a simple idea very common in experimental modal
analysis [7] is to build two shifted Hankel matrices encompassing, in a compressed form, the system state matrices. According to (2a), we have:

$$H_{1_{ij}} = \Theta_{i}^T \Phi_{j} \quad H_{2_{ij}} = \Theta_{i}^T \Phi_{j} \Theta_{j}$$

(3)

If \(i >> n = \text{dim}(A)\), the singular value decomposition (SVD) of \(H_{1_{ij}}\) can be partitioned as follows:

$$H_{1_{ij}} = [U_{no} \, U] \, \Sigma_{o} \, [V_{n1}^T \, V_{n2}]$$

(4)

where \(\Sigma_{o}\) contains the \(n\) dominant singular values and \(UU_{n}^T =VV_{n}^T = I_{n}\), with \(I_{n}\), the \(n\)-dimensional unit matrix. Comparing the decompositions of \(H_{1_{ij}}\) in (3) and (4), it follows that minimal-dimension estimates of the observability and controllability matrices are given by:

$$\Theta_{i} = U_{n} \Sigma_{n}^{1/2} \quad \Phi_{j} = \Sigma_{n}^{1/2} \quad V_{n}$$

(5)

Using the expression of \(H_{2_{ij}}\) in (3) and thanks to the orthogonality of \(U_{n}\) and \(V_{n}\), the state matrix is obtained:

$$A = \Theta_{i}^T H_{2_{ij}} \Theta_{j} = U_{n} \Sigma_{n}^{1/2} \Sigma_{n}^{1/2} V_{n} \Sigma_{n}^{1/2}$$

(6a)

where the symbol \(^T\) denotes the pseudo-inverse. Using (3), the input matrix is given by the first \(m\) columns of \(\Phi_{j}\) while the output matrix equals the first \(p\) rows of \(\Theta_{i}\). Formally, we have

$$B = U_{n} \Sigma_{n}^{1/2} E_{n0}^{T} \quad C = E_{n} \Sigma_{n}^{1/2} V_{n}$$

(6b)

where \(E_{n0} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad O \quad \text{and} \quad E_{n0}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad O\) are two special matrices consisting of unit and null matrices of appropriate dimensions. Finally, recall that, when necessary, \(D \Theta_{i} \quad O\), although with proper data shifting, it is always possible to force \(D \Phi_{j} \quad O\).

Implementation of this algorithm may require Hankel matrices of impractically large dimensions when the system is stiff, \textit{i.e.} containing very low and very fast frequencies. The reason for this is that we then need a small sampling interval for the fast mode, combined with a comparatively long observation window for the slow mode. To lessen these heavy memory and computational requirements without compromising the structure of the realization algorithm, Juan and Pappa [6] proposed the use of a generalized Hankel matrix, which allows nonconsecutive data samples. This matrix is given by
\[ H_{A,(i,j)} = \begin{bmatrix}
H_k & H_{k+j} & H_{k+j} & \cdots & H_{k+j} \\
H_{k+j} & H_{k+2j} & H_{k+2j} & \cdots & H_{k+2j} \\
H_{k+2j} & H_{k+3j} & H_{k+3j} & \cdots & H_{k+3j} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
H_{k+1} & H_{k+j} & H_{k+j} & \cdots & H_{k+j}
\end{bmatrix} = \varrho_{rs}^{k-j}\mathcal{Q} \]  

(7a)

where \( \{i, l = 1,2, \ldots, r\} \) and \( \{j, l = 1,2, \ldots, s\} \) are arbitrary integers and \( \varrho_{rs} \) and \( \mathcal{Q} \) are generalized observability and controllability matrices:

\[ \varrho_{rs} = \begin{bmatrix} C \\ CA^i \end{bmatrix}, \mathcal{Q} = \begin{bmatrix} B & AB & A^{j-1}B \\ CA^{i-1} \end{bmatrix} \]  

(7b)

We notice that the formulation (2a-b) is a generalization of (7a-b) above. To assess the benefit yields for stiff systems, consider the settings: \( i_n = nl \) and \( i_{n+1} = nl \). The width of the data window involved in the generalized Hankel matrix (13) is then \( N_w = n_i \) \( N_j \). When \( n_i > 1 \) and \( n_j > 1 \), this observation window is much wider than that involved in the normal contiguous Hankel matrix (2a) which implicitly assumes \( n_p = 1 \) with \( N_w = n_r \) as a by-product. For the same sampling interval and size of \( H_{A,(i,j)} \) (i.e., memory requirement), the generalized Hankel matrix thus efficiently incorporates the larger observation windows required by systems with very slow inter-area oscillations. This point was discussed in some depth in [10, 20]. The MATLAB robust control toolbox [41] now offers an implementation of the MIMO realization algorithm in its basic formulation (i.e., using the contiguous Hankel matrix) but, from experience, its computational requirements make it impractical for systems as stiff as Hydro-Québec’s [9].

**A2.3.2 Subspace Identification from Random Data**

Although several variants of subspace identification methods have emerged in recent years, they all seem to be more or less related to the following fundamental algebraic relationship between Hankel matrices of input and output data [12-14]:

\[ Y_{k-1,j} = \varrho_{rs}^{k-1} X_k^{l} + \Gamma_{i} U_{k-1,j} + \Gamma_{i} W_{k-1,j} + V_{k-1,j} \]  

(8a)

The extended state-matrix is defined as

\[ X_k = \begin{bmatrix} x_{i+1} & x_{i+j} \end{bmatrix} \]  

(8b)

While
\[
\Gamma_i = \begin{bmatrix}
O \\
CO \\
CA^{\hat{\Sigma}} \\
CA \\
O
\end{bmatrix}
\]  
(8c)

and

\[
\Gamma^d_i = \begin{bmatrix}
H_0 \\
HH \\
H_{w^e} \\
H_0 \\
CA^{\hat{\Sigma}} B \\
CA B \\
D
\end{bmatrix}
\]  
(8d)

are lower block triangular Toeplitz matrices. Note that the latter are built from the system impulse responses \( \{H_k\} \). On the other hand, the block Hankel matrices of input and output-data vectors \( U_{i|k-1,j} \) and \( Y_{i|k-1,j} \) are defined in (2c) while \( V_{i|k-1,j} \) and \( W_{i|k-1,j} \) are defined similarly from the noise sequences \( w_k \) and \( v_k \).

Recalling that the first step in eigensystem realization was to find some estimates of the observability and controllability matrices from the Hankel matrices of the signals, it may come as no surprise that the same strategy also applies to N4SID. In this mathematical setup, subspace identification consists in estimating first the extended observability matrix and then the model parameters \( (n,A,B,C,D,Q,R,S) \).

Similarly to the ERA method, the extended observability matrix is estimated in the first stage through SVD decomposition of certain data matrix obtained by manipulating equation (8a) through orthogonal projections defined below.

**Definition:** Let us denote the orthogonal of the matrix \( B \in \mathbb{R}^{q \times j} \) by:

\[
B^\perp = I - B^T(BB)^+B = I - \Pi_B
\]  
(9a)

where \( (\cdot)^+ \) is the Moore-Penrose pseudo-inverse of the matrix \( \cdot \). The projection of the matrix \( A \in \mathbb{R}^{p \times j} \) on the row space of matrix \( B \) is

\[
A\hat{B} = AB^T(BB)^+B = A\Pi_B
\]  
(9b)

A direct corollary of this definition is that:

\[
A\hat{B} = AAB^T(BB)^+B = A\Pi_B
\]  
(9c)

In the stochastic framework, the projection operation is redefined by virtue of the substitution \( AB^\perp \leftarrow \Phi_{[AB]} \), where \( \Phi_{[AB]} \) is the covariance of \( (A, B) \). For example,

\[
A\hat{B} = \Phi_{[AB]} \Phi^{\perp}_{[AB]}B \quad \hat{A}B = A - \Phi_{[AB]} \Phi^{\perp}_{[AB]}B
\]  
(9d)
How can we extract an estimate \( \hat{\phi}_i X_{ij} \) from the fundamental equation (8a)? For this, we need the previously defined notion of orthogonal projection. By projecting the row space of \( Y_{k-1,j} \) into the orthogonal complement \( U_{k-1,j}^\perp \) of the row space of \( U_{k-1,j}^\perp \) we find:

\[
Y_{k-1,j}/U_{k-1,j}^\perp = \hat{\phi}_i X_{ij}/U_{k-1,j}^\perp + \Gamma' W_{k-1,j} + V_{k-1,j}
\]

(10a)

Given that the following relationships hold when the noise is uncorrelated with inputs:

\[
W_{k-1,j}/U_{k-1,j}^\perp = W_{k-1,j} \quad V_{k-1,j}/U_{k-1,j}^\perp = V_{k-1,j}
\]

(10b)

Equation (10a) can be further simplified by weighting it to left and right with matrices \( W_c \) and \( W_r \):

\[
W_r \left( Y_{k-1,j}/U_{k-1,j}^\perp \right) W_c = \Psi_c \left( X_{ij}/U_{k-1,j}^\perp \right) W_r + W_c \left( \Gamma' W_{k-1,j} + V_{k-1,j} \right) W_r
\]

(11)

In the N4SID approach of Overshee and De Moor [33-34], the following input-output data related weighting matrices are selected essentially to cancel term 3 above:

\[
W_c = I_n, \quad W_r = \left( Z_{ij}^\perp / U_{k-1,ij}^\perp \right) Z_{ij}^\perp
\]

(12a)

where

\[
Z_{ij}^\perp = \begin{bmatrix} Y_{ij}^\perp \\ \eta_{ij}^\perp \end{bmatrix}
\]

(12b)

An SVD of the right term, which is completely defined by the input-output measurements, then allow us to determine the extended observability and state matrices as follows:

\[
W_r \left( Y_{k-1,j}/U_{k-1,j}^\perp \right) W_c = \left[ U_n \quad U_o \right] \Sigma \left[ V_n^T \\ V_o^T \right]
\]

(13a)
With
\[
\vartheta_i = U Y_{n}^T = \vartheta_{i}^c
\]
\[
X_d = \Sigma_n V_n^T = X_d / U_{d(\tau = -1, j)} W_r
\] (13b)

From (2b), the matrix C is the first $n_y$ lines of $\vartheta_i$, while A is determined using the shift invariance property of $\vartheta_i$:
\[
A = \vartheta_i \vartheta_i^\perp, \quad C = \vartheta_i (1:n_y,:)
\] (14)

in Matlab notations, with $\vartheta_i$ and $\vartheta_i^\perp$ denoting $\vartheta_i$ without the first $n_y$ and last $n_y$ lines respectively and $\vartheta_i^\perp = \vartheta_i$:
\[
\vartheta_i = \begin{bmatrix} \vartheta_i \\
CA \end{bmatrix} = \begin{bmatrix} C \\
\vartheta_i \\
CA \end{bmatrix}^{-1}
\] (15a)

After determining A and C, the system matrices B and D are computed from the input-output equation (8a) as follows:
\[
\vartheta_i \mathbb{U}^\perp_{d(\tau = -1, j)} \mathbb{A} \mathbb{U}^\perp_{d(\tau = -1, j)} = \vartheta_i \cdot \Gamma^d
\] (15b)

Here again, the noise is cancelled out due to the assumption that the input $u_k$ is uncorrelated with the process or measurement noise. Observe that with known matrices $\mathbb{A} \mathbb{C}$, $\vartheta_i^\perp$, $U_{d(\tau = -1, j)}^\perp$, and $Y_{d(\tau = -1, j)}$ this equation is linear in B and D. These matrices can thus be obtained through a least-squares solution of (15b). The residuals are given by [12]:
\[
\rho = \begin{bmatrix} U_{d(\tau = -1, \perp)} \\
Z_{d(\tau = -1, j)} \\
X_d \end{bmatrix} \begin{bmatrix} \mathcal{V}_{d(\tau = -1, j)} \\ d(\tau = -1, j) \end{bmatrix}
\] (16a)

where
\[
\mathcal{Z} Y_{d(\tau = -1, j)} = X_d / U_{d(\tau = -1, j)}
\] (16b)
and $W_{d_{k,j}}$ and $V_{d_{k,j}}$ are block Hankel matrices with entries $w_k$ and $v_k$. The noise covariance matrices are finally determined from the residuals as follows:

$$
\frac{1}{j} [\Phi] \Phi^T = \begin{bmatrix}
Q_S \\
S \tilde{R}
\end{bmatrix}_{\eta_p}
$$

(16c)

### A2.4 Linear SIMO Signal identification

Without any loss of generality, a time series given in the form $\{y_k, k = 1,2, \ldots, N\}$, with $y_k$, a column vector of dimension $n_y$, can be considered the output of a linear filter with suitable input excitation [5]. If the signal is deterministic, such as a ringdown power system response for example, a convenient input is the discrete impulse. On the other hand, if the signal is stochastic, like the ambient power system response to random load switching, the input is naturally a random noise excitation. Whatever the assumption, it is convenient to assume a single input for the multiple signals in order to derive a more compact transfer function representation with a single denominator. The signal will therefore share the same poles but for each pole, the residue will vary with respect to the output.

![Figure A2-8: Application of ESA on multi-component AM-FM signals.](image)

For illustration, consider the following ringdown signal:

$$
y(t) = Ae^{-\sigma t} \cos(\omega t + \psi), \quad t \geq 0
$$

(17a)

which is the impulse response of the following transfer function:

$$
H(s) = \frac{(sd^2 + \omega^2) \cos(\psi)}{(sd + \omega^2)}
$$

(17b)

with $s$, the Laplace Transform symbol [38]. Using the Tustin method of discretizing a continuous time transfer function, the relationship between $H(s)$ and $H(z)$ in Figure A2-8 becomes obvious.
A2.4.1 Deterministic SIMO

Assuming the transfer functions in Figure A2-8 are proper, i.e. the numerator polynomials orders are strictly less than the order of the denominator polynomial), the deterministic SIMO model of the system is equivalent to the following state-space model: \( \{ABCD, = 0\} \). If \( \{y_k, k = 1, 2, \ldots, N\} \) are assumed to be measurements of the impulse responses describing the system \( \{ABCD, = 0\} \), this state model can be identified using the ERA method described in section A2.3.1. Therefore, the only assumption here is that the measurements can be generated by some hypothetical linear time-invariant system.

A2.4.2 Stochastic SIMO

The state-space representation of the SIMO stochastic model of the signals is obtained by setting the forced input \( \{u_k, k = 1, 2, \ldots, N\} \) equal to zero in (1):

\[
\begin{align*}
\begin{bmatrix} x_{k+1} \\ y_k 
\end{bmatrix} &= \begin{bmatrix} A & W_k \\ C & V_k \end{bmatrix} \begin{bmatrix} x_k \\ v_k 
\end{bmatrix} \\
\{x_0, y_0\} &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}
\end{align*}
\]  

(18)

The process noise \( w_k \) is the input that drives the system dynamics whereas the measurement noise is the direct disturbance of the \( n \times 1 \) system response vector \( \{y_k, k = 1, 2, \ldots, N\} \), which is therefore a mixture of the observable part of the state and some noise modeled by the measurement noise \( v_k \). The state matrices \( \{AB = 0, CD = 0\} \) can be determined using a simple modification of the algorithm in section A2.3.2. To avoid confusion, we will name it the stochastic subspace state-space identification (SSSID). Basically, it consists in replacing the fundamental equation (8a) by the following [18]:

\[
\begin{align*}
W_r \left( \frac{A_{k+1,j}}{Y_{d_{k+1,j}}} \right) W_r &= \Psi_{r,j} \left( \frac{X_{d_{k+1,j}}}{Y_{d_{k+1,j}}} W_r \right) \\
&\quad + W_r \left( \Gamma_{r}\left(\frac{W_{d_{k+1,j}}}{V_{d_{k+1,j}}} + V_{d_{k+1,j}}\right) W_r \right) \\
&= \Phi_{r,j} \left( \frac{Y_{d_{k+1,j}}}{Y_{d_{k+1,j}}} \right) W_r \\
W_c &= I_{m}, \quad W_r = Y_{d_{k+1,j}}^T \Phi^{-1}_{d_{k+1,j}} \left[ \begin{bmatrix} \Phi_{d_{k+1,j}}^{-1} \end{bmatrix} \right]_{k+1,j} Y_{d_{k+1,j}} \\
\end{align*}
\]  

(19a)

where the input-output data-related weighting matrices are selected essentially in order to cancel term 3 above:

\[
W_r = I_{m}, \quad W_r = Y_{d_{k+1,j}}^T \Phi^{-1}_{d_{k+1,j}} \left[ \begin{bmatrix} \Phi_{d_{k+1,j}}^{-1} \end{bmatrix} \right]_{k+1,j} Y_{d_{k+1,j}} \\
\]  

(19b)

The matrices A and C result again from the SVD of the data matrix \( W_r \left( \frac{A_{k+1,j}}{Y_{d_{k+1,j}}} \right) \). Then, the noise covariance matrices are derived from the residuals, as in section A2.3.2, by cancelling all terms involving the forced input excitation:
A2.5 Modal Characteristics from (A,B,C,D)

The first step is to convert the discrete time state space model of the previous sections into an equivalent continuous time model using the inverse bilinear transform or any other suitable mapping function between the two domains. For instance, the following textbook formulas can be used:

$$\hat{A} = \frac{\log(A)}{\Delta t}, \quad B = F_{x} \left( \hat{A}^{-1} (B - I) \right)^{t},$$
$$\hat{C} = C \ast \Delta t, \quad D = D \ast \Delta t$$

where $\Delta t$ is the sampling rate and the matrix logarithm is computed as in [35]. Naturally, Matlab contains all the necessary ingredients to achieve this process. In the sequel we will assume for the sake of presentation simplicity that the system is proper, i.e. $D=0$. In addition to being verified in most cases of interest, this assumption on the static gain between input and output has no impact on the system’s modal characteristics. Transforming the continuous time system into modal space results in the following alternate representation:

$$\left\{ \begin{array}{c} \hat{A} \\ \hat{C} \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \hat{A} \\ \hat{C} \end{array} \right\}$$

where $\Lambda = \text{diag}(\lambda_{s}, \ldots, \lambda_{s})$ is a diagonal matrix of eigenvalues and $\Psi_{y}$ a matrix whose columns are the corresponding eigenvectors. $\Sigma = \Psi^{-1}B$ is the modal shapes matrix and $\Pi = C\Psi$, the initial modal amplitudes matrix [6-7]. From these definitions, the residues associated with the $k^{th}$ mode can be collected in a $n_{k} \times n_{k}$ dimensional matrix $R_{k}$ with entries

$$r_{y}(k) = \Pi (ik) \times \Sigma (k, j) = A_{k} \phi_{k} \Psi_{y}$$

where $i \bar{F} 1, 2, \ldots, j \bar{F} 1, 2, \ldots, k \bar{F} 1, 2, \ldots$, and $n$ is the number of distinct natural modes of $\Lambda$, with damping $\sigma_{k}$ (sec$^{-1}$) and natural frequency $\omega_{k}$ (rad/sec). The corresponding transfer function matrix takes the form:

$$\mathbf{G}(s) = \sum_{k=1}^{n} \sum_{s=1}^{n} \Lambda_{s} \phi_{s} \left[ \frac{1}{s - \sigma_{k} \pm j \omega_{k}} \right] \mathbf{B}_{y}(s)$$

where $Z_{k}^{2 \pi \tau_{1}, j} = Y_{k}^{2 \pi \tau_{1}, j} / Y_{0}^{2 \pi \tau_{1}, j}$.
To facilitate modal recognition and interpretation, the results in the sequel are expressed mainly in terms of the zero-initial condition impulse response components (inverse Laplace transform of $g_s(t)$) according to the following formula [38]:

$$h_i(t) = \sum_{k=1}^{\hat{K}} 2A_k \epsilon^{s_i k} \cos (\omega_k t - i_j(k))$$

(24)

Besides its direct involvement in the above Prony decomposition of the impulse responses of the system, the residue matrix allows us to assess the strength of a signal or the performance of a controller with respect to a given mode “$k$” using the residue-based measures of controllability $\bar{m}_k$ and observability $\bar{m}_k$, expressed in a normalized form as follows [10]:

$$\begin{align*}
\bar{m}_k &= \frac{\|R_i(\cdot, \cdot)\|}{\|R_i\|} (im^1, \ldots, ) \\
\bar{m}_k &= \frac{\|R_j(\cdot, \cdot)\|}{\|R_j\|} (jm^1, \ldots, )
\end{align*}$$

(25)

If $\bar{m}_k = 0$, then mode “$k$” is uncontrollable from input $i$. If $\bar{m}_k = 0$, then mode “$k$” is unobservable from output $j$. The joint observability/controllability measure, conventionally defined as $m_k = m_k \times \bar{m}_k$ [9-10, 28], is a system-wide quantity, not to be confused with $\|R_i(j, i)\|$, even though they are related to each other. When $R_i(j, i)$ vanishes, we can conclude that the control input $i$ has no influence on the $k$th mode response if the feedback controller relies on the measured output signal $j$. In practice, the residue $R_i(j, i)$ will not vanish completely but will diminish gradually to a small value for which the system is still observable/controllable using larger and larger feedback gains in order to counteract the weakness of the residue. In fact, the well-documented detrimental effect of small residue in feedback control can be understood by considering the sensitivity of the $k$th eigenvalue to the gain $K$ of a proportional feedback controller:

$$\frac{d\lambda_k}{dK} = \|C_{ef} \bar{R}_i \bar{B} \| \|R_i\|$$

(26)

It happens that the residue is simply the modal sensitivity of the constant gain controller, meaning that a small joint observability/controllability measure or residue norm creates the need for large feedback gains, which are generally associated with gain and phase margin limitations [10, 28], increased sensitivity to time-delays, noise and, in some situations, control saturation during large disturbances.
A2.6 Multi-Band Modal Analysis (MBMA) for Improved Accuracy and Resolution

Although stochastic state-space methods can deal with process and measurement noise, their performance can still be improved on ambient noise responses, when band-pass pre-filtering is applied to the incoming signal before the identification stage. Recently, a new concept of linear filter bank pre-processing was proposed by the authors to deal with this issue in the context of closely spaced natural modes embedded in significant noise [20]. In addition, filter bank processing makes it possible to determine the energy channel by channel, using the TKEO concept surveyed in an appendix (Section A2.11) to test the level of within-band energy or activity before even attempting any modal identification on that channel. This approach, which performs the modal identification separately for each frequency band provisional to the within-band energy exceeding a given threshold, was shown to be effective in minimizing the ambiguity of modal parameters in the presence of significant noise, while offering a relatively fast response time to modal parameter changes. In this section, we will review the MBMA approach of [20], assuming the linear filter bank in the appendix (Section A2.11) and ERA or SSSID as our identification engines.

A2.6.1 Outline of the MBMA method

The power system response signals can generally be described as the sum of the combined amplitude (AM) and frequency (FM) modulated primitive signals:

\[
x(t) = \sum \sum t_k \cos(\Theta_k(t)) = a_k e^{\sigma_k t} \cos(\omega_k(t + \phi_k))
\]  

(27)

where \(a_k\), \(\omega_k\), \(\sigma_k\) and \(\phi_k\) are respectively the amplitude, frequency, damping and phase of the \(k\)th modal component. This is essentially a single signal version of (24). It should be stressed that all parameters \(a_k\), \(\omega_k\), \(\sigma_k\) and \(\phi_k\) can be slowly time-varying without posing any significant complication to the ongoing development. Figure A2-9 illustrates the overall concept of first decomposing the multi-component power system signal (27) using a filter bank, and then performing a time-frequency analysis on each channel component using ESA (cf. appendix, Section A2.12) or alternatively, the Hilbert transform [24, 37].
The ESA provides the amplitude and frequency of the dominant modes with decent accuracy after filter-bank preprocessing (as in Figure A2-9) but when accurate damping information is required or the filter bank output signal is not monochromatic enough, a more detailed modal analysis, using a parametric method for instance, should be preferred to the ESA. The overall scheme is shown in Figure A2-10, assuming that ERA is the electromechanical-mode identification tool, even though any alternative identification method could fit just as well [23-26]. An optional DHT [23] is first applied to the incoming signal to reject quasi-steady state components more efficiently. The filter bank then splits the possibly multi-component signal, as in (27), into \( N=9 \) essentially orthogonal components.

After decomposing the incoming signal, each channel’s energy is computed using TKEO and \( N_d \) (typically \( N_d \leq 4 \)) energy-dominant signals are selected for parametric modal analysis based on an energy threshold test. The modal analysis is then performed on sliding non-overlapping data blocks whose size can be adjusted for each channel, according to the channel center frequency.
For instance, a 5-s data window suffices for filters centered in the range 1 to 3 Hz while a 20-s data window may be preferable for analyzing the output signals of filters in the range 0.1 to 1 Hz.

**A2.7 Software issues**

Since the early nineties, the Planning Department at Hydro-Québec has been using an in-house ERA and N4SID-based software for various purposes including eingenstructure discovery [10, 28], wide-area measurements-based damping controller design [32], conventional PSS tuning [42], and automated stability simulations analysis [9, 27]. The typical study framework is illustrated in Figure A2-11.

![Figure A2-11: Conceptual view of the modal analysis study process](image)

The process comprises three main steps as follows:

1. A stability interface (Figure A2-12a) is first used to set the system configurations in terms of the load-flows patterns and control systems included (e.g. PSS in or out). The test conditions are also defined in terms of pulse location, duration and magnitude. Hydro-Québec has developed graphical interfaces for its internal stability program ST600, PSS/E (from Siemens-PTI) and EMTP [27]. In this paper, all the studies reported were performed using a non-graphical SimpowerSystems interface procedure.

2. A power system simulation is then performed using the appropriate software and the selected results exported to Matlab. It should be noted that the approach is power system simulation software agnostic as it works equally well for PSS/E, EMTP or SimPowerSystems.

3. A Matlab GUI (Figure A2-12b) is then invoked (automatically upon completion of the necessary power system stability simulations) in order to perform the MIMO state-space identification. As shown in Figure A2-11, this last stage offers extensive model validations, along with many state-space model post-processing tools to visualize the underlying damped sinusoid parameters, observability/controllability mode shapes, frequency domain plots, etc.
In the above implantation approach, N4SID and ERA are programmed in Matlab, taking full advantage of its powerful matrix computation capabilities. While the system identification toolbox contains a version of the N4SID algorithm at a cost, free Matlab codes abound also, with no tie to any commercial toolbox [13, 30-31, 33]. A very simple ERA code is available on the Matlab file exchange website but, to our knowledge, there is no MIMO version of the ERA method. In some applications, the Matlab heritage becomes a burden. SLICOT is a standalone FORTRAN 77 package which can be used to address the issue of system identification itself using the N4SID approach [34] but even so post-processing the results and presenting them convivially will remain a big issue because SLICOT is a “plain vanilla” numerical engine.

(a) Stability Interface (Siemens-PTI PSS/E example)

(b) Matlab interface for ERA based MIMO identification

Figure A2-12: Software shot screens
A more promising approach for dealing with the portability issue was explored recently in [27] where all the code necessary to perform the ERA-based state-space identification and matrix post-processing was implemented in C and then interfaced to a powerful host program, ScopeView, which is used by EMTP-RV as companion waveform visualization software. In Figure A2-13, the specialized algorithm (PRONY/ERA) is built on top of a standard open-source matrix package LAPACK. A key issue easily resolved in Matlab using the function \textit{d2c.m} but which is actually a difficult numerical problem is that of computing the logarithm of the state matrix (c.f. the \textit{logpack} module in Figure A2-13), required to convert the system from the discrete to a continuous time domain (20).

Figure A2-13: Portable state-space modal analysis tool (amod.c) embedded in a commercial host program [27].
Table A2-I: Malin-round Mountain Modified Natural Response

<table>
<thead>
<tr>
<th>Mode No</th>
<th>$\alpha_i$ (%)</th>
<th>$\zeta_i$ (%)</th>
<th>$f_i$ (Hz)</th>
<th>$\phi_i$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37805</td>
<td>10.938</td>
<td>0.35366</td>
<td>161.95</td>
</tr>
<tr>
<td>2</td>
<td>0.14787</td>
<td>6.2974</td>
<td>0.64921</td>
<td>177.38</td>
</tr>
<tr>
<td>3</td>
<td>0.32807</td>
<td>17.668</td>
<td>0.69102</td>
<td>159.93</td>
</tr>
<tr>
<td>4</td>
<td>0.06310</td>
<td>1.2010</td>
<td>0.75888</td>
<td>201.78</td>
</tr>
<tr>
<td>5</td>
<td>0.08225</td>
<td>8.2685</td>
<td>0.82130</td>
<td>138.29</td>
</tr>
<tr>
<td>6</td>
<td>0.27956</td>
<td>13.320</td>
<td>1.0715</td>
<td>35.74</td>
</tr>
<tr>
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<td>1.3944</td>
<td>-71.68</td>
</tr>
<tr>
<td>8</td>
<td>0.09780</td>
<td>6.9829</td>
<td>1.5954</td>
<td>-1.475</td>
</tr>
</tbody>
</table>

A2.8 Applications

A2.8.1 ERA-based Modal Analysis of a Single Ringdown Signal

The conventional ERA-based modal parameter estimation will be tested first on a noise-free multi-component signal with several closely spaced natural modes. The signal is the result of the Prony analysis of a measured power system response used in [7, 20] as a typical case for assessing modal analysis methods. The parameters shown in Table 1 contribute to the signal as follows:

$$x(t) = \sum_{i=1}^{n} a_i \exp\left(2\pi f_i t\right) \cos\left(\sqrt{1-\zeta_i^2} 2\pi f_i t + \phi_i\right) \tag{28}$$

with obvious relationships between the modal parameters in (24) and (28), assuming a single-input single-output signal ($m_h = 1$) system. The results of a conventional ERA analysis of this signal (28) are presented in Figure A2-14. The sampling frequency is 40 Hz, with an assumed system order $n=20$. After Prony decomposition of the state-space model as in (24), a filtering is performed to reject as non-relevant all modes with damping higher than 30% and amplitude lower than 0.1% of the maximum modal amplitude. Under these assumptions, conventional ERA-based modal analysis is quite accurate in spite of the many closely spaced modes present in the signal (28).
A2.8.2 MIMO System Identification of the 9-Bus Test System

This application considers the MIMO identification of the \((A,B,C,D)\) model required for designing wide-area measurements-based PSS for the 9-bus test system. To this end, we first apply the ERA method to the pulse responses described in section A2.3.1. A 0.1% pulse of 0.25-s duration was applied sequentially to the AVR’s summation point of each generator 1, 2, 3, and 4. Application of the ERA method to the 4x4 sequence of pseudo-Markov parameters recorded this way resulted in an accurate 12-order 4-input/4-output model. Given the discrepancy between the 0.25-s pulse and a perfect discrete impulse, the correction procedure described in [10] was applied to the original ERA-based state-space model \((A,B,C)\). Figure A2-15 illustrates the goodness-of-fit achieved for a selected input-output pair. The other combinations were similarly accurate. The caption of the Bode plot lists the dominant modes in the Prony decomposition of the time-response signal shown (with the highest amplitude mode listed first). The filtering criterion was to: retain only complex modes with a relative damping less than 0.35 and amplitude greater than 1/50th that of the strongest mode. On this signal response, the criterion resulted in the section of four dominant modes (illustrated in the figure) even though the model was of order 12. Next, the N4SID algorithm in section A2.3.2 was applied to the same 4-input/4-output configuration with the random excitations in Figure A2-5. The model performance for a system order \(n=12\) and \(n=50\) is illustrated in Figure A2-16. Two performance aspects are considered: the simulation performance which consists in replaying the model \((A,B,C,D)\) in a deterministic manner and the prediction performance, which takes into account the Kalman filter component. It happens that a low-order model can provide quite accurate predictions while the simulation is somewhat inaccurate. This problem is well known in subspace identification where the models from many proposed variants of the N4SID are often weak in simulation mode [31]. However, when the order is increased to \(n=50\), the simulation performance improves drastically at the expense of the physical significance of the model.
Figure A2-15: Controllability mode shapes without PSS. Input (1,2,3) = voltage reference of generator (1,2,3,4).

Figure A2-16: Controllability mode shapes without PSS. Input (1,2,3) = Voltage reference of Generator(1,2,3,4).

In Figure A2-18, the 4x4 ERA and N4SID models with orders n=12 and n=50 respectively are compared in the frequency domain. Interestingly, the two models largely agree at high frequency (> 0.06 Hz) but they are not consistent at very low frequency. Normally, as discussed in [10], the ERA should be better at modeling the DC behavior because the spectrum of the 0.25-s pulse is relatively strong at very low frequencies.
Figure A2-17: Observability mode shapes without PSS: Output (1,2,3,4) = speed Deviation of generator (1,2,3,4).

Figure A2-18: Controllability mode shapes without PSS. Input (1,2,3,4) = voltage reference of generator (1,2,3,4).
The application of residue-based controllability and observability measures on the ERA-based 4x4 MIMO model results in the mode shapes in Figures A2-18 and A2-19. Two local modes are clearly identifiable: the mode at 1Hz, involving generator 3, and the mode at 1.4 Hz, involving generator 2. The inter-area mode at 0.24 Hz involves generators 1 against 4 while the last mode at 0.06 Hz, is essentially a so-called global oscillation mode, equally observable at all generators, but better controllable at the largest generators 1 and 4. According to Figure A2-16, the modes at 0.24 and 1.4 Hz are unstable. Therefore, local PSSs should be installed in priority at generators 1 and 2 which show the best joint controllability/observability (or equivalently the highest residue’s magnitude) for modes 0.24 and 1.4 respectively. Given that the generator with the highest controllability also has a good observability of the given unstable mode, this analysis shows that there is actually no need for a wide-area information exchange in order to damp the unstable modes through excitation modulation.

A2.8.3 Multiple Ringdown Signals Analysis

Since the test system is unstable, we installed IEEE4B PSS at generators 1, 2 and 4. This was enough to stabilize all the system modes. A single set of PSS parameters (cf. appendix, section A2.13) was assigned to all locations. A three-phase fault was then applied on line C at bus 7 for 0.5 s, followed by the outage of the faulted line. The system remained stable under such conditions and the speed deviations of the four generators were recorded for determining the post-fault modal characteristics of the system. Applying the SIMO version of the ERA method to these ringdown signals resulted in an accurate single-input four-outputs state-space representation (A,B,C) with an order n=14.
Figure A2-20: Prony decomposition of speed deviation of generators 2 and 4 in response to a 0.5-s fault at bus 7 with IEEE PSS4B at generators 1, 2 and 4.

A sample illustration of this SIMO model goodness-of-fit is shown in Figure A2-20 for the responses of generators 2 and 4. The simulation is simply the impulse response of the ERA model (A,B,C) starting from zero initial conditions. On each plot, the modal parameters are shown in decreasing amplitude and the filtering is done according to the same rule as in the previous section: retain a mode if its damping is less than 0.3 or its amplitude is greater than 1/50 that of the top mode in the list. To further check the SIMO model accuracy, the next figure, Figure A2-21, superimposes the power spectrum density of the actual and reconstructed fault responses. This amounts to comparing the spectrum of the two signals in each plot of the previous figure. The results of Figure A2-21 confirm that the SIMO model with 14 eigenvalues can simultaneously reproduce the four post-fault speed spectra in the 0-5-Hz frequency range.

Figure A2-21: Normalized PSD of actual versus modeled speed responses to a three-phase fault at bus 7.
The last illustration (Figure A2-22) pertaining to this application shows the observability mode shapes for the six dominant complex eigenvalues of the SIMO model. The common frequency mode (0.06 Hz) is observed with the same phase and magnitude at all generators, while the inter-area mode, which is shifted from 0.22 Hz to 0.12 Hz after the line outage, involves generators 1 and 2 against 3 and 4. The post-PSS local mode of generator 2 is 1.36 Hz while the local mode of generator 3 (without PSS) remains unchanged at 1 Hz. The mode at 1.92 Hz involves generators 2 and 3 while the last mode, 2.74 Hz, sited at generator 2, seems too weak to have any special significance.

A2.8.4 Estimation of Synthetic Time-Varying Signals in Noise

To better establish the performance of the stochastic state-space based modal analysis on time-varying ambient signals, we will first consider the following synthetic time series:

$$\gamma(t) = \sum_{k=1}^{3} A_f(t) \sin(2 \pi f_k(t) \times t + \theta_k(t) \times \pi/180) + \nu(t)$$

(29a)

where $\nu(t)$ is a white noise process and

$$A_f(t) = \hat{A}_f \ p.u. \ \theta(t) \ 0 \deg$$

$$f_k(t) = f_0, \ \theta(t) = 0 \deg \ \forall f \geq 0$$

$$A_f(t) = \hat{A}_f \ pu \ f \ 1.5 \ Hz$$

(29b)

The first component is a frequency chirp, ramped from 0.15 Hz to 0.4 Hz in 400 s:

$$f_1(t) = \begin{cases} 
0.15, & t < 40 \\
0.25 + (t - 40) \cdot 0.15, & 40 \leq t \leq 440 \\
0.40, & t > 440 
\end{cases} \ Hz$$

(29c)
The second component is a fixed un-damped sine (at 1Hz), whose amplitude ramp-modulated starting at 260s:

\[
\begin{align*}
A_1(t) &= 0.5, & 260 \\
A_2(t) &= \frac{0.5}{200} (t - 260) + 0.5, & 260 \leq t \leq 440 \\
A_2(t) &= \frac{0.5}{200} \times 220 + 0.5 \times 0.71667, & t \geq 440
\end{align*}
\] (29d)

The last term is fixed un-damped sine (at 1.5 Hz) whose phase is ramp-modulated beginning at 260 s:

\[
\begin{align*}
\theta_1(t) &= 0, & 260 \\
\theta_2(t) &= t - 260, & 260 \leq t \leq 600 \\
\theta_2(t) &= 40, & 600
\end{align*}
\] (29e)

Figure A2-23 displays the above time series (top) and its time-varying spectrum (bottom). Random white noise was added so as to achieve a 15-dB signal-to-noise ratio. The power spectral density is computed every 15 s using a non-overlapping data block on which a Kaiser window is applied before the Fourier transform.

The linear filter bank described in the appendix (Section A2.12) is first applied to the signal. According the TKEO energy threshold criterion the dominant channel output signals are shown in Figure A2-24 along with their energy. Figure A2-24 provides the time-varying frequency and amplitude according to the TKEO theory included as a reminder in appendix (Section A2.11). Even under a 15-dB signal-to-noise ratio, the results shown are clearly in accordance with the analytic signal definitions (29). However, the results for the low-frequency channels, FB1 and FB2, need some discussion. Initially, the lowest-frequency component was 0.15 Hz, which is the center frequency of FB1 while at t = 300 s, the same frequency has been ramped up to 0.3 Hz, which is now the center frequency of FB2. Therefore, the TKEO amplitude of FB1 smoothly transitions to zero while the amplitude of FB2 increases to its maximum value of 1/3. The multi-bank analysis relies on TKEO energy to detect the time for channel change, and this is reflected in the frequency plot at the top of Figure A2-24: the frequency estimate is from channel 1 (black) up to about t=200 s and thereafter, it is from channel 2 (magenta).
The SSSID in section A2.4.2 was applied to 15-s consecutive non-overlapping blocks of noisy data filtered through the 9-channel filter bank described in appendix (Section A2.12). A threshold was applied to the TKEO energy [20], in order to select just the four dominant channels in each given 15-s data block under analysis for further modeling. The SSSID algorithm was performed on a SISO basis, for each selected channel, using the parameters i=25 and n=3. After identifying matrices A and C, the extraction of modal information according to section A2.5 yields the results illustrated in Figure A2-26. It is obvious that, despite the low 15-dB SNR and the multiple time-
varying parameters of the signal, the estimates of mode frequency are very accurate. The amplitude and damping are more uncertain during periods of rapid frequency change, especially when the dominant channel changed from FB1 to FB2. Interestingly, the damping and amplitude are not significantly affected by the phase modulation within the ramping rate considered (1 deg/s). It should be noted that, in Figure A2-26, amplitude attenuation due to the pass-band response of the filter bank is not compensated (in contrast to [20].)
A2.8.5 Analysis of Simulated Ambient Power System Response

Now that we have verified the method effectiveness in tracking known time-varying modal characteristics, we will apply it blindly to the analysis of the signal in Figure A2-7, which mimics a power system ambient response to ambient noise. The same MBMA algorithm using the SSSID algorithm for signal modeling was applied on the four channel filters with the highest TKEO-based energy, assuming the parameters \(i=25\) and \(n=3\). Figure A2-27 illustrates the distribution of frequency, damping and amplitude obtained throughout the 640-s time-frame. Using a time-series representation, Figure A2-28 presents the same result as in Figure A2-27.

While Modes 1, 2, and 4 are easily related to the closed-loop responses in section A2.8.3, Mode 3...
seems to result from the random load-switching process. The key observation overall is that the frequency estimates are remarkably stable while the damping and amplitude variances are high. However, assuming time-invariance of the modal parameters over time windows longer than 15 s, it is possible to smooth these results by averaging four consecutive values to produce a single reading every minute.

Figure A2-27: Simulated ambient noise from the nine-bus test system: distribution of the dominant electromechanical modes over a 640 s time frame.

Figure A2-28: Simulated ambient noise from the nine-bus test system: features time-series of the dominant electromechanical modes over a 640-s time frame.
A2.9 Conclusions

State-space identification has been maturing for almost two decades. While still less accurate than the classical prediction errors method [13, 41], they have many advantages, making them well suited for electromechanical modal identification; Non-iterative, linear matrix computations-based, MIMO-enabled methods, they can effectively tackle large dimensional problems without breaking down answerless [34].

This paper attempted, successfully we hope, to achieve three main objectives:

Unified presentation of the two main state-space identification methods hitherto used in power systems applications, namely the ERA, which targets pulse responses in deterministic setups, and the N4SID, which addresses random forced or unforced responses with measurement and process noise eventually included.

Unified approach to measurement- and simulation-based model and signal identification, considering within the same conceptual framework various engineering problems justifying the need for electromechanical modal identification, such as black-box modeling for power system eigenstructure discovery or damping controller tuning, Prony decomposition of ringdown response signals for damping assessment, real-time monitoring of small-signal stability conditions from ambient load-switching noise, etc.

Illustration of all the above facets of the electromechanical modal identification problem with the same simple 9-bus test system [21], scheduled to be available as a demo in a future release of the SimpowerSystems, a Matlab Simulink module developed in partnership with Hydro-Québec’s Research Institute [41].

Although real-time monitoring based on wide-area measurements has attracted a lot of interest recently, most researchers seem to favor identification approaches based on transfer function models [8, 23-26]. With this paper, we hope to generate as much activity in matrix approaches by demonstrating how off-the-shelf state-space software based on ERA or SSSID, combined with efficient filter-bank preprocessing, results in robust electromechanical identification schemes. These so-called multi-band modal analysis (MBMA) schemes are shown to be able to update the modal estimates at faster rates than correlation-based Prony analysis methods [19] without the strict convergence monitoring required by recursive least-squares methods [40].

A2.10 Appendix 2 References


A2.11 Appendix 2A: the Teager-Kaiser Energy Operator

As introduced in [20, 36] the Teager-Kaiser energy operator is highly effective for assessing the energy and detecting AM and FM signals in (27):

\[ \Psi_x[x(t)] = [x(t) \int x(t) dt] \]

with \( x = dx/dt \) and \( x = dx/dt \) the first and second derivatives of \( x \), while \( \Psi_x[x] \) is the instantaneous energy of \( x \). In fact, denoting by \( x_i(t) \) the \( i^{th} \) component of the AM-FM signal in (27) and assuming a constant initial phase \( \Phi \), it is proven that [36]:

\[ \Psi_x[x_i(t)] \approx A_i^2 \Omega_i(t) \]

with a negligible approximation error under quite general and realistic conditions. This motivated our use of the energy separation algorithm (ESA) for tracking the instantaneous amplitude and frequency, as follows:

\[ A_i(t) = \frac{\Psi_x[x_i(t)]}{\sqrt{\Psi_x[x(t)]}} \quad \text{and} \quad \Omega_i(t) = \frac{\sqrt{\Psi_x[x_i(t)]}}{\sqrt{\Psi_x[x(t)]}} \]

Note that, in the case of a monochromatic signal with constant \( A_i \) and \( \Omega_i \), the relationships (5) are exact. When the continuous energy operator (3) is sampled with a sampling period \( T \), the following equivalent discrete energy operator is obtained [36]:

\[ \Psi_d[x_i(n)] = x^2(n) - x_{i-1}x_{i+1} + 1 \]

This produces

\[ \Psi_d[x_i(n)] = A_i^2 \Omega_i(n) \sin \left[ \Omega_i(n) \right] \]

with \( \Omega_i(n) \) \( \Omega(nT) \). By applying \( \Psi_d \) to both \( x(t) \) and its backward difference, \( y_i(n) = x_i(n) - x_i(n-1) \), the discrete-time ESA was developed in [13,14]:
\[ \Omega_{x}(n) = \arccos \left( -\frac{\Psi_{x}[y_{j}(n)] + \Psi_{x}[y_{j}(n + 1)]}{4\Psi_{a}[x(n)]} \right) \]
\[ A(n) = \sqrt{1 + \left( \frac{\Psi_{x}[y_{j}(n)] + \Psi_{x}[y_{j}(n + 1)]}{4\Psi_{a}[x(n)]} \right)^2} \]

(34)

ESA can provide the instantaneous amplitude and frequency with a single sample delay in sharp contrast with the DHT [24, 37] and other block-processing schemes. However, involvement of the signal derivative in (34) also points to a high sensitivity of the frequency estimate to noise.

### A2.12 Appendix 2B: Linear Multi-band Signal decomposition

In order to design a realistic narrow-band filter bank for decomposing power system signals such as (27) with closely spaced modes, let us consider the following definition, which comes from the cosine-modulated filter bank theory [20]:

\[ H_{ij} = \frac{4}{\sqrt{K}} h_{i}(n) \cos \left( k \frac{1}{24} \frac{2\pi n}{K} \left( n + \frac{K + 2}{2} \right) \right) \]

(35)

where \( k = 0,1,\ldots,2K-1; n = 1,\ldots,2K \) and \( h_{i} \) is the impulse response coefficients of a linear-phase low-pass FIR prototype filter. Furthermore, the number \( K \) of filter channels is selected so that \( 2K \) is the length of the prototype filter. If properly chosen, the scaling factor \( \gamma_i \) and the center frequency of the \( i^{th} \) filter can be located at a pre-specified frequency. The final impulse response data of the \( i^{th} \) band-pass filter, after a re-scaling for a unit magnitude and zero phase at the center frequency, is:

\[ H_{ij}(n, \omega_{i}) = \frac{H_{ij}(n)}{H_{ij}(0)} \], \( n = 0,1,\ldots,2K \)

(36)

where \( H_{ij}^{i}(\omega_{i}) = G_{ij} e^{-j\theta_{j}} \) is the response of the \( i^{th} \) filter cell at its center frequency. The filter bank gain is shown in Figure A2-29(a). The corresponding low-pass prototype is based on the following 8-term cosine window:

\[ h_{ij}(n) = \frac{1}{2K} \sum_{s=0}^{K} (-1)^{s} \cos \left( \frac{\pi n}{2K} \right), n = 0,1,\ldots,2K-1 \]

(37)

with the filter coefficients being:

\[ a_0 = 1.0 \quad a_1 = -1.03538 \quad a_2 = 0.0824936 \]
\[ a_3 = -0.00116197 \quad a_4 = -0.00188862 \quad a_5 = -0.00123387 \]
\[ a_6 = -0.000671595 \quad \text{and} \quad a_7 = -0.000275885 \]
Given a 40-Hz sampling rate, the prototype window has a 400-sample length for filters 1 to 4 and 200 samples for filters 5 to 9. These numbers were selected as a trade-off between filter delay and narrow-band behavior. It should be noted that the component filters are essentially orthogonal as they overlap at their -6-dB attenuation frequency. For instance, the -6-dB crossing frequencies of the third filter are 0.4 Hz and 0.6 Hz on the low and high sides respectively. To improve the DC rejection of the filter bank, especially that of the first filter, a DHT pre-filter can be applied to the signal [23]. The effect of a 100-sample FIR-based DHT designed in Matlab is shown in Figure A2-29 (b), where it is seen that the attenuation at 0.01 Hz has increased from 20 dB to 40 dB.

**A2.13 Appendix 2C: IEEE4B PSS Parameters**

The main characteristics of the IEEE PSS4B are shown in Figures A2-30 and A2-31. Two speed deviation transducers are required to feed the three-band structure used as lead-lag compensation.

Figure A2-32 zooms on the differential filter displaying a pass-band characteristic. What is truly special with the PSS2B is that lead-lag compensation bands are based on differential filters that may be used in different ways. Figure A2-34 zooms on the high band to illustrate its characteristics. However, in this paper, we use the simplified tuning method described in STD 423.5 and available in a standard block of the Matlab Simulink SimpowerSystem blockset [41]. Basically, the IEEE4B is re-framed as three symmetrical bandpass filters that provide inherent DC wash-out, zero gain at high frequency and phase leading up to the center-frequency (Figure A2-34). It is then possible to set the PSS with only two high-level parameters per band. Doing so, the whole lead-lag compensation circuit is specified with six parameters, namely the three filter central frequencies $F_L$, $F_I$, $F_H$ and gains $K_L$, $K_I$, $K_H$. Being plain band-pass filters, only the first block in each branch is involved.
Figure A2-30: The IEEE PSS2B structure (from IEEE Std. 421.5)

Figure A2-31: IEEE PSS2B Speed Deviation Transducers

Figure A2-32: Differential filter realizing a pass-band characteristic.

Time constants and gains are derived from simple equations as shown below for the high-band case.

\[ K_{H11} = K_{H17} = 1 \]  \hspace{1cm} (lead-lag blocks) \hspace{1cm} (40a)

\[ T_{H2} = T_{H7} = \frac{1}{2\pi F_H \sqrt{R}} \]  \hspace{1cm} (40b)
Central time constants $T_{H2}$ and $T_{H7}$ are directly derived from the filter central frequency $F_H$ (in Hz) while the symmetrical time constants, $T_{H1}$ and $T_{H8}$, are computed using the constant ratio $R$. Equation (40e) is used to derive branch gains $K_{H1}$ and $K_{H2}$ corresponding to a unit gain for the differential filter. The band gain is therefore equal to $K_H$. Using these notations the PSS used in this paper has the following settings for generators 1, 2 and 4:

- $K_L = 5.0, F_L = 0.025$ Hz, $V_{Lmax} = V_{Lmin} = 0.07$ pu
- $K_I = 25.0, F_I = 0.80$ Hz, $V_{Imax} = V_{Imin} = 0.15$ pu
- $K_H = 145.0, F_H = 12.0$ Hz, $V_{Hmax} = V_{Hmin} = 0.15$ pu
- with $K_{L1} = K_{L2} = K_{I1} = K_{I2} = K_{H1} = K_{H2} = 66, R = 1.2, V_{STmax} = V_{STmin} = 0.1$ pu.

The corresponding Bode locus, is plotted in Figure A2-34 for the three bands and for total PSS output. With respect to their central frequencies, the band filters show symmetry for the gain response and asymmetry for the phase response.

Figure A2-33: MB-PSS simplified model
Figure A2-34: Frequency responses of the PSS2B stabilizer assuming a rotor speed input and including the transducer models in Figure A2-34.
3.1 Introduction

Most transient processes in power systems are nonlinear and fall outside the domain of traditional linear analysis methods. Power system transient phenomena, in addition, are governed by multi-scale processes and are inherently non-stationary due to time-dependent control actions and nonlinear dynamics. To obtain more localized information, several enhancements have recently been introduced to essentially stationary time-series models.

Recent developments in the application of nonlinear and non-stationary time-frequency analysis techniques have provided mechanisms to adaptively analyze complex oscillatory phenomena in power systems. They are able to characterize non-stationary processes more completely than other approaches and have the potential for online application.

In the analysis of nonlinear, time-varying behavior, both models based on adaptive and non-adaptive methods have emerged as useful tools. This chapter provides a concise introduction to nonlinear and non-stationary time-frequency analysis techniques. The basic theory of the main types of nonlinear/non-stationary analysis techniques is reviewed, and a wide range of applications is also provided. Various other extensions to this analysis are briefly discussed, and references are provided for further details.

The potential advantages and limitations of these approaches are illustrated by processing simulated and measured time series using different methods and concluding recommendations are given. A few problems of importance in the further development of the method are stated. A summary and conclusions are presented in the final section.

3.2 Hilbert-Huang Transform Analysis

The Hilbert-Huang Transform (HHT) is an empirically based data analysis method for nonlinear and nonstationary time series analysis. Given a nonlinear and non-stationary time series, the HHT finds a set of nearly orthogonal basis functions with a set of associated instantaneous attributes [1]-[3]. The localized nature of these attributes characterizing temporal behavior can be very useful since each basis function contains phenomena with differing temporal scales and thus can be isolated.

The HHT consists of two main steps. In the first step, a nonlinear and non-stationary signal is decomposed into a finite number of time-varying oscillating components that can be associated with different time scales using a procedure called empirical mode decomposition.

The second stage involves forming a complex signal and then extracting instantaneous attributes associated with the oscillating components using Hilbert analysis.
Hilbert analysis has been introduced to the power system community only recently and its applications are still in its infancy [4]. The method has contributed much in advancing our knowledge of system dynamics and has led to the development of other methods that give a time-frequency-energy display of nonlinear and non-stationary data.

In what follows, the empirical mode decomposition and the Hilbert spectrum are briefly reviewed and recent developments and extensions are outlined. Guidelines for the investigation of nonlinear and non-stationary behavior are provided in the context of their application to the study of power system dynamic phenomena.

3.2.1 The Empirical Mode Decomposition (EMD)

The EMD method introduced by Huang provides an analytical basis for the nonlinear decomposition of a signal $x(t)$ into a finite set of essentially band-limited components or basis functions called Intrinsic Mode Functions (IMFs) [1, 5].

Conceptually, EMD has its foundations in the notion that any oscillatory signal consists of two parts: a slowly varying trend or residue, and a fast component superimposed on the slow component [6]. This decomposition can be represented mathematically by

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t)$$

where $c_j(t)$ is the $j$th IMF, $n$ is the number of IMFs, and $r_n(t)$ is the residue. Distinct from previous methods, the transformation is complete, nearly orthogonal, adaptive and total (the original signal may be recovered by summing the IMF components). Completeness, in particular, depends on the accuracy of the extraction process. In addition, orthogonality is also critical in isolating and identifying local timescales.

3.2.1.1 Sifting Process

Central to the computation of efficient basis functions is the extraction technique. As highlighted in the previous section, the EMD is based on the simple physical assumption that any signal $x(t)$ consists of the sum of different simple IMFs.

More precisely, an IMF is defined as a wave for which:

- In the whole time span of the signal, the total number of extremes, namely maxima and minima, $N_{\text{max}}, N_{\text{min}}$, and the number of zero crossings, $N_{\text{zeros}}$, must be equal or differ at most by one, i.e.

$$N_{\text{max}} + N_{\text{min}} - N_{\text{zeros}} = \pm 1$$

- At any time instant, the mean value of the amplitudes defined by the local maxima $e_{\text{max}}$ and minima $e_{\text{min}}$ must be zero:

$$\left( e_{\text{max}}(t) + e_{\text{min}}(t) \right)/2 = 0$$

Because of physical constraints, the mean value of the IMFs is never zero since this involves the definition of a local timescale.

The IMFs are found by using a recursive procedure called sifting that generates the highest-frequency IMF first. The basic EMD algorithm to extract the IMFs can be summarized as follows [1, 7]:
Step 1) Starting with the original signal $x(t)$, set $r_0(t)=x(t)$, and $j=1$

Step 2) Extract the $j$th IMF using the following iterative sifting procedure

a) Set $h_o(t)=r_j(t)$ and $i=1$

b) Identify the successive local maxima and the local minima. The time spacing between successive maxima is defined to be the time scale of the successive maxima

c) Interpolate the local minima and the local maxima with a cubic spline or other similar techniques. Form an upper envelope $e_{\text{max}_j}(t)$ and a lower envelope $e_{\text{min}_j}(t)$ for the whole data span

d) Compute the instantaneous mean of envelopes $m_{i-1}(t) = (e_{\text{max}_j}(t) - e_{\text{min}_j}(t))/2$ and subtract it from $h_i(t)$. Determine a new estimate $h^i_q(t), q=1,...,n$ using the recursive relations

$$h^i_q(t) = h^i_{q-1}(t) - m^i_q(t), q=1,...,n$$

for $i=1,...,n$, where $h^i_q(t)=r_o(t)=x(t)$, for $i=1$, such that $e_{\text{min}_i}(t) \leq h_i(t) \leq e_{\text{max}_i}(t)$ for all $t$. Set $i=i+1$.

e) Repeat the above procedure until $h_i(t)$ satisfies a predetermined stopping criterion. Then, set $c_j(t)=h_i(t)$

Step 3) Obtain an improved residue $r_j(t)=r_{j-1}(t)-c_j(t)$. Repeat the above steps with $j=j+1$ until the number of extrema in $r_j(t)$ is less than 2. When successful, the result of this procedure is a residual $r_i(t)=r_{i-1}(t)-c_i(t)$, with $c_i(t)=h^{i}_{n_i}$, that contains information about higher frequency components. The residual $r_j(t)$ is then treated as a new signal and the process is repeated for the new signal $(i=i+1)$. The process concludes when there are no longer any maxima or minima in the residual.

Central to this process is the criteria used to stop the sifting. In its original formulation, the process of sifting for an IMF stops if the value of the normalized square difference between successive values of $h^i_q(t)$ is smaller than a pre-set value:

$$SD = \sum_{i=0}^{n} \left[ \frac{h^i_q(t) - h^i_{q-1}(t)}{(h^i_{q-1}(t))^2} \right] \leq \text{threshold}$$

A critical assessment of the performance of this criterion to satisfy the above criteria along with a review of recent extensions is provided in [5]. Reference [6] describes other criteria used in publicly available software.

The sifting process serves mainly two purposes: (i) to eliminate riding waves, and (ii) to make the wave profiles more symmetric. Each of these steps is discussed briefly in subsequent sections.
Referring to Figure 3-2, the iterative process can be written in the recursive form

\[ h_{i_q}^i(t) = h_{i_q}^i(t) - \sum_{q=1}^{n_i} m_q^i(t) \]  

(3)

where the mean values, \( m_q^i(t) \), are obtained using an interpolation technique. As pointed out in [2], the \( h_q^i, q = 1, \ldots, n_i \), represent proto-IMF components that may contain riding waves, i.e. there may exist multiple extrema between successive zero-crossings. To eliminate these effects the process is repeated as many times as necessary to eliminate all components that are not needed.

Several algorithms have been proposed to obtain the upper and lower envelopes. Broadly speaking, these methods can be grouped into three main categories [8]:

1. Cubic spline (third-order polynomials) and other interpolation methods
2. Radial basis functions
3. Hermite piece-wise polynomial interpolations

Each method has advantages and limitations that should be evaluated in more detail in future work.

In many physical applications, only a set of IMFs contain information relevant to system behavior. As a result, the signal can be reconstructed in the form

\[ x(t) = \sum_{j=1}^{p} c_j(t) + \sum_{i=p+1}^{n} c_i(t) + r(t), \quad |r(t)| \leq \text{tolerance} \quad (4) \]

where the terms \( c_j(t), j = 1, \ldots, p \) contain the physical behavior of interest, and the remaining \( n-p \) terms contain uninteresting, non-sinusoidal characteristics; \( r_n(t) \) is a zero-mean low-order polynomial remainder. In practice, a predetermined criterion is used to stop the algorithm [1, 5, 9].

A crucial factor to the successful implementation of the technique is the degree of orthogonality of the extracted functions. Neglecting the residual term, and squaring expression (1) we obtain

\[ x^2(t) = \sum_{j=1}^{n} c_j^2(t) + 2 \sum_{k=1}^{n} \sum_{l=1}^{n} c_k(t)c_l(t) \]

An index of orthogonality, \( IO \), can then be defined as [1]

\[ IO = \sum_{i=0}^{T} \left[ \frac{2 \sum_{k=1}^{n} \sum_{l=1}^{n} c_k(t)c_l(t)}{x^2(t)} \right] \quad (5) \]

where \( 0 \leq IO \leq 1 \). Recent numerical experience suggests that the orthogonality index for practical power system time series can be made arbitrarily small by using selective empirical mode decomposition [7].

Although efficient for most types of signals, the above basic approach has a number of drawbacks [9]:

- The spline fitting procedure used in the algorithm for upper and lower envelopes can create problems at the end of the data that can propagate into the data series
- Execution times are dependent on the signal length and content
- Basic EMD implementations may result in mode mixing and other artifacts
- It may be difficult to identify and isolate physically meaningful modes

To help overcome these limitations and facilitate easier interpretation, a number of methods and techniques have been developed and tested during the last few years. Among these methods, masking signal EMD techniques seem appropriate for the analysis of various physical processes in power systems.
3.2.1.2 Masking Signal Empirical Mode Decomposition

As highlighted in the introductory section, extraction of signal components from a data set is very challenging and may involve various complications including mode mixing and the generation of modes that are physically meaningful for various types of signals. These problems can cause difficulties in interpreting system behavior, especially when the observed oscillations exhibit closely spaced modes.

To overcome the previously identified difficulty and address the problem of mode mixing, EMD with masking technique was introduced by Deering and Kaiser [10, 11] and later extended by various authors [12-15]. In these works it was suggested and proved that the masking signal method could be used to separate components that are similar in frequency.

The key idea in this procedure is to insert a masking signal, to prevent lower frequency components from being included in the IMF. Among the various approaches proposed, methods based on the EMD itself are of particular interest since they do not rely on any external information.

Assume, in order to introduce these ideas, that the EMD is applied once, and let $A_i(t)$ and $f_i(t)$ be the instantaneous amplitude and frequency respectively of the first, raw IMF. As noted in [14], the first IMF is expected to contain the highest frequency component of the signal.

Figure 3-3 shows a conceptual view of the masking-signal EMD method. As suggested in this diagram, the masking signal can be exogenous (i.e. derived from FFT analysis of the signal or other approach) or internal, i.e. be obtained from the EMD procedure itself.

The masking signal EMD method can be summarized as follows:

1. Perform EMD on the original signal $x(t)$. Use only the first IMF, $c_1(t)$ which is expected to contain the highest frequency component of the signal, $f_{\text{max}}(t)$. Obtain $A_1(t)$ and $f_1(t)$ using Hilbert analysis (or any other approach that computes the instantaneous amplitude and frequency)

2. Compute the energy weighted mean of $f_1(t)$ over $L$ samples using

$$ f_{\bar{f}}(t) = \frac{\sum_{i=1}^{L} A_i(t)f_{\bar{i}}^2(i)}{\sum_{i=1}^{L} A_i(t)f_i(i)} $$

3. Construct the masking signal

$$ mask_1(t) = M_1 \sin \left( 2\pi (mf_{\bar{f}})t \right) $$  \hspace{1cm} (6)

4. Perform EMD on $x_+(t) = x(t) + mask_1(t)$ and $x_-(t) = x(t) - mask_1(t)$. Obtain the IMFs for $c_{\mu_+}(t)$ and $c_{\mu_-}(t), i=1,\ldots,n$, and the residues $r_{\mu_+}(t), r_{\mu_-}(t)$. The IMFs and residues of the signal are then given by

$$ c_i(t) = \frac{c_{\mu_+} + c_{\mu_-}}{2}, \quad i = 1, 2, \ldots, n $$

$$ r_i(t) = \frac{r_{\mu_+} + r_{\mu_-}}{2} $$
5. Use the next masking signal to perform steps 2)-4) iteratively, using each masking signal while replacing \( x(t) \) with the residue obtained at each iteration until \( n-1 \) IMFs containing the frequency components \( f_2, f_3, \ldots, f_n \) are extracted.

See Senroy, et. al. [12], Ribeiro [13], and Laila, et al. [14] for extensions of this idea and a detailed explanation of numerical algorithms to choose the masking signals. The total effect of these three operations is to separate the low frequency components from the high frequency modes.

It has been noted that the selection of the masking signal is not unique [14]. Moreover, the choice of the signal amplitude, \( A_o \) can affect the performance of the algorithm, although numerical experience suggests that reasonable results can be obtained using values which are not much larger than the highest frequency.

![Figure 3-3: Conceptual view of the masking-signal EMD method (single IMF). The dashed line indicates the use of EMD to generate the masking signal](image)

Simulation studies conducted to illustrate the performance of the method are discussed next.

### 3.2.1.3 Analytical Example

As an illustrative example consider, an unstable multi-component signal from a transient stability simulation of a stressed power system (see Figure 3-4). This is an unstable oscillation in which three closely spaced dominant modes are seen to interact nonlinearly.

The EMD-based masking method was used to adaptively extract the dominant modes of oscillation. Analysis of the simulated data using the HHT gives 5 IMFs; Figure 3-5 shows the first step in the application of the EMD-based algorithm while Figure 3-6 depicts the extracted instantaneous amplitudes and frequencies.

Simulation results in Figure 3-6 demonstrate that this approach can be efficiently used to separate closely spaced modes under severe transient conditions. While not explicitly discussed in the analysis, refinements to this method based on the recursive computation of masking signals are expected to enhance the performance of the algorithm.
Figure 3-4: Simulated time series showing upper and lower envelopes and the instantaneous mean.

Figure 3-5: First step in the application of the EMD-based method showing the signals $x_+(t)$ and $x_-(t)$. 
1. IMFs

2. Instantaneous amplitude and frequency

Figure 3-6: Instantaneous attributes extracted using the EMD-based masking signal method.

Recent developments in EMD analysis include improvements in frequency calculation methods, the determination of confidence limits of IMFs and the development of the ensemble EMD, among others. See [5] for further discussions on these approaches.

Once the signal has been decomposed into a set of IMFs, the Hilbert transform or an energy operator can be applied to each component to construct energy-time-frequency distributions.

3.2.2 Hilbert Spectral Analysis

The EMD allows the definition of uniquely defined time-varying signals using the notion of an analytic signal. Given a data series \( x(t) \), a complex signal \( z(t) \) can be constructed by adding an imaginary signal to the original function
The phasor rotates at a time varying frequency with amplitude and phase of the local time-varying wave, and

\[
\tilde{x}(t) = H[x] = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau
\]

is the Hilbert transform of \( x(t) \), and \( \text{PV} \int_{-\infty}^{\infty} \) means the Cauchy principal value

\[
\text{PV} \int_{-\infty}^{\infty} = \lim_{\epsilon \to 0} \left[ \int_{-\infty}^{-\epsilon} + \int_{\epsilon}^{+\infty} \right]
\]

The original signal can then be expressed as the real part of the complex expansion

\[
x(t, A, \Omega) = \text{Re} \left[ \sum_{j=1}^{n} A_j(t) e^{i \varphi_j(t)} \right] = \text{Re} \left[ \sum_{j=1}^{n} A_j(t) e^{i \int_{t}^{\infty} \Omega(\tau) d\tau} \right]
\]

which defines a generalized form of the Fourier spectra with time-varying amplitudes and phases.

Differentiating (7) it can be shown that [7]

\[
\dot{z}(t) = \frac{d}{dt} \left[ A(t) e^{i \varphi(t)} \right] = A(t) e^{i \varphi(t)} \left( i \frac{d}{dt} \varphi(t) + e^{i \varphi(t)} \dot{A}(t) = A(t) e^{i \varphi(t)} \left[ \frac{\dot{A}(t)}{A(t)} + i \omega(t) \right] \right] = A(t) e^{i \varphi(t)} \left[ \frac{\dot{A}(t)}{A(t)} + i \omega(t) \right] z(t)
\]

and

\[
\ddot{z}(t) = \frac{d}{dt} \left[ A(t) e^{i \varphi(t)} \left( \omega(t) + e^{i \varphi(t)} \dot{A}(t) \right) \right] = A(t) e^{i \varphi(t)} \left[ \frac{\ddot{A}(t)}{A(t)} - \omega(t)^2 + i \left( 2 \frac{\dot{A}(t)}{A(t)} + \dot{\omega}(t) \right) \right] = A(t) e^{i \varphi(t)} \left[ \frac{\ddot{A}(t)}{A(t)} - \omega(t)^2 + i \left( 2 \frac{\dot{A}(t)}{A(t)} + \dot{\omega}(t) \right) \right] z(t)
\]

where \( \Omega(t) = d\varphi(t)/dt \) is the instantaneous frequency.

The analytic signal represents a time-dependent phasor in the complex plane \( \{ x(t), i \tilde{x}(t) \} \) with amplitude \( A(t) = \sqrt{x(t)^2 + \tilde{x}(t)^2} \), and instantaneous phase \( e^{i \varphi(t)} = \arctan \left( \frac{\tilde{x}(t)}{x(t)} \right) \).

The phasor rotates at a time varying frequency \( \Omega(t) \), defined as

\[
\Omega(t_k) = \frac{\varphi(t_k) - \varphi(t_{k-1})}{t_k - t_{k-1}}
\]
In the limit $\Delta t = t_k - t_{k-1} \to 0$, Equation (12) defines the instantaneous frequency. Refer to Figure 3-7. Further, as suggested in [15], the phase differences $\Phi(t_k) - \Phi(t_{k-1})$ can be used to define modal coherency in time domain.

![Figure 3-7: Time-varying vector in complex plane.](image)

Because an analytical signal is computed for each IMF, both local and global instantaneous characteristics can be derived as discussed below.

Combining (7) and (10) gives

$$
\frac{\dot{z}(t)}{z(t)} = \left[ \frac{\dot{A}(t)}{A(t)} + i\omega(t) \right]
$$

(13)

Physically, $\frac{\dot{z}(t)}{z(t)} = [\frac{\dot{A}(t)}{A(t)} + i\omega(t)] z(t)$ represents the first-order sensitivity of a mono-component analytic signal (7) to changes in time: the real part of (13) has units of s$^{-1}$ and has an interesting interpretation in terms of damping as noted below; the second part, $\omega(t) = \text{Im}[\frac{\dot{z}(t)}{z(t)}]$, is the instantaneous frequency. It is to be noted that the same analysis is valid for the $j$th complex signal.

### 3.2.3 Local HHT Implementations

#### 3.2.3.1 Local Empirical Mode Decomposition

Recently, several local EMD approaches have been implemented and tested in real time. Motivation for these approaches springs from the observation that cubic-spline-based interpolation requires a minimum of five maxima and five minima [6]. This suggests that the extraction of a mode could be done block-wise without knowledge of the whole signal or the previous residual.

A discussed in [1], the sifting process relies on interpolation between successive extrema applied to the full length signal. This can result in over-decomposition of the signal and other undesirable effects.
One way to circumvent these limitations is the use of a sliding-window based HHT. The method works by locally decomposing time-varying signals into IMFs using a sliding-window analysis technique.

Reference [16] describes the testing and development of this approach using measured power system data. The basic idea behind this method is to divide the data into segments to which a local HHT is applied. In implementing the method, however, various decisions have to be made on an apriori basis. Two vital assumptions invoked are:

1. The number of sifting steps for each window has been proposed to be fixed apriori in order to avoid inconsistencies in the application of the method
2. It is assumed that there is no time overlap between data windows

The practical application of this approach requires a local implementation of the Hilbert transform that can be applied to the window-based EMD.

3.2.3.2 Local Calculation of the Hilbert Transform

The Hilbert transform is usually implemented using the Fourier transform. This transform, however, has a global character and hence it is not suitable for characterization of local signal parameters. Further, this approach is subject to problems associated with Fourier analysis such as end effects and aliasing.

Several approaches to compute the Hilbert transform have been recently derived. These methods are discussed below.

Assume, in order to introduce these concepts, that \( x(t) \) is real and is defined for \(-\infty < t < \infty\). In the discrete case, the Hilbert transform can be obtained by applying the trapezoidal rule to \( x(t) \) as [16-18]

\[
H \left[ x(t) \right] = \frac{2}{\pi} \sum_{k=-\infty}^{\infty} \frac{x(t + (2k + 1)\Delta t)}{2k + 1}
\]  

(14)

Application of this expression to discrete time series, \( x(n), n = 0, \pm 1, \pm 2, \ldots \) yields

\[
H \left[ x(t) \right] = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k + 1} \left( x_{r+2k+1} - x_{r-2k-1} \right)
\]  

(15)

In practice, the Hilbert transform can be obtained by truncating the infinite sum in (15). Another interesting approach to perform this truncation is to use a convolution filter of the form

\[
H \left[ x(t) \right] = \sum_{k=-L}^{L} h(t) x(t - n)
\]  

(16)

where \( h(t) \) is the filter

\[
h(n) = \begin{cases} 
\frac{2}{\pi n} \sin \frac{\pi n}{2} & n \neq 0 \\
0 & n = 0
\end{cases}
\]  

(17)

with unit amplitude response and 90° phase shift.
As \( L \to \infty \), (16) yields an exact Hilbert transform; this represents a filtering operation upon \( x(t) \) in which the amplitude of each Fourier spectral component remains unchanged while its phase is advanced by \( \pi/2 \).

For practical use, the Hilbert operator is usually applied in a modified truncated version. Barnet [18] suggested that \( 7 \leq L \leq 25 \) provides adequate values for the filter response. Similar experiences are reported in [19].

Experience shows that low values of \( L \) are often adequate to approximate the Hilbert transform. Also, it is to be noted that, using the convolution method with finite \( L \) requires losing information at each end of the time series.

Other interesting approaches to compute the Hilbert transform are based on window-based FIR filters [20]. In these approaches, the Hilbert transform can be implemented by using an Finite Impulse Response (FIR) approximation to the infinite impulse response \( h(n) \). This is discussed in section 3.2.9 of this Chapter in the analysis of methods to compute instantaneous frequency.

The key point to emphasize is that, by combining a local implementation of the EMD with a filter design obtained via the window method, the HHT can be used to analyze complicated oscillations locally in time for which the assumptions of linearity or stationarity may not apply. Open issues include the choice of the data windows and the order of the approximation.

### 3.2.4 The Hilbert Spectrum

As highlighted in section 3.2.2, the original signal can be expressed as the real part of the complex expansion

\[
x(t) = \text{Re} \sum_{i=1}^{N} z_i(t) = \text{Re} \left[ \sum_{i=1}^{N} x_i(t) + jH[x_i(t)] \right] = \text{Re} \left[ \sum_{i=1}^{N} A_i(\Omega, t) e^{j\int_0^t \omega_i(t) dt} \right]
\]

where Equation (18) indicates that the amplitude \( A(t) \) is a function of \( \omega \) and \( t \).

By analogy with the Fourier amplitude spectrum, the time-frequency distribution of the amplitude, designated as the Hilbert amplitude spectrum can be defined in terms of the IMFs instantaneous amplitudes as [1]:

\[
H(\Omega, t) = \left[ \sum_{i=1}^{n} A_i(\Omega, t) \right]
\]

Alternatively, the squared Hilbert amplitude spectrum gives the temporal evolution of the energy distribution.

The marginal Hilbert spectrum \( \hat{h}(\Omega) \) can then be defined as [5]

\[
\hat{h}(\Omega, t) = \left[ \sum_{i=1}^{n} A_i(\Omega, t) \right]
\]

where \([0,T]\) is the temporal domain within which the data is defined. As discussed by Huang et al. [2],[5], the Hilbert marginal spectrum can be considered as the accumulated amplitude (energy) over the entire data span and is associated with the fraction of time that a given
frequency can be observed in the system. It offers a measure of the total amplitude (energy) contribution from each frequency value.

### 3.2.5 Quasi-Harmonic Behavior

It has long been understood that system dynamics are principally driven by quasi-harmonic behavior. In many practical cases, a good understanding of system dynamics is possible by using simple basic functions. As a first step toward characterizing this behavior, assume for simplicity, that an observed signal, \( x(t) \), consists of a family of \( n \) oscillatory functions, with time-varying amplitudes \( A_j \), and phase \( \Phi_j \) in the form

\[
x(t) = \sum_{j=1}^{n} A_j(t) \cos(\Phi_j(t))
\]

where \( n \) is the number of modal components, \( A_j(t) \) is the slowly varying instantaneous amplitude, and \( \Phi_j(t) = \Omega_j(t) + \int_0^t \omega_j(\tau) d\tau + \Theta_j \) is the instantaneous phase. Taking the Hilbert transform of (21) gives

\[
z(t) = \sum_{j=1}^{n} z_j = A_1(t) e^{i\Phi_1(t)} + \ldots + A_n(t) e^{i\Phi_n(t)} = A(t) e^{i\Phi(t)}
\]

Now, noting that \( \overline{x}(t) = -iA(t) \sin(\Phi(t)) \) and using the definition of the Hilbert transform, it can be shown that

\[
A(t) = \sqrt{\left( \sum_{j=1}^{n} A_j(t) \cos(\Phi_j(t)) \right)^2 + \left( \sum_{j=1}^{n} A_j(t) \sin(\Phi_j(t)) \right)^2} = \sqrt{\sum_{j=1}^{n} A_j^2(t) + \sum_{k} \sum_{j \neq k} 2A_k A_j \cos(\Phi_{kj}(t))}
\]

as well as

\[
\omega(t) = \dot{\Phi}(t) = \frac{x(t) \ast \dot{x}_H(t) - \dot{x}(t) \ast x_H(t)}{A^2(t)} = \text{Im} \left[ \frac{\dot{z}(t)}{z(t)} \right]
\]

In terms of these variables we write the derivative of the analytic signal as

\[
\dot{z}(t) = \sum_{j=1}^{n} \left[ A_j e^{i\int_0^t \omega_j(\tau) d\tau} \dot{x}_j(t) + A_j e^{i\int_0^t \omega_j(\tau) d\tau} \dot{x}_j(t) \right]
\]

and

\[
\frac{\dot{A}(t)}{A(t)} = \text{Re} \left[ \frac{\dot{z}(t)}{z(t)} \right]
\]

Further, use of (26) in (22) yields the instantaneous frequency
\[
\omega(A, t) = \text{Im} \left\{ \sum_{j=1}^{n} A_j e^{i \phi_j(t)} \right\} 
\]

Given the instantaneous amplitude and phase of the modal components, it is straightforward to obtain analytical expressions for the amplitude and frequency of the composite oscillation. An interesting particular case arises when, for a physical system, the system response can be approximated by a few slowly varying functions or modal components. The approach below follows Feldman’s treatment of nonlinear freely vibrating systems [21].

### 3.2.6 Low-Dimensional Representation

In order to get insight in understanding the dynamic behavior involved, it is useful to attempt to understand the Hilbert representation in terms of simplified models. The analysis serves to introduce some of the problems associated with the analysis of complex signals.

Following Feldman [21], [22] consider a two-tone signal

\[
x(t) = A_1 \cos(\Omega_1 t) + A_2 \cos(\Omega_2 t)
\]

Use of Hilbert analysis results in the complex representation

\[
z(t) = x(t) + \tilde{x}(t) = A_1(t)e^{i\varphi_1(t)} + A_2(t)e^{i\varphi_2(t)} = A(t)e^{i\varphi(t)}
\]

with

\[
\varphi_i(t) = \int_0^t \Omega_i(\tau) d\tau \quad , \quad i = 1, 2
\]

Setting \( \dot{A}_j(t) \approx 0 \) in (25), and using (22) one obtains,

\[
A^2(t) = A_1^2 + A_2^2 + 2A_1A_2 \cos((\Omega_2 - \Omega_1)t)
\]

and

\[
\Omega(t) = \frac{(\Omega_1 + \Omega_2)}{2} + \frac{1}{2} \frac{(A_1^2 - A_2^2)}{A_1^2 + A_2^2 + A_1A_2 \cos(\Omega_1 - \Omega_2)t} (\Omega_2 - \Omega_1)
\]

In addition, the instantaneous phase can be written in the form

\[
\varphi(t) = \tan^{-1} \left[ \frac{A_1(t) \sin \Omega_1 t + A_2(t) \sin \Omega_2 t}{A_1(t) \cos \Omega_1 t + A_2(t) \cos \Omega_2 t} \right]
\]

The following properties can be easily verified:

- Equation (29) has two solutions corresponding to the upper and lower envelopes. As noted by Feldman [21], [22] the first two terms represent the slowly varying part associated with the amplitudes squared while the third term describes a rapidly varying part, oscillating with a frequency equal to the difference between the individual component frequencies.
- For \( A_1 \neq A_2 \) the instantaneous frequency is time varying and exhibits asymmetrical deviations about the average frequency \((\Omega_1 + \Omega_2)/2\).
Another point worthy of note is that the extrema of the frequency variations (peaks and troughs) coincide with the time instants the signal’s amplitude has a minimum or maximum, respectively. Low amplitude values may result in noise or spikes in frequency behavior.

The unwrapped instantaneous phase exhibits a dominant linear trend of the form $\Phi(t) = \omega_o t + \Delta\phi(t)$ where $\omega_o$ is the reference frequency and $\Delta\phi(t)$ is the fluctuating part; changes or breaks in the slope indicate fluctuations in system frequency.

Amplitude (energy) variations help to localize the time variations of the signal energy and point out the instant of maximum energy occurrence as well as the starting and ending time of the transient signal.

Figure 3-8 illustrates the use of Hilbert analysis for modal extraction of the two-tone signal $x(t) = A_1(t)\cos(2\pi f_1 t) + A_2(t)\cos(2\pi f_2 t)$. In these plots, the top panel shows the upper and lower envelopes computed using (29); the lower panels show the instantaneous frequency and phase of the signal obtained using (30) ad (31).

![Figure 3-8: Hilbert analysis of a two-tone signal.](image)

$A_1(t) = 1.0, A_2(t) = 0.3, f_1 = 1.0 \text{Hz}$ and $f_2 = 3.0 \text{Hz}$

### 3.2.7 Hilbert-based Modal Parameter Identification

In order to get insight in understanding the dynamic behavior involved, it is useful to characterize system behavior in terms of decoupled harmonic oscillators. Consider to this end an unforced damped oscillator [23]

$$\ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = 0 \quad (32)$$

where $\xi$ is the damping ratio and $\omega_n$ is the natural frequency.

Using the concept of an analytic signal let the measured data be expressed as

$$z(t) = \tilde{x}(t) + i\hat{x}(t) = A(t)e^{i\Phi(t)}$$

where $\tilde{x}(t) = iA(t)\cos\Phi(t)$. 
Taking the Hilbert transform of (32), the following association can be made
\[ \dddot{x} + 2\zeta_0^\omega \dot{x} + \ddot{\omega}_o^2(t)x = 0 \]  
(33)

Adding (32) and (33), one obtains
\[ \dddot{x} + 2\zeta_0^\omega (A) \dot{x} + \ddot{\omega}_o^2(A)z = 0 \]

It remains to express nonlinear damping and stiffness in terms of the instantaneous amplitude and frequency. Using (10) and (11) it can be proved that
\[
2\zeta_0^\omega = h(t) = -2 \frac{A}{\omega} \frac{\dot{\omega}}{\omega} \\
\ddot{\omega}_o^2(t) = \omega^2 - \frac{A}{\omega} \frac{\dot{A}}{\omega} = \omega^2 - \frac{\ddot{A}}{\omega} + 2 \frac{\dot{A}}{\omega} + \frac{A}{\omega} \frac{\ddot{\omega}}{\omega}
\]

With the instantaneous amplitude \(A(t)\) and frequency \(\omega(t)\) computed using Hilbert analysis, the energy in the oscillator can be estimated. Let the solution of (32) be expressed as \(x(t) = A \cos \omega t\). The total energy of the system is the sum of the potential and kinetic energy:
\[ \dddot{x} + 2\zeta_0^\omega (A) \dot{x} + \ddot{\omega}_o^2(A)z = 0 \]  
(34)

This equation provides a criterion to assess the importance of each IMF in the signal and corresponds to the energy displayed in the HHT spectra. Section 3.5 discusses two possible applications of this notion: a) developing thresholding criteria, and b) determining the total energy contribution from each IMF.

3.2.8 Analysis of Multi-Component Signals

The proposed time-domain solution can be readily extended to multi-component signals. Once a signal is decomposed into its essential basis functions, each of these functions can be analyzed individually. Note that for the estimate to be accurate, the basic functions have to be nearly orthogonal.

Figure 3-9 shows a schematic of the multicomponent case. In this representation, each modal coordinate is governed by the equation of a SDOF oscillator with nonlinear damping.
This approach forms the basis of existing HHT-based nonlinear identification methods [24].

### 3.2.9 Frequency Computation

Frequency computation using HHT and other analysis techniques is numerically sensitive. As discussed above, in Hilbert analysis, the calculation of instantaneous frequency using (24) requires two differentiations.

To overcome these limitations, several approaches that approximate instantaneous frequency have been developed that are faster to compute. Beginning with first-order, these approaches can be continued recursively to higher and higher order using finite difference methods. Table 3-I summarizes some existing methods to estimate the instantaneous frequency [25-27].

Other approaches to compute the instantaneous frequency that have been used in power system studies include the Wigner-Ville distribution [28] and the Teager operator [29, 30] as discussed later in the document.
The first observation is that some of these approaches lose information at the beginning and end of the records and may be sensitive to the amplitude of the oscillations. The comments apply to all of the methods for estimating derivatives whether in frequency analysis or in the time or frequency domain.

Estimates of the instantaneous frequency obtained using the methods outlined in Table 3-I are shown in Fig. 3-10. For comparison, the frequency estimates obtained using two FIR Hilbert transformers implemented via the window method are shown in Fig. 3-11 [20].

In particular, a Park-McMillan and a Kaiser-based FIR filter are investigated; a window size of 20 samples was used in all simulations.

Simulation results in Fig. 3-11 suggest that local HHT implementations have the capability to extract modal characteristics accurately. In particular, comparison of the window-based FIR filter results with the Barnes approximation appear to compare favorably with essentially off-line frequency estimation models.

The results are not conclusive and sensitivity studies are needed to fully assess the potential utilization of these approaches in near-real-time implementations.
3.2.10 Hilbert Spectrum via Wavelet Projections

Alternatives to Hilbert spectral analysis have been recently discussed in the literature. Among them, wavelet analysis is an attractive alternative to Hilbert analysis that has been applied to the analysis of various power system phenomena.

3.2.10.1 The Wavelet Transform

The wavelet transform can be used to analyze time series that contain non-stationary power at many different frequencies [31],[32]. Following Daubechies [31], the wavelet transform (WT) of a time series $x(t)$ is defined by

$$ W_s (s) = s^{-1/2} \int_{-\infty}^{\infty} x(t') \psi \left( \frac{t-t'}{s} \right) dt' $$

(35)
which is the convolution of $x(t)$ with a family of functions $\psi_{st}(t')$, given by

$$\psi_{st}(t') = s^{-1/2} \psi \left( \frac{t-t'}{s} \right)$$

where the asterisk indicates the complex conjugate, $s>0$ represents dilation and corresponds to frequency, $-\infty<t<\infty$ represents translation in time, and $\psi$ is the mother wavelet. See [31] for mathematical details.

Olhede and Walden [33] suggested that non-stationary signals could be analyzed by wavelet-based projections onto the time-frequency plane giving a set of monocomponent signals. These signals could then be converted into analytic signals using the Hilbert transform.

In [7] it was shown that the Hilbert amplitude spectrum could be computed using wavelet analysis. The approach consists of the following steps:

1. Compute the wavelet decomposition of the signal
2. The wavelet coefficients at important scales for a given mode are then summed together to produce the wavelet model for all scales
3. Compute instantaneous amplitude and frequency (damping) by applying the analytic signal to the wavelet model

The relationship between the EMD and wavelet analysis can now be noted. Steps 1 and 2 are effectively equivalent to the EMD approach above and can be used to compute the wavelet spectrum directly or the Hilbert spectrum via Hilbert analysis.

In the following, HHT and wavelet analysis are used to characterize the temporal behavior of measured data. Comparisons are provided with other analysis techniques.

### 3.2.10.2 Numerical Example

As a second example, Hilbert analysis and wavelet analysis are used to characterize the time evolution of non-stationary power system oscillations. Simulation studies below are based on PMU data. Figure 3-12 gives the time evolution of a selected tie-line time-series from a real event.

Measurements were recorded over a period of 245 s at a rate of 20 samples per second for a total of 4999 samples.

Figure 3-13 gives the first four IMFs produced by EMD, using the masking signal method in section 3.2.1.2 whilst Fig. 3-14 shows the captured energy, $E(t)$, using (34). As seen in Fig. 3-13, IMF 1 essentially captures high frequency noise whilst IMF 2 is seen to capture the dominant temporal features of the dynamic process.
A contour plot of the signal’s energy as a function of frequency and time obtained from the Hilbert marginal spectrum of the data in (30) is shown in Fig. 3-15. The scale on the right provides the relative spectral values.

An interesting feature in this plot is the presence of a second mode at about 0.26 Hz that becomes of importance at about 100 s into the simulation.

In this analysis, the metric

$$rp_i = E_i \left/ \sum_{k=1 \atop k \neq i}^k E_k \right., \quad i = 1,\ldots,n$$

is introduced as an objective criterion to measure the total energy contribution from each IMF. Somewhat more general alternatives to this simple criterion are to be explored in future stages of this study.

Figure 3-13: Intrinsic mode functions.
To confirm the results obtained by Hilbert analysis and provide a basis for comparison with other approaches, wavelet analysis was conducted. Based on Torrence and Compo [32], the wavelet transform of an equally-spaced time series $x_n$, $n=0,\ldots,N-1$, with respect to a mother wavelet $\psi$ is given as

$$W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi} \ast (s \Theta_k) e^{i\theta_k s} \delta_j$$  \hspace{1cm} (36)$$

where $\hat{x}_k$ is the DFT of $x_n$, the asterisk indicates the complex conjugate, $s$ represents dilation and corresponds to frequency, and $k=0,\ldots,N-1$ is the frequency index. Figure 3-16 shows the wavelet spectra. For further reference, Fig. 3-17 shows the FFT of the individual wavelets.
As can be seen in this plot, each wave captures a different range of energy with the highest order waves containing the slowest scales of the data:

Figure 3-16: Wavelet power spectra.

Figure 3-17: Fourier spectra of the wavelet components.

Comparison of the Hilbert and wavelets spectra shows that both techniques are able to extract the underlying phenomena of interest. In both cases, a mode with a frequency of about 1.09 Hz is identified in the main interval of interest, 0-250 s. Also of relevance, the analysis shows a second mode at about 0.25 Hz that becomes unstable at about 170 s into the simulation. The two modes are seen to coexist for several seconds prior to system instability.

To make a fair comparison between both approaches, weighting factors were removed and the scale-averaged wavelet power was used. Referring to Fig. 3-17, wavelets (data) 1-11 were chosen because they have an important contribution to the 1.09 Hz and 0.26 Hz modes and the total power was obtained by adding up the individual magnitudes.
Figure 3-18 compares the total power of the wavelets with the amplitude of IMF2. The good correlation between the spectra and the instantaneous amplitudes shows that time-frequency energy representations can be used to characterize typical non-stationary behavior in measured data.

![Figure 3-18: Comparison of instantaneous amplitudes.](image)

### 3.3 Approaches Based on Energy Tracking Operators

Another recent approach that attempts to characterize nonlinear and/or non-stationary behavior is based on the Teager-Kaiser energy operator introduced by Teager [29],[34].

The Teager nonlinear energy operator, $\Psi$, is defined in the continuous and discrete domains, as

\[
\Psi_c = \left[ x(t)^2 \right] - x(t)\ddot{x}(t) \tag{37}
\]

\[
\Psi_d = x^2(n) - x(n + 1)x(n - 1) \tag{38}
\]

where $x(n)$ is the sampled signal, and the overdots indicate time differentiation.

As noted in [29], the operator was originally derived to track the energy of a linear undamped oscillator by following a procedure similar to that in section 3.3.2. The accompanying paper by Barocio et al., discusses extensions to this approach based on physical and mechanical concepts.

Based on the Teager operator, Maragos [35] derived the following Discrete Energy Separation Algorithm (DESA) to estimate the instantaneous frequency, $\Omega$, and amplitude $A$ of a general AM-FM signal

\[
\Omega(n) = \cos^{-1}\left( 1 - \frac{\Psi_d[x(n) - x(n - 1)]}{2\Psi_d[x(n)]} \right) \tag{39}
\]
\[ |A(n)| = \frac{\Psi[x(n)]}{\sqrt{1 - \left(\frac{\Psi[x(n) - x(n-1)]}{2\Psi[x(n)]}\right)^2}} \]  

(40)

Principal advantages of this method are its simplicity and its ability to characterize temporal behavior using only three (or more) data points. The application of the Teager energy operator to multi-component signals, however, has been found to contain significant cross terms. It has also been noted that the definition of instantaneous frequency only has physical meaning when the signal has no frequency and amplitude modulation over the global domain [5].

Additional sources of error are the sensitivity to noise in the computation and the use of numerical approximations to estimate instantaneous frequency. Also, the issue of nonlinearity should be taken into account in the analysis of complicated phenomena.

To be of practical use, the signal of interest has to be decomposed first into essentially narrow-band components. The first alternative is to use EMD before using the Teager-Kaiser operator. Another interesting alternative is to use linear (possibly nonlinear) multi-band signal decomposition. See Kamwa et al. for details about this approach [30].

Apart from these techniques, there exist various other methods for the calculation of instantaneous parameters, based on wavelet analysis and filtering methods that could be used previous to the utilization of the nonlinear operator, including empirical orthogonal functions or a combination of various methods.

### 3.4 Damping Characterization

#### 3.4.1 Hilbert-based Approaches

Hilbert analysis provides a natural approach to estimate instantaneous damping. In this procedure, the instantaneous amplitude is rewritten in the form \( A(t) = \Lambda(t) \exp[\alpha(t)] \), where \( \alpha(t) = \int_0^t \sigma(t) \, dt \) is an exponential factor characterizing the time-dependent decay, and \( \sigma(t) \) is the associated instantaneous damping [36],[37]. Use of this assumption in (7) results in

\[
\frac{\dot{z}(t)}{z(t)} = \left( -\sigma(t) + \frac{\dot{\Lambda}(t)}{\Lambda(t)} \right) + i\Omega(t) \tag{41}
\]

Noting that \( \frac{\dot{A}(t)}{A(t)} = \text{Re} \left[ \frac{\dot{z}(t)}{z(t)} \right] \), and splitting (41) into real and imaginary parts yields

\[
\sigma(t) = -\frac{d\alpha(t)}{dt} = -\frac{\dot{A}(t) - \dot{\Lambda}(t)}{A(t) - \Lambda(t)} \tag{42}
\]

We point out that for measured data, \( \Lambda(t) \) is time-dependent and unknown. The process can be applied to multiple recordings in order to remove its effects but no general technique has so far been devised. Other alternatives to compute a better approximation to damping may include the use of higher order approximations.
3.4.1.1 Demodulation-based Approaches

Several variations to the above approach are possible. Palmer [28], for example, has suggested the following implementation:

Given the analytic function \( z_j(t) = A_j(t) e^{-\sigma_j(t)} e^{\varphi_j(t)} \),

- demodulate the signal according to \( v_j(t) = z_j(t) e^{\varphi_j(t)} A_j(t) e^{-\sigma_j(t)} \)
- Compute the derivative with respect to time \( \dot{v}_j(t) = -\sigma_j A_j(t) e^{-\sigma_j(t)} + e^{-\sigma_j(t)} A_j(t) \)
- Assuming \( \dot{A}_j(t) = 0 \), damping can be computed using the expression \( \sigma_j = \frac{\dot{v}_j(t)}{v_j(t)} \)

Generalizations to the above models are needed as complex non-stationary processes, may involve some degree of amplitude or frequency modulation.

3.4.1.2 Synthetic Example

To build insight into how the above method works, we consider an amplitude-modulated frequency-modulated synthetic signal of the form

\[
x(t) = A(t) \cos(\omega_c t + A_{fm} \cos(\omega_{fm} t))
\]

with

\[
A(t) = (1.0 + A_{am} \cos(\omega_{am} t)) e^{-\sigma_i} = \Lambda(t) e^{-\sigma_i}
\]

And \( \sigma = 0.02, A_o = 1.0, A_{am} = 0.10, f_{am} = 0.10 \text{Hz}, f_{fm} = 0.30 \text{Hz}, \) and \( f_c = 1.00 \text{Hz} \), where \( \omega_c \) is the center frequency of the process, \( A_{am}, A_{fm} \) give the degree of amplitude and frequency modulation, respectively, and \( \Lambda(t) e^{-\sigma_i} \) is the time-varying amplitude-modulated envelope.

Empirical mode decomposition of this signal, results essentially, in one IMF, \( n = 1 \) in (43a), i.e.

\[
x(t) = \sum_{i=1}^{n} IMF_i(t) = A_i(t) e^{\int f_i(t) dt}
\]

Figure 3-19 depicts the time evolution of the synthetic signal showing the instantaneous amplitude obtained from Hilbert analysis. Also shown, is the instantaneous amplitude computed from (43b). Table 3-II gives the corresponding Prony analysis results for the signal.
As suggested in this plot, both the envelope and frequency of IMF 1 are amplitude-modulated sinusoidal functions.

The analysis of instantaneous frequency in Fig. 3-20, on the other hand, shows an oscillating curve centered at 1.0 Hz. Prony analysis results (PRS) in Tables 3-II and 3-III show that the amplitude can be expressed as a sinusoidal function of the form \[ A(t) = e^{-\sigma t} \cos(\omega t + \varphi) \] with an amplitude \( A_0 = 1.0 \), frequency \( f = 0.10 \text{Hz} \) and damping \( \sigma = 0.02/(2\pi) = 0.0032 \). Similarly, PRS in Table 3-IV disclose a frequency-modulated function of the form

\[ \omega(t) = \omega_c t + A_{fm} \cos(\omega_{fm} t) \]

with \( \omega_c = 1.0 \text{Hz} \), \( A_{fm} = 0.20 \), and \( \omega_{fm} = 0.30 \text{Hz} \). These results are consistent with the parameters chosen for the synthetic data.
Table 3-II: Prony Analysis Results on the AM-FM Signal

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
<th>Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.0031</td>
<td>0.9900</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>1.300</td>
<td>0.0031</td>
<td>0.0995</td>
<td>180.00</td>
</tr>
<tr>
<td>3</td>
<td>0.700</td>
<td>0.0031</td>
<td>0.0995</td>
<td>180.00</td>
</tr>
<tr>
<td>4</td>
<td>0.900</td>
<td>0.0031</td>
<td>0.0495</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>1.100</td>
<td>0.0031</td>
<td>0.0496</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 3-III: Prony Analysis Results on the Envelope of IMF1

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
<th>Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.0031</td>
<td>0.9900</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>1.300</td>
<td>0.0031</td>
<td>0.0995</td>
<td>180.00</td>
</tr>
<tr>
<td>3</td>
<td>0.700</td>
<td>0.0031</td>
<td>0.0995</td>
<td>180.00</td>
</tr>
<tr>
<td>4</td>
<td>0.900</td>
<td>0.0031</td>
<td>0.0495</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>1.100</td>
<td>0.0031</td>
<td>0.0496</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

With the instantaneous amplitude computed using Hilbert analysis, damping can be obtained directly by computing the decay rate of (43b).

Taking the natural logarithm of (43b), we have

$$\sigma = \left( \ln A(t) - \ln \Lambda(t) \right)/t$$  \hspace{1cm} (45)

Figure 3-21 provides a comparison of the exponential term obtained from the data $e^{-0.2t}$ with that obtained from (44) using the HHT approximation $e^{-\sigma t} \approx a_i(t)/(A_v + A_{am} \cos(f_i t))$.

Insight into the nature of the Hilbert approximation can be gained by noting from (43b) damping can be expressed in the form

$$\sigma = \frac{A_j(t)}{\Lambda(t)} - \frac{\Lambda_j(t)}{\Lambda(t)} = A_j(t) + \frac{A_{am} \Omega_{am} \sin \Omega_{am} t}{(A_v + A_{am} \cos \Omega_{am} t)}$$  \hspace{1cm} (46)

Two interesting facts arise from this simple analysis:

- Since $\Lambda(t)$ is not known, use of (45) results in an approximation to the true damping
- Analysis of (45) suggests that the true damping is the mean value of (44)
- For the simple case of constant amplitude, the second term in (45) vanishes, and the problem of damping estimation reduces to conventional analysis. For more general cases, the signal amplitude must be determined using Hilbert analysis.

Table 3-IV: Prony Analysis Results on Instantaneous Frequency

<table>
<thead>
<tr>
<th>Mode</th>
<th>Amplitude</th>
<th>Frequency (Hz)</th>
<th>Damping/2%</th>
<th>Relative energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1000</td>
<td>0.3000</td>
<td>0.003183</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>0.0000</td>
<td>0.003183</td>
<td>0.0050</td>
</tr>
</tbody>
</table>
To further illustrate the potential usefulness of the methods, we consider, as a final example, the analysis of measured data in section 3.2.10.2. The analysis focuses on IMF 2 that captures most of the energy in the signal.

Figure 3-22a gives the instantaneous frequency whilst Fig. 3.22b gives the instantaneous damping for IMF 2 in Figure 3-13. The solid dashed line in Fig. 3-22a gives the mean instantaneous frequency.

In close agreement with Hilbert and wavelet analysis, simulation results in Figure 3-22 show a nearly stationary mode whose damping decreases from a relatively high value to a condition in which the mode becomes unstable.

Of particular relevance, the method identifies the onset of instability at about 75 s and the time at which the mode is made stable by control actions in the system. The analysis indicates the potential of the method for developing real-time monitoring systems.

Similar conclusions can be drawn regarding the temporal evolution of IMF 3 – not shown.

Together, modal features can be used to initiate control actions to stabilize the system.
3.4.2 Wavelet-based Approaches

Among the alternative non-stationary formulations, wavelet analysis has proven to be particularly useful for modal identification. We next briefly review this method in the context of recent applications to power system modal identification. The reader is directed to [38]-[40], for more details about these formulations.

3.4.2.1 Frequency Estimation with Complex Continuous Wavelet Transform

In this approach discussed in [38] and [39], the mode frequency is estimated in a specified time window and frequency band with the complex continuous wavelet transform. First, the analyzed signal, \( y(t) \), is wavelet transformed:

\[
C(ab_n) = \frac{1}{\sqrt{a_i}} \int_{-\infty}^{\infty} y(t) \frac{\psi^*_{a_i}}{a_i} dt
\]

(47)

where parameters \( a_i \) are the wavelet scales from the lower frequency bound to the upper frequency bound with dense enough spacing to achieve high enough resolution for frequency estimation. Parameters \( b_j \) are the wavelet positions from the beginning of the time window to
the end of the time window with the spacing of signal sampling period. Time is \( t \), \( \psi_{f.e.} \) is the wavelet function used in the frequency estimation, and \( C(a_i, b_j) \) are the wavelet coefficients with the specific scale and position parameters.

The wavelet scale \( a_m \) that produces the highest average wavelet coefficient modulus, \( a_m = \left\{ \{a_i \max \left| C(a_i, b_j) \right| \}_{b_j} \right\} \) (48), is selected as the scale of the mode. The scale is converted to the mode frequency, \( f_m \):

\[
f_m = \frac{f_c(\psi_{f.e.})}{a_m(\psi_{f.e.}) \cdot \Delta}
\]

where \( f_c(\psi_{f.e.}) \) is the center frequency of the wavelet function \( \psi_{f.e.} \) and \( \Delta \) is the signal sampling period. The center frequency of the wavelet function, \( f_c \), is defined as the frequency that maximizes the Fourier transform of the wavelet function \( \psi \):

\[
f_{c.e.} = \left\{ f \left[ \max \psi(f) \right] \right\}
\]

where \( f \) is frequency and \( \hat{\psi} \) is the Fourier transform of the wavelet function.

### 3.4.2.2 Mode Extraction with Real Continuous Wavelet Transform

Having estimated the mode frequency, \( f_m \), the mode is extracted from the analyzed signal, \( y(t) \), with the real continuous wavelet transform by calculating the resulting wavelet coefficients:

\[
C_m(a_m(\psi_{m.e.}), b) = \frac{1}{a_m(\psi_{m.e.})} \int_{-\infty}^{\infty} y(t) m.e. \left\{ \frac{t \Delta}{a_m(\psi_{m.e.})} \right\} dt
\]

The wavelet coefficients, \( C_m \), are (approximately) linearly dependent on the instantaneous value of the mode at different time instances, \( b \) [31]. Therefore, the mode damping information is (approximately) preserved during the mode extraction. The mode extraction resolution is limited by Heisenberg’s uncertainty principle (frequency resolution vs. time resolution). The parameter \( a_m(\psi_{m.e.}) \) is the wavelet scale corresponding to the estimated mode frequency, \( f_m \), with the equation

\[
a_m(\psi) = \frac{f_c(\psi)}{f_m \cdot \Delta}
\]

where \( f_c(\psi) \) is the center frequency of the mode extraction wavelet function, \( \psi_{m.e.} \), and \( \Delta \) is the signal sampling period. Damping characterization.

### 3.4.2.3 Impulse Response from Ambient response with the Random Decrement Technique

After extracting the mode of interest from the signal, the result is the power system’s ambient response at the mode frequency. Because it is not possible to determine the damping of the mode directly from the single-mode ambient response, the random decrement technique (RDT) is used to estimate the approximate single-mode impulse response from the single-mode ambient response. From the estimated impulse response, the damping of the mode can be estimated.
Under the assumption that the power system is linear and excited with Gaussian distributed random variations, the RD auto signature, $D_{YY}(\tau)$, is proportional to the free decay or impulse response of the system [41]. Therefore, the approximate impulse response, $r_{\text{imp,a}}(t)$, is

$$r_{\text{imp,a}}(t) \approx D_{YY}(\tau) = \frac{1}{N} \sum_{s=1}^{N} C_{m}(t_{s}\tau + \tau)$$  \hspace{1cm} (53)$$

where $N$ is the total number of samples collected using the threshold, $s$ is the sample number, $t_{s}$ is the time instance when the single-mode ambient response, $C_{m}(\cdot\cdot\cdot)$, crosses the threshold, $\tau$ is the length of each sample (and corresponding approximate impulse response).

The main assumptions of the random decrement technique are that the power system dynamic behavior is linear and that the system is excited with random variations that are Gaussian distributed. The assumptions can be considered valid when the system is operating under the ambient conditions. Then the oscillations are small and approximately linear. The random load variations can be considered approximately Gaussian distributed based on the central limit theorem [42] because the number of loads in a power system is large.

### 3.4.2.4 Mode Damping Estimation from the Approximate Impulse Response with Complex Continuous Wavelet Form

Wavelet analysis provides an alternative to damping computation. The damping of the mode is estimated from the approximate impulse response, $r_{\text{imp,a}}(t)$, utilizing the complex continuous wavelet transform. At first the complex wavelet coefficients, $C_{\text{imp}}(\cdot\cdot\cdot)$, at different time instances, $b$, are calculated:

$$C_{\text{imp}}(a_{m}(\psi_{\text{d.e.}}, b)) = \frac{1}{\sqrt{a_{m}(\psi_{\text{d.e.}})}} \int_{-\infty}^{\infty} r_{\text{imp,a}}(t) \cdot a_{m}(\psi_{\text{d.e.}}) \cdot \psi_{\text{d.e.}} \cdot dt$$ \hspace{1cm} (54)$$

where $a_{m}(\psi_{\text{d.e.}})$ is the wavelet scale corresponding to the estimated mode frequency, $f_{m}$, calculated with Equation (52), $\psi_{\text{d.e.}}$ is the wavelet function used in the damping estimation.

The damping ratio of the mode, $\zeta_{m}$, is finally calculated by using wavelet coefficients, $C_{\text{imp}}(\cdot\cdot\cdot)$, from two different time instances:

$$\zeta_{m} = \frac{100}{2 \cdot \pi \cdot f_{m} T_{a}} \ln \left| \frac{C_{\text{imp}}(t_{sp}) + T_{a}/2}{C_{\text{imp}}(t_{sp}) - T_{a}/2} \right|$$ \hspace{1cm} (55)$$

where $T_{a}$ is the difference between the positions (or time instants), $b$, of the wavelet coefficients $C_{\text{imp}}$ in the damping calculation, and $T_{sp}$ is the time instant from the beginning of the approximate impulse response (selection point), needed for the damping calculation. The absolute value of the complex wavelet coefficient corresponds to the amplitude of the approximate impulse response at a certain time instant.

1 The central limit theorem states that the sum of independent and identically distributed random variables with finite mean and variance approaches the normal distribution (Gaussian distribution) when the number of random variables increases, irrespective of the distribution of the random variables.
Figure 3-23 summarizes the damping estimation method. The method consists of four blocks:

- mode frequency estimation,
- mode extraction,
- estimation of the mode’s approximate impulse response, and
- mode damping estimation from the approximate impulse response.

Alternative formulations to modal damping estimation are given in [43] and [44].

The following subsections discuss the experience in the analysis of measured ambient data in the Nordic power system.

### 3.4.2.5 Application to Measured Data

In the measured data cases, the power system operates under ambient conditions; then the inputs are the power system ambient excitations: mainly the ever present load fluctuations in the grid. The measured output signals that are used in the damping estimation are the phasor measurement quantities of the Finnish wide-area monitoring system. The physical measurement quantities are voltages, currents, local frequency, and local rate of change of frequency [45]. Active, reactive, and apparent power is calculated from the voltages and currents.

The dominant 0.3 Hz mode is well observable in Southern Finland when voltage magnitude, frequency, or rate of change of frequency is analyzed. The mode is also well observable when current, or active power flow of the AC interconnection path between Finland and Sweden is analyzed. In addition, the voltage angle difference between the oscillating generator groups in Finland and Sweden has a high observability of the mode [46, 47]. Therefore, these measurement locations are used.
The signals which have a high observability of the mode of interest are preferred in the damping and frequency estimation because the results are more reliable compared to the signals with poor observability of the mode. The observabilities can be calculated with the linear analysis of the power system dynamic model [46, 47].

3.5 Nonlinear Approaches to Trend Identification and Denoising

Extracting from measured data a system representation of the form (1) is a challenging problem. Complex oscillatory problems may contain noise, trends and other artifacts that can prevent the extraction of special features of interest, such as localized events in time.

Detrending refers to the separation of low-frequency components (i.e. the time-varying instantaneous mean) from high-frequency components [48]. Extraction of high-frequency fluctuations, in turn, involves solving two conflicting and difficult problems: 1) retaining physical components showing no hint of noise (denoising), and 2) identifying noise displaying no hint of the underlying signal or fluctuations.

A number of studies in power system dynamic analysis have estimated nonlinear trends by 1) removing linear trends, and 2) using low-and high-pass filters. The procedures, however, may fail to take into account temporal characteristics. In addition, the use of filters may modify the signal characteristics.

Experience with the analysis of complicated oscillations shows that the system response can often be described by the general nonlinear model

$$x(t) = m(t) + h(t) + \varepsilon(t)$$

where $m(t)$ is the time-varying instantaneous mean or underlying trend that results from control actions, topology changes or other effects; $h(t)$ is the fluctuating variation of the signal; $\varepsilon(t)$ represents noise effects.

Several techniques designed to analyze complicated system oscillations have been proposed including linear, nonlinear, and time-varying approaches and their combinations. Zhou, et al. [49] designed a trend identification algorithm to identify and remove slow trends within the data. The use of EMD as a nonlinear and non-stationary filter is discussed below.

3.5.1 EMD as a Nonlinear and Non-stationary Filter

Selective EMD of the data offers a natural way to extract from the original signal an instantaneous time-varying mean. It can actually be considered a time-domain detrending tool.

In [48], a procedure to detrend nonlinear signals was set out. The procedure was motivated by noticing that at each step of the local mean computation, the signal $x(t)$ or any subsequent residual can be represented as the sum of a time-varying mean $m_q(t), q = 1,\ldots,n_i$ representing a slow component, and a fast, high-frequency component, $h_q^i(t)$ , as

$$h_q^i(t) = h_{q-1}^i(t) - m_q^i(t)$$

(56)
We recall from previous analysis that the sifting process can be written in recursive form

\[ h_n^i(t) = h_n^i(t) - \sum_{q=1}^{q_n} (m_q^i(t)) \]

where

\[ h_n^i(t) = r_{i-1}(t); \]
\[ h_{i+1}^i = h_{i+1}^i(t) - m_{i+1}^i(t) \]
\[ \vdots \]
\[ h_{i_n}^i = h_{i_n}^i(t) - m_{i_n}^i(t) \]

Noting that \( \bar{x}_i(t) = r_{i-1}(t) \), (58) may be rewritten as

\[ \bar{x}_i(t) = r_{i-1}(t) = h_n^i(t) = h_n^i(t) + \sum_{q=1}^{q_n} (m_q^i(t)) = h_n^i(t) + (m_1^i(t) + m_2^i(t) + \cdots + m_{i_n}^i(t)) \]

and

\[ M_{i_r}(t) = (m_1^i(t) + m_2^i(t) + \cdots + m_{i_n}^i(t)) \]

is termed the total local mean envelope (the slow-varying trend), and \( h_{i_n}^i \) is the fast oscillating component.

The analysis suggests that the local means can be used to detrend a nonlinear signal in a systematic manner.

Referring back to Fig. 3-1, one has from (58) and (59),

\[ r_1(t) = x(t) - h_{i_1}^1 = x(t) - c_1(t) = M_1(t) \]
\[ r_2(t) = r_1(t) - h_{i_2}^2 = r_1(t) - c_2(t) = M_2(t) \]
\[ \vdots \]
\[ r_n(t) = r_{n-1}(t) - h_{i_n}^n = r_{n-1}(t) - c_n(t) = M_n(t) \]

as expected from (1).

The physical meaning of the decomposition becomes evident. As shown in (60), at each step of the process, the residue \( r_i(t) \) becomes the local mean envelope of the previous step. Although these local mean signals carry only partial information, they can be used to describe overall system motion.

The following general non-stationary model has been proposed to describe the time evolution of the measured data \( t \),

\[ \hat{x}(t) = M(t) + h(t) \]
Where \( h_f(t) \) is the fast total fluctuating component superimposed on a slow time-varying mean, and \( M(t) \) is the slow, time-varying mean of the signal.

It has been observed that because successive approximations to system behavior result in smoother representations, finer local means may carry no physically meaningful interpretation, and they should be excluded from the model (61). This is discussed below in the context of numerical applications.

Several algorithms to estimate the instantaneous time varying mean can be derived within this framework. These models can be cast in the general form [48]

\[
M(t) \approx \frac{\sum_{k=1}^{r} (p_k(t)M_{kT})}{\sum_{k=1}^{r} p_k(t)}
\]

(62)

where \( r \) is a subset of the local means in (61), and the \( p_k(t) \) are suitable weight coefficients. Particular cases of this model are amplitude (energy), weighted instantaneous means, and simple, time-averaged arithmetic means.

### 3.5.2 EMD-based Filtering

Recent work which applies EMD techniques to measured data has shown that general arbitrary signal corrupted with noise can be decomposed into the general form [48]

\[
x(t) = \sum_{j=1}^{N} c_j(t) = \sum_{j=1}^{N} c_{j,HFC}(t) + \sum_{k=r+1}^{N} c_{k}(t) + \sum_{i=L}^{M} c_{i}(t)
\]

(63)

The first summation on the right-hand side corresponds to higher frequency components (HFC) or noise while the last summation involves uninteresting, non-sinusoidal-like characteristics.

By discarding insignificant or uninteresting behavior, the underlying phenomena of interest can be selected as

\[
\hat{x}(t) = x(t) - \sum_{i=L}^{M} c_{i}(t)
\]

(64)

where \( \hat{x}(t) \) is the filtered signal, and the indices \( L, M \) represent a subset of the IMFs obtained during the sifting process. Two applications of the method have been envisaged:

- **Noise reduction.** Subtracting the higher frequency components in (64), noise can be eliminated or reduced in a systematic manner
- **Data-adaptive smoothing or filtering.** Selected temporal frequency scales can be removed by subtracting from (61) frequency components of concern in the signal.

One crucial aspect of this overall approach is the processes to identify the intrinsic timescale in an IMF. Two procedures have been discussed in power system literature: (a) the use of general analytical models based on simplified system representations, and (b) the use of thresholding methods.
3.5.3 Wavelet-based Denoising

A remarkably successful approach for denoising signals is the wavelet shrinkage method introduced in [50, 51]. Consider the problem of recovering a function \( f(.) \) from noise contaminated observations,

\[
y_i(t_i) = f(t_i) + \sigma z_i, \quad i = 1, \ldots, n
\]

without assuming any particular parametric structure on its form, where \( y_i \) is the observed data point, \( f(.) \) is the unknown function of interest, the \( t_i \) are equally spaced points, \( z_i \) is standard Gaussian white noise, and \( \sigma \) is a noise level that may, or may not, be known. The problem now becomes that of estimating \( f \) with small mean-square error.

Let \( f = f(t_i)_{i=1}^n \) and \( \hat{f} = \hat{f}(t_i)_{i=1}^n \) denote the vectors of true (the uncontaminated signal) and estimated sample values, respectively. Minimizing the difference between these two vectors is accomplished by optimizing the mean-squared error, i.e. the \( L_2 \) norm risk, \( R \),

\[
R(\hat{f}, f) := E\left[\left\| \hat{f} - f \right\|_2^2\right] = \frac{1}{n} \sum_{i=1}^{n} E(f_i - \hat{f}_i)^2
\]

subject to the condition that with high probability, \( \hat{f} \) is at least as smooth as \( f \).

The whole process can be summarized as follows [36]:

1. **Compute the one-dimensional wavelet transform of the data**, \( w = Ty \), where

\[
w = [w_1 \cdots w_n]^T
\]

is the vector of wavelet coefficients, and \( T \) is the wavelet transform matrix

\[
T = \begin{bmatrix}
h_{11} & \cdots & h_{1n} \\
\vdots & \ddots & \vdots \\
h_{n1} & \cdots & h_{nn}
\end{bmatrix}
\]

2. **Compute an estimate of the standard deviation \( \hat{\sigma} \) of the noise in (65) from the wavelet coefficients** \( w \)

3. **Apply a hard or soft threshold function \( \delta_{\lambda_k} \) to the wavelet coefficients** \( w_k \)

\[
\hat{w}_k = \delta_{\lambda_k} \left( \frac{w_k}{\hat{\sigma}} \right)
\]

(67)

4. **Having determined the thresholds, compute the inverse fast wavelet transform of the resulting coefficients by using**

\[
\hat{f}_k = T^{-1} \hat{w}
\]

(68)

where \( \hat{w} = [\hat{w}_1 \cdots \hat{w}_n]^T \).
Step 3 in the above algorithm aims at determining which wavelet coefficients are indicative of the signals and should be retained, and which ones are likely the reflection of the noise and should be set to zero. This step is called thresholding or denoising.

It should be emphasized that, in principle, the same approach could be used to determine the physically meaningful signals of interest in EMD-based methods. This, however, has not been explored in the power system literature.

Figure 3-24 compares the mean values obtained from HHT and wavelet shrinkage analysis for the signal in section 3.2.10.2 as a function of the decomposition level. In these simulations, the matlab script \textit{wdcbm} was used to obtain level-dependent thresholds as well as the numbers of coefficients to be kept for denoising. Thresholds are obtained using a wavelet coefficients selection rule based on Birge-Massart strategy.

Both techniques are found to provide a good approximation to the time-varying mean, but considerable structural differences exist between the two methods that need to be evaluated in practical applications.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3-24}
\caption{Comparison of instantaneous mean estimates.}
\end{figure}
3.5.4 EMD as an Oscillation Detector

EMD can be used as adaptive basis to filter signals. An interesting extension of time-domain filtering system dynamic analysis that has not been explored is its use as a swing detector. Figure 3-25 suggests a possible implementation of HHT as an oscillation detector based on the implementation in [52].

The envisaged processing structure involves tree main processing steps: (a) time domain filtering, b) modal extraction, and (c) signal comparison. In the first step, the signal is decomposed into slow and fast components using EMD. As noted earlier, this amounts to differentiating, squaring and time-averaging in existing disturbance monitor implementations [52]. The second step involves feature extraction and modal identification using Hilbert analysis. Finally, in the latter step the oscillation detector compares the input signal with a preset level and issues an alarm flag. Recent work [53] discusses approaches to derive an early warning system based on the above techniques.

![Figure 3-25: EMD-based oscillation trigger.](image)

Figure 3-25 illustrates the application of this approach to the IMF2 in Section 3.2.10.2. It is seen that the method accurately identifies the periods of interest thus providing a basis for assessing modal properties. A critical assessment of the performance of EMD-based detectors in power system applications is, however, needed.
3.6 Concluding Remarks and Directions for Future Research

Nonlinear and non-stationary analysis techniques have proven useful in a number of applications including feature and modal extraction, detrending and denoising. HHT and other formulations are able to capture nonlinear phenomena associated with various transient processes and have the potential to be applied on line.

For large-scale application involving measured data, however, several numerical enhancements and improvements are needed for the development of reliable wide-area measurement systems.

These include the development of accurate local implementations and the development of criteria to identify and isolate physical phenomena of interest.

Other aspects of interest are the effects of missing data, linear and nonlinear trend and closely spaced frequency components on the performance of the methods.

Methods are also needed that take advantage of the multi-scale nature of the data. In particular, statistical methods are needed that combine the high temporal resolution of nonlinear and non-stationary methods with ability of statistical approaches to compress system information.

3.7 References for Chapter 3


Paul F. Ribeiro (Editor), Time-Varying Waveform Distortions in Power Systems, John Wiley and Sons Ltd, 2009.


Modal Identification of Transient and Ambient Data Oscillations using Local Empirical Mode Decomposition and Teager-Kaiser Energy Operator

Authors: E. Barocio, Bikash Pal, A.R. Messina

A3.1 Introduction

Automatic detection and characterization of transient oscillations is an important utilization of wide-area measurement, for system control purpose [1]. Time-synchronized wide-area measurement systems facilitate the integration of real-time information from the key selected system locations. However, real-time non-linear and non-stationary signal processing is very challenging [1, 2]. For accurate and timely assessment of system security, critical parameters governing the dynamics of the system have to be extracted and evaluated in near real-time [3-7]. The detection of instantaneous or locally-occurring transient oscillations is crucial to protection and control strategies [1]. Algorithms with the ability to extract modal information in the presence of noise and changing operating conditions are being developed and tested using measured data [5-8].

A variety of techniques have been proposed for automatic extraction of dynamic features from ambient system data including parametric and non-parametric mode estimation algorithms. Among these, mode-meter block processing techniques have been used for extracting and characterizing system dynamic features [8]. In parallel to this effort, recursive mode-meter algorithms with the ability to estimate power system electromechanical modes using a combination of new data points and past estimates, have been developed [8, 9]. Extending these approaches to extract modal parameters in near real-time, however, is very challenging, and may require long records of data. As pointed out in [1], the accuracy of any mode estimation technique is limited and may be affected by the very nature of system dynamic behavior as well as the approximations made in the computation.

In recent years, modifications to obtain more localized features of system behavior have been introduced [8]. Approaches based on the Hilbert-Huang Technique (HHT) have been extensively applied to the analysis of to near real-time tracking of local variations in damping and frequency of measured system data [10, 11]. The methods have been extended in several directions to further improve their performance [12-13]. A recent paper by Kamwa et al. [14] proposes an alternative algorithm to estimate the instantaneous amplitude and frequency based on the use of energy operators and linear filter decomposition that can overcome some of the limitations of HHT analysis. While these technique promise to yield a far more detailed understanding of transient behavior, several issues remain unsolved.

Characterizing abrupt changes in system behavior is the first essential step in developing oscillation and alarm triggers and control actions. In this paper, an alternative method that combines a local Empirical Mode Decomposition (EMD) of the data computed as a function of time with a short sliding window, with a nonlinear energy operator is proposed to extract temporal features from measured data. When combined with a sliding window based change detection algorithm, the method allows near real-time tracking of the evolving dynamics of the underlying oscillatory process and the onset of system instability.
Because of its instantaneously adaptive nature, the proposed technique enables identification and extraction of instantaneous modal information of general nonlinear and/or non-stationary signals, and is thus ideally suited to the study of time varying features that can be associated with complex oscillations.

A3.2 Low Frequency Oscillation Model

Low frequency electromechanical oscillations are inherent in interconnected power system operation. In many cases, these oscillations manifest, because of the perturbations of the operating equilibrium of the system. Experience with the analysis of complex measured data suggests that most oscillatory processes can be modelled as a linear/nonlinear combination of a small number of sinusoidal signals that are slowly time-varying in both amplitude and frequency [10].

In this research, a non-stationary model for modal characterization of non-stationary oscillations of the general form

\[
x(t) = \sum_{i=1}^{M} A_i(t) \cos(\phi_i(t) + f_i(t))
\]

is assumed, where is the time-varying amplitude, is the phase of the oscillation, and is a non linear time-varying function that captures trends, noise or other time-varying quantities. The model is reasonably general to accommodate single and composite oscillations and amplitude/frequency modulated (AM-FM) signals. Algorithms to detect and extract the dynamic features of the observed phenomena described by models of the form (1) are described next.

A3.3 Detection of Abrupt Changes Based on Energy Tracking

A3.3.1 The Teager-Kaiser Energy Operator

The Teager-Kaiser Energy Operator (TKEO) is a nonlinear operator developed to track the instantaneous energy content of a signal based on mechanical and physical considerations [17, 18]. Consider, to introduce this idea, a simple mass-spring system described by the mass normalized equation of motion

\[
c_i + (b/m)c_i + (k/m^2)c_i + (d/m) = 0
\]

where, and denote displacement, velocity and acceleration, respectively, is a spring constant, is mass and is the damping coefficient.

Assuming that nonlinear forces are relatively small, the solution of (2) is given by

\[
c_i(t) = a(t) \cos(\omega t + \theta(t))
\]

\[
c_i(t) = a(t) \sin(\omega t + \theta(t)) + \cos(\omega t + \theta(t))
\]

where is the time-varying amplitude, and is the energy dissipation rate.

Let now the mass normalized instantaneous energy in the system be defined by
Substitution of (3) into (4), yields

\[ E_T(t) = \frac{1}{2} \omega_i^2 c_i^2(t) + \frac{1}{2} c_i(t) \tag{4} \]

For a continuous signal \( c_i(t) \), the TKEO, \( \Psi(c_i(t)) \), is defined as [18]

\[ \Psi(c_i(t)) = (c_i(t))^2 - c_i(t) \langle c_i(t) \rangle \tag{5} \]

where the operator has the units of energy.

Several practical criteria for computing the TKEO have been discussed in the literature [17, 18]. We briefly review two of the most-widely used approaches and introduce the approximation used in this paper.

**A3.1.1.1 Discrete Teager-Kaiser Energy Operator**

For a discretized temporal signal, the TKEO can be expressed in time domain as

\[ \Psi(c_i(k)) = (c_i(k))^2 - c_i(k-1)c_i(k+1) \tag{7} \]

where \( k \) denotes the sample number. Higher-order approximations follow along the same lines and are not discussed here [18]. Of interest here, the nonlinear energy operator is an instantaneous feature that depends on three consecutive signal samples, is symmetric, and is independent of the initial phase of the signal.

**A3.1.1.2 Analytic representation of the TKEO**

An alternate representation of the TKEO for oscillatory signals can be obtained from (5) and is expressed as:

\[ \Psi(c_i(k)) = (A_\omega e^{-\tau})^2 - A_\omega e^{-\tau} \approx A_\omega^2 c_i(t)^2 \tag{8} \]

As shown in (8), the output of the TKEO is proportional to the instantaneous amplitude and frequency of the input signal. It can be shown that, if one restricts the value of \( \omega_i \) to \( \omega_i < \frac{\pi}{4} \) i.e., \( \left(F_i F_s\right) < 1 \) , where \( F_o \) is the frequency of oscillation and \( F_s \) is the sampling frequency, one can guarantee a relative error below 11% [16],[17]. Thus for instance, for a 0.27 Hz oscillation and a typical PMU sampling rate of 10 Hz, the above criterion is easily met.

There are several features intrinsic to the TKEO procedure that merit discussion. First, the method is applicable only for essentially noise-free harmonic signals. Second, because of discretization errors, the accuracy of the method can be destroyed by measurement and/or signal-processing noise. Moreover, the signal must not exhibit amplitude and frequency modulation over the global for the instantaneous frequency to have physical meaning.

In what follows, a method that combines a masking signal EMD method with a local EMD decomposition is proposed to extend the nonlinear energy operator approach to the analysis of multi-component AM-FM signals.
A3.4 Near Real-time Characterization of Modal Features

A3.4.1 Local Empirical Mode Decomposition (EMD)

The EMD has been proposed recently, as an adaptive time-frequency data analysis method to decompose a signal into essentially mono-component intrinsic mode functions (IMFs). As discussed by various authors, however, the sifting process relies on interpolation between successive extrema applied to the full length signal; this can result in over-decomposition of the signal and other undesirable effects [15, 16].

One way to avoid this is by defining a local EMD for a sliding window around a time interval of interest. With this approach, each set of IMFs is computed by applying the EMD procedure successively to sets of short data windows (slices) distributed continuously through the time series.

Three main considerations are introduced in this formulation:

1. The same number of sifting steps is applied to all windows in order to avoid possible discontinuities
2. It is assumed that there is no time overlap between segments.
3. The number of sifting operations is fixed a priori.

In addition, a masking signal method operating on top of this decomposition [13] is used here to effectively decompose a multi-component signal into essentially decoupled second-order nonlinear oscillators. The use of a moving window limits the generation of cross-terms in the application of energy operators and results in physically-motivated temporal modal information.

More precisely, at each observation window \( t_w \), the observed signal \( \hat{x}(t_w) \) is decomposed into \( N \) mono-component IMFs as

\[
\hat{x}(t_w) = \sum_{i=1}^{M} c_i(t_w) r_i(t_w)
\]

in which \( r_i(\cdot) \) is the residue and \( c_i(t_w) \) is a time-varying temporal component of the form

\[
c_i(t_w) = a_i(t_w) \cos(\phi_i(t_w))
\]

where \( a_i(\cdot) \) and \( \phi_i(\cdot) \) are the amplitude and phase of the \( i \)-th time-varying component, respectively.

A3.5 Extraction of Dominant Components Based on Energy Content

Once a signal is decomposed into a set of mono-component IMFs, the TKEO can be expressed in the form

\[
\Psi(\hat{x}(t_w)) = \sum_{i=1}^{M} \psi(c_i(t_w)) + \Psi(\hat{x}(t_w))
\]

or

\[
\Psi(\hat{x}(t_w)) = \sum_{i=1}^{M} \psi(c_i(t_w)) + \sum_{j=p+1}^{N-1} \psi(c_j(t_w)) + \psi(\hat{x}(t_w))
\]
where the terms \( \Psi(t_w) = \sum_{i=1}^{p} \Psi_i(t_w) \) contain the highest energy content among the IMFs, and the remaining \( M - p \) terms contain uninteresting non-sinusoidal characteristics. The metric
\[
\mathcal{R}_i(t_w) = \left( \frac{\Psi_i(t_w)}{\Psi(t_w)} \right)^{i=1,2,\ldots,M}
\]
is used as a thresholding criteria to measure the relative energy contribution from each IMF, where \( \| \) represent the norm 2 of the energy operator associated with each IMF. By discarding contributions below a given threshold, the physically-meaningful components are retained.

### A3.5.1 Threshold Determination for Online Detection of Emerging Oscillations

Assuming that the signal characteristics remain constant or vary slowly over time, a simple algorithm to capture transient events locally in time is proposed. In this approach, a sliding-window based non-parametric Cumulative Sum (CUSUM) method [20] is used to detect abrupt changes in the signal’s statistical properties as well as to minimize false alarm rates.

Starting with an initial time observation window \( t_w \), the transient energy, \( \Theta \), averaged over the time window is calculated as
\[
\Theta = \frac{1}{t_w} \sum_{k=1}^{t_w} \Psi(t_k)
\]
where \( t_w \) is the number of samples in the observation window, and \( \Psi(t_k) \) is the sum of the individual energies in (12). The window is then slid forward in time, and the difference between the transient energy, averaged over the new time window and that of the threshold value \( \Theta \) is computed.

The cumulative sum, \( S_i \), is then defined as
\[
S_i = S_{i+1} + (\Psi(t_k) - \Theta)
\]

From the above analysis, the time intervals in which the CUSUM score is less than a given threshold (i.e. a straight path), indicate normal, ambient operation in which the statistical characteristics of the signal remain invariant; if the cumulative sum exceeds the threshold, the criteria will indicate a change in statistical properties, and therefore a transient change in system behavior.

Using this simple idea, an energy threshold level in the TKEO output is defined as
\[
\Gamma = \beta \Theta
\]
in which \( \beta \) is a factor to be determined and \( \Theta \) is the average value of the energy threshold determined by CUSUM analysis (i.e. under ambient conditions). The onset time of transient behavior is determined as the time instant where \( S_i \) exceeds the preset threshold. A better alternative currently being investigated is to dynamically select the threshold value to identify the start of a transient period by using the standard deviation of the energy components within a given window.

To reduce the possibility of false alarms and to reduce the subjective nature of this metric, a normalized energy index, \( Y(t_k) \) is then proposed as follows:
This energy index can be used to raise an alarm flag when its value exceeds a threshold value, i.e.,

\[ \text{Flag}(k) = \begin{cases} 1.0 & \text{if } Y(k) > 1.0 \\ 0 & \text{otherwise} \end{cases} \]  

A decision threshold higher than 1 (Flag = 1.0) indicates "the onset of transient behavior". More generally, if the warning flag, Flag(k), exhibits a high value around 1.0 over several (τ) seconds, a sudden change in system behavior is detected and the algorithm initiates a process for determining modal properties in near real-time using an extended Discrete Energy Separation Algorithm (DESA). In real-time situations, this algorithm is expected to capture emerging transient processes or locally-occurring events associated with major system changes and reduce false alarms.

### A3.5.2 Energy Operator Demodulation Algorithm

Using the above sinusoidal model, a simple algorithm to extract modal features from measured data is proposed. Assuming that the amplitude \( a_k \) and the instantaneous frequency \( \omega_{\text{inst}} \) do not vary too fast in time or too greatly compared to their average value, we can write \[17\]:

\[ a_k = \frac{|\Psi(c_k)|}{\Gamma} \]  

and

\[ \omega_{\text{inst}} = \cos^{-1}(1 - \rho_k) \]  

where

\[ \rho_k = \frac{\Psi(y_k) + \Psi(y_{k+1})}{4\Psi(y_{k/2})} \]  

and \( y_k = c_k - c_{k-1} \).

The frequency in Hertz can then be defined as:

\[ F_k = \frac{\omega_{\text{inst}}}{2\pi} \]  

As is apparent from eqns. \[19\] through \[21\], tracking of a signal’s amplitude and frequency requires only three (or more) recent data points. The accuracy in the amplitude and frequency estimates depends on both, the EMD and the sampling rate. In addition, the signal should be free from noise for enhanced results.

### A3.5.3 Estimation of Energy Dissipation Rate using TKEO Information

Drawing upon the above discussion, a simple approach to estimate the damping or energy dissipation rate (\( \sigma \)) using the TKEO is proposed. Let now the \( i \)th IMF be expressed in the form \[10\]. Application of the TKEO to this component yields
\[ \Psi(c_i(k)) \left( A e^{-\sigma_i^k} \right) \left( \omega_i(k) \right) \]  

(22)

Taking the natural logarithm on both sides of (22) and solving for \( \sigma \) yields

\[ \sigma_i(k) = \frac{1}{2k} \ln \left( \frac{\Psi(c_i(k))}{A_k^k \omega_i(k)} \right) \]  

(23)

Equations (19), (21) and (23) are used in the following to extract modal parameter in a near real-time environment. Figure A3-1 gives a conceptual representation of the adopted method.

---

**A3.6 Application to a Synthetic Time Series**

Improved understanding of the nature of the proposed framework can be obtained from the analysis of synthetic signals. We next explore the potential application of the TKEO to track in near real-time the energy content of signal as well as local variations of the original signal using a local EMD example.

**A3.6.1 Example: Noiseless Multi-component Signal**

As a first example, consider a multi-component, noise-free signal of the form given in (1), which is designed to approximate the behavior of a time-varying two-tone signal,

\[
\begin{align*}
    x(t) &= \begin{cases} 
        c_1(t) = 0.0 & \text{for } -30 \leq t \leq 0 \text{ s} \\
        c_2(t) = c_3(t) + c_4(t) & \text{for } 0 < t \leq 30 \text{ s}
    \end{cases}
\end{align*}
\]  

(24)

where:

\[
\begin{align*}
    c_1(t) &= 0.5e^{-0.1t} \sin(2\pi(1.2)t + 1.0) \\
    c_2(t) &= 0.1e^{-0.1t} \sin(2\pi(0.27)t + 1.0)
\end{align*}
\]  

(25)
In this representation, \( c_1(t) \) represents a zero amplitude component; subsequent terms \( (c_2(t), c_3(t)) \) represent two periodic components with frequencies of oscillation of 1.2 Hz and 0.27 Hz, respectively.

For completeness, the signal was then contaminated with white Gaussian noise to yield a SNR of 20 dB. The noise was generated using the Matlab function “awgn”; a sampling frequency of 20 Hz is used in the analysis. The synthetic time series \( x(t) \), the TKEO, \( \Psi \) and the decision threshold, \( \Gamma \) are shown in Figure A3-2.

Following the procedure outlined in the previous section the transient period was identified using statistical analysis. The procedure was applied with a 5-second window intended to separate natural variability from the oscillatory components. The low-frequency (0.27) Hz and high-frequency (1.2) Hz response and the temporal nature of system response are clearly distinguishable from the zero-amplitude behavior in Figure A3-2(b) showing that the proposed index accurately identifies the onset of system oscillations.

![Figure A3-2: Detection and identification of transient period](image)

Once that the dynamic event has been detected the output of the TKEO is used to extract modal content. Application of the energy index \( \Re \) in (13) reveals that motion is governed by the periodic components \( c_2 \) and \( c_3 \) in (25). Figures A3-3b and A3-3c show the instantaneous frequency and damping extracted using the proposed procedure. For completeness, the IMFs are also plotted in Figure A3-3a. It is of interest to note, the signal noise was suppressed by the local EMD with masking technique. The example demonstrates that the local EMD procedure can be used to filter out noise beneath the signal.

An interesting aspect of this algorithm is the ability to estimate the modal parameters even when the oscillation decreases rapidly. This can be seen in the last 10 sec (20 to 30 sec) of the records in Figures A3-2a and A3-3a.

This observation opens the possibility to studying the dynamics of slow fluctuations associated
with ambient operating conditions.

**A3.6.2 Sensitivity to Window Length and Sampling Frequency**

In this section, a statistical analysis to examine the sensitivity of the TKEO method to the length of the observation window \( t_w \) and sampling frequency \( F_s \) is conducted.

To illustrate the performance of the proposed method we consider the synthetic time series

\[
\hat{x}(t) = c_2 \hat{\theta} + c_3 \xi + \xi
\]

where \( c_2 \hat{\theta}, c_3 \theta \) are given in (24), and \( \xi \) is a Gaussian white noise process. This model enables the ability of the TKH method to separate and estimate the instantaneous parameters to be evaluated.

In order to verify the accuracy and statistical performance of the proposed method in the presence of noise, a similar procedure to that in [4] was used to evaluate the statistical performance of the method. The procedure can be summarized as follows:

1) Add white Gaussian noise to the vector signal \( c_{mix} \theta \) using the Matlab function `awgn`
2) Generate multiple data sets by adding independent noise to model (25).
3) Compute modal information using (19)-(23)
4) Estimate the mean square error (MSE) of the approximation for each data set by using the metric

\[
MSE_{it}(l) = \frac{1}{WL} \sum_{k=1}^{WL} (\hat{\theta}_k - \theta_{true})^2
\]

where \( WL = t_w \cdot F_s \) denotes the total window length, \( \hat{\theta}_k \) is the instantaneous parameter estimated by the TKH method and \( \theta_{true} \) is the known true parameter.
5) Compute the Global Average Error (GAE) using the metric

\[
GAE_{it} = \frac{1}{N} \sum_{l=1}^{N} (MSE_{it}(l))
\]

where \( N \) is the total number of data sets.
In this study, 50 runs ($N=50$) for different $t_w$ (5, 10, 15, 20, 25 and 30 sec) and various $F_s$ (20, 15, 10 and 5 Hz) were performed. In all cases, Gaussian noise of 20 dB was added to each set.

For clarity, attention is focused on the 0.27 Hz component; an energy dissipation rate of 0.06 was considered. This corresponds to component $c_3(t)$ in signal $c_{\text{mix}}(t)$ in (25).

Figures A3-4 and A3-5 give the $GAE$ for this case as a function of the window length and sampling frequency.

From these plots, several general observations can be made:

- The algorithm is sensitive to both, the width of the window applied, and the sampling frequency. For short $t_w$ (5 sec) and low $F_s$ (5 Hz), the error is low (the GAE is estimated to be 0.3).
- Because the method acts like a natural filter to noise at low sampling frequencies, $F_s$, the local EMD accurately decomposes the signal into its constituents components (mono-component signals).
- Increasing both the sampling frequency and the signal’ noise decreases the accuracy and robustness of the proposed method. This motivates the use of adaptive filtering algorithm to supress the signal’ noise.

Figure A3-4: GAE of the reconstructed time series relative to the 0.27 Hz component as a function of the time window and sampling frequency.
Figure A3-5: GAE of the reconstructed time series relative to the 0.06 energy dissipation rate component as a function of the time window and sampling frequency.

Figure A3-5 shows the corresponding GAE relative to the energy dissipation rate. It is seen that the approximation (26) yields better estimates at low sampling frequencies, $F_0$ (10 and 5 Hz) and short analysis window (5 sec). From this study, we can conclude that the TKH method exhibits a good performance under short window lengths and slow sampling rates.

We remark that proper choice of the analysis window is an important issue in online tracking of oscillations; in addition, a low sampling frequency is compatible with typical sampling frequencies used by modern PMUs.

**A3.7 Application to Real Measured Data**

The practical applicability of the proposed technique to extract the temporal behavior of electromechanical modes of oscillations from time-synchronized PMU measurements is discussed. Synchronized phasor measurements from actual field measurements in various systems under both, transient and ambient conditions are used to develop and test the algorithms. The measured data were collected by PMUs with different sampling frequencies. Two system events are analyzed in this section.

**A3.7.1 Analysis of Transient Event in the Mexican Interconnected System (MIS)**

The first data sets (signals MIS) represent nodal frequency from the January 1, 2004 event in the Mexican interconnected system. Oscillations from this event involved severe frequency, voltage and power changes throughout the northern systems in the MIS and resulted in load shedding and the disconnection of major equipment [21]. Signals MIS are chosen to test the ability of the method to track the time evolution of this mode following large disturbances.
Figure A3-6: Map and time traces of MIS disturbance recordings used in the study.

Figure A3-6 depicts the time traces of the observed oscillations measured by PMUs showing time intervals of abrupt, transient behavior. These measurements are chosen to identify specific temporal behavior likely to be found in measured data. It is important to note that oscillations present complex dynamic behavior related to multi-component and non-linear signals.

A3.7.2 Determination of Threshold Settings

Since the exact choice of thresholds used during the detection stage depends on the magnitude of the initiating event, studies were conducted to verify their performance under various window lengths. Figures A3-7a, A3-7b and A3-7c show the CUSUM charts as a function of different window lengths, $t_w$. 
Figure 7 (a): CUSUM for different time window observation MIS_S1

Figure 7 (b): CUSUM for different time window observation MIS_S2

Figure 7 (c): CUSUM for different time window observation MIS_S3

Figure A3-7: CUSUM charts as a function of different window lengths, $t_w$.

From these results, the time observation windows selected were 25 for all the MIS signals. This selection was based on the minimum of the values obtained with the index $\max \left( \left( \frac{s_i}{\omega} \right) \right)$. These observation windows guarantee that the statistical characteristics of the signal are not strongly time dependent, i.e., the energy values remain close to the average of the time observation window. Figure A3-8 shows the number of false alarms as a function of weight factor ($\beta$). As shown in this plot, increasing the threshold value $\beta$ may cause the detector to miss potential events of interest, while decreasing its value tends to have
more false alarms. Efforts to automate these criteria are underway.

![Figure A3-8: Number of false alarms for different weight factors](image)

Table A3-I summarizes the threshold settings used in the studies. Weight factors, $\beta$, of 3 was found to perform well for the scenarios under study. These threshold settings are deemed sufficient for this study, but not necessarily optimal for all applications.

Table A3-I: Threshold Settings for Dynamic Events

<table>
<thead>
<tr>
<th>Signal</th>
<th>Time window/Sampling frequency</th>
<th>Weight Factor $\beta$</th>
<th>Alarm Value $\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIS_S1</td>
<td>Window length of 250 samples corresponding to 25 seconds time duration and $F_s$ of 5 Hz</td>
<td>3.0</td>
<td>0.1150</td>
</tr>
<tr>
<td>MIS_S2</td>
<td>Window length of 250 samples corresponding to 25 seconds time duration and $F_s$ of 5 Hz</td>
<td>3.0</td>
<td>0.2405</td>
</tr>
<tr>
<td>MIS_S3</td>
<td>Window length of 250 samples corresponding to 25 seconds time duration and $F_s$ of 5 Hz</td>
<td>3.0</td>
<td>0.2405</td>
</tr>
</tbody>
</table>

**A3.7.3 Transient Event Detection**

Once the threshold settings ($\beta, \Gamma$) have been determined, the proposed detection method was applied to the test data sets. Inspection of the nonlinear energy information in Figure A3-9 clearly indicates that the periods of greater activity are accurately identified (transient periods TP$_1$ and TP$_2$).
Figure A3-9: Time evolution of the nonlinear energy operator.

Figure A3-10 shows the output of the CUSUM method corresponding to the instantaneous energy in Figure A3-9. These plots illustrate different peculiarities during the various stages of the time evolution of the observed signals. For normal, ambient conditions, the cumulative sum, $S_i$, is either constant or increases steadily with time (time intervals 0-125 sec for signal MIS). Abnormal transient operation, on the other hand, is detected if the CUSUM deviates from the mean value. Of particular interest in these plots, is the ability to characterize both the onset and offset time of transient behavior as illustrated in Figure A3-10.

Figure A3-10: Output of the CUSUM method showing the start of transient activity.

Once the onset time of transient behavior has been detected, the extended DESA algorithm is triggered to compute the damping system in an automatic manner.
A3.7.4 Estimation of Modal Parameters

Figure 11 depicts the instantaneous frequency of the dominant IMFs extracted using the proposed algorithm. The remaining modes contain noise and other featureless characteristics of little relevance and are not discussed here.

Simulation results in Figure A3-11 allow confirming, that the proposed approach can be used to characterize quasi-stationary behavior and identify the evolving frequency content arising from control actions or severe system perturbations. This is especially true for transient periods TP1 and TP2 in Figure A3-9, while some fluctuations are present under quasi-stationary conditions. These fluctuations are likely to be caused by numerical errors in the estimation process and require further analysis. It is important to note that the sliding window approach results in a smoother representation of temporal behavior as compared to Hilbert transform demodulation algorithms [21] and provides facilities to characterize transient operating conditions in near-real-time.

A3.7.5 Comparison with Other Modal Extraction Methods

Table A3-II compares Prony analysis results with the proposed technique. For completeness, numerical estimates obtained using the Hilbert transform demodulation algorithm in [12]. For direct comparison to Prony analysis, instantaneous values are averaged over selected time windows. Results compare well with other approaches.
Table A3-II:

**COMPARISON METHODS DURING TRANSIENT EVENTS**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Time window (sec)</th>
<th>Prony</th>
<th>Hilbert</th>
<th>EMD/TKEO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$F_0$</td>
<td>$\sigma$</td>
<td>$F_0$</td>
</tr>
<tr>
<td>MIS_S1</td>
<td>125-142</td>
<td>0.234</td>
<td>0.006</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
<td>142-160</td>
<td>0.501</td>
<td>0.015</td>
<td>0.517</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.612</td>
<td>0.040</td>
<td>0.601</td>
</tr>
<tr>
<td>MIS_S2</td>
<td>126-138</td>
<td>0.231</td>
<td>-0.028</td>
<td>0.241</td>
</tr>
<tr>
<td></td>
<td>138-160</td>
<td>0.621</td>
<td>-0.038</td>
<td>0.623</td>
</tr>
<tr>
<td>MIS_S3</td>
<td>122-137</td>
<td>0.241</td>
<td>0.026</td>
<td>0.243</td>
</tr>
<tr>
<td></td>
<td>137-160</td>
<td>0.271</td>
<td>0.23</td>
<td>0.261</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.462</td>
<td>0.016</td>
<td>0.473</td>
</tr>
</tbody>
</table>

The bolt numbers in the Table A3-III shows correctly the periods when oscillations are coming unstable. The three methods present similar results.

**A3.7.6 Analysis of Ambient Measurements of Finland System (FIND_S)**

In the literature has been applied successfully several parametric algorithms to analyze the ambient operations conditions of the power systems using measurements of the PMU [10]. These algorithms has been denominated in the literature as block-processing algorithms, which operate on blocks of the PMU data, usually using from 5 to 20 minutes. Following a similar philosophy the EMD-TKEO is applied to analyze the ambient conditions data. To illustrate the performance of the method, field measurement data from the Finish system is used [22, 23]. This set of data corresponds to frequency nodes from 2 different locations in the Finnish system. All frequency measurements present natural deviations around of frequency system (50 Hz).
The PMU data obtained from two different substations at different day let us analyze the effect of loading system into the dynamic modal parameters estimation. Therefore, data from FIND_S1 were obtained during 06-09-08 at 9 a.m. and one week later 13-09-08 at 17 p.m. The data set for FIND_S2 were obtained two month later at 05-11-08 at 3 am and 5 am respectively. Both data information let us know the impact of load at different time in the modal parameter system.

**A3.7.6.1 Data Pre-processing Information for Data Sets**

As is well known any algorithm could estimate modes that are not true system modes. To discriminate the artificial modes is developed a preliminary study using a conventional approach based in Fast Fourier Transform analysis. This study let us know more about the true modes. Therefore, the PMU data mentioned in the Figure A3-13 are pre-processed for analysis. Bus frequency signals were obtained from numerical derivative of the voltage angle. First, the signal’s trend is removed from the signals using a suitable detrending algorithm [11]. After the low-pass filter with cut-off frequency of 3 Hz is applied to signals to remove high frequency components beyond this cut-off frequency. Thus let us remove the signal’s trends and noise from PMU data. Figures 14 show the bus frequency under study and their respectively FFT analysis.
From frequency analysis, we can see a dominant frequency of 0.35 Hz as well as a frequency of 0.50 Hz. Although, the spectrum essentially tells us which frequencies are contained in the signal, as well as their corresponding amplitudes, but does not provided any information at which times these frequencies occur. In the next section this data set are evaluated since point of view of time-frequency analysis.

**A3.7.7 Time-Frequency Analysis for Data Sets**

In this section is validating the performance of proposed method. The time-frequency resolution aspect is evaluated and compared with other similar techniques such that Short-Time Fourier Transform (STFT) and Continuous Wavelets Transform (CWT). These methods were applied to data records showed in the Figure A3-13.

**A3.7.7.1 Data Pre-processing Information for Data Sets**

The results in the Figure A3-14a show all estimates for FIND_S1 at different time. Figures A3-14b, A3-14c, A3-14f and A3-14g show the Spectrograms obtained from the application of STFT and CWT for the signal FIND_S1. The red colors represent maximum values and the blue colors represent minimum values of the power spectrum density [dB]. On the other hand the estimates obtained from EMD/TKEO algorithm employ the instantaneous amplitude and frequency in a complementary way to estimate the relevant frequencies content into the FIND_S1 signal.

Following with the analysis, note the effect of loading time. Figures A3-14a, A3-14b, and A3-14c present just a dominant frequency of 0.35 Hz, meanwhile in the Figures A3-14d, A3-14f and A3-14g appear two dominant frequencies of 0.35 and 0.54 Hz. Although the methods found similar results, the EMD/TKEO presents a better time-frequency resolution that STFT and CWT. It is important remark that the methods also detect the presence of higher energy on the FIND_S1
signal. For example in the Figures A3-14a, A3-14b and A3-14c we can see this issue in the period of 0 to 1000 seconds. In particular the EMD/TKEO algorithm presents a better time frequency resolution.

A) EMD/TKEO for FIND_S1 at 06-09-08

b) STFT for FIND_S1 at 06-09-08
c) CWT for FIND_S1 at 06-09-08

d) EMD/TKEO for FIND_S1 at 13-09-08

f) STFT for FIND_S1 at 13-09-08
g) CWT for FIND_S1 at 13-09-08

Figure A3-14: Instantaneous frequencies of the FIND_S1 data obtained by using -
EMD/TKEO method; the STFT method and CWT method.

A3.7.7.2 Time-Frequency Analysis for FIND_S2 Data Record

The results in the Figure A3-15 show all estimates for FIND_S2 at different time. Figures 15b, 15c, 15f and 15g show the Spectrograms obtained from the application of STFT and CWT for the signal FIND_S2. Figures 15a, 15b, and 15c present three dominant frequencies 0.35 Hz, 0.86 Hz and 1.22 Hz, meanwhile in Figures 15d, 15f and 15g appear only two dominant frequencies of 0.36 Hz and 1.22 Hz. Although the methods found similar results, the EMD/TKEO presents a better time-frequency resolution that STFT and CWT.
A method for identifying temporal behaviour from measured data has been presented. The method combines the ability of a local empirical decomposition to capture fast temporal behaviour with that of energy operators to track modal properties and identify suitable changes in signal’s energy. By combining a local method with an energy operator, the method overcomes the burden of analyzing long system records and provides a smoother representation of temporal behavior as compared to other representations.

A3.8 Conclusions

A method for identifying temporal behaviour from measured data has been presented. The method combines the ability of a local empirical decomposition to capture fast temporal behaviour with that of energy operators to track modal properties and identify suitable changes in signal’s energy. By combining a local method with an energy operator, the method overcomes the burden of analyzing long system records and provides a smoother representation of temporal behavior as compared to other representations.

A3.9 References


[22] FINGRID operational PMU data, provided under Non disclosure agreement between Imperial, ABB, and FINGRID.


A3.10 Appendix 3A: The Teager-Kaiser Energy Operator Application on Chirp Linear Signal

The tracking capability of Teager-Kaiser Algorithm is tested using a Chirp signal. The Chirp signal is described with the following equation:

\[ c_2 \theta = \cos(2\pi f(t)) \]

with:

\[ f(t) = f_o + \beta t \]

where:

\[ \beta = \frac{f_1 - f_0}{t_1} \]
where:

- $f_0$: is the instantaneous frequencies at time 0
- $f_I$: is the instantaneous frequencies at time $t_1$.

Figure A3-16: Instantaneous modal information obtained from Teager-Kaiser Algorithm.
4.1 Introduction

The objective of this chapter is (i) to review the experience to date of the Electricity Supply Industry (ESI) in the Identification of Electromechanical Modes, and (ii) to learn of the issues in the application and operation of a continuous modal estimation system in practice. Furthermore, the current usage of staged single-shot methods such as braking resistor insertion tests, and non-staged events such as system disturbances, are of interest. For a system operator who is considering the introduction of a continuous modal estimation system the steps followed in setting up and installing a system may be of value. To identify the various issues a questionnaire was prepared and circulated to a number of system operators during the first quarter of 2011. The questionnaire covered the following topics.

A copy of the relevant parts of the questionnaire is included in the Appendix to Chapter 4.

Of those invited to respond to the Questionnaire the responses of ten system operators and a commercial software vendor were received. The respondents are listed in Table 4-I.
Table 4-I: Respondents

<table>
<thead>
<tr>
<th>Organization Name (and abbreviation)</th>
<th>Department/Unit</th>
<th>Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australian Energy Market Operator (AEMO)*</td>
<td>Network Models – Planning</td>
<td>Paul Ravalli</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Joseph Leung</td>
</tr>
<tr>
<td>Hydro-Quebec Research Institute (IREQ) (responded to sections 4.4 &amp; 4.7 to 6)</td>
<td>Power Systems &amp; Mathematics</td>
<td>Mathieu Perron</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Innocent Kamwa</td>
</tr>
<tr>
<td>National Grid, UK (NG)</td>
<td>Network Operations</td>
<td>Andrew Kensley</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Alex Carter</td>
</tr>
<tr>
<td>NTNU ** / Stattnett</td>
<td></td>
<td>Kjetil Uhlen</td>
</tr>
<tr>
<td>Powerlink Queensland (PQld)*</td>
<td></td>
<td>Tom Anderson</td>
</tr>
<tr>
<td>Swissgrid ag</td>
<td>System Operation Support</td>
<td>Walter Sattinger</td>
</tr>
<tr>
<td>TransGrid (TG)</td>
<td>Network Planning &amp; Performance/System Planning &amp; Analysis</td>
<td>Don Geddey</td>
</tr>
<tr>
<td>New south Wales, Australia*</td>
<td></td>
<td>Colin Parker</td>
</tr>
<tr>
<td>Transpower New Zealand Ltd. (TPNZ)</td>
<td>Grid Development</td>
<td>Richard Sherry</td>
</tr>
<tr>
<td>Bonneville Power Administration &amp; Supporting Organizations (BPA)</td>
<td>WECC*** Technical Support PNNL Laboratory</td>
<td>William A. Mittelstadt</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(BPA ret)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>John Hauer (BPA ret., PNNL)</td>
</tr>
<tr>
<td>Commercial Software Vendor: Psymetrix Limited</td>
<td>Power Systems Group</td>
<td>Douglas Wilson</td>
</tr>
</tbody>
</table>

* References to ‘Australia’ generally refer to matters concerning the Eastern Australian Grid.
** Norwegian University of Science and Technology: Answering for the Norwegian Transmission System Operator, Stattnett
*** Western Electricity Coordinating Council

Note: In the following sections, the number ahead of the paragraph, i.e. Q1, Q2,... corresponds to the question number in the survey.

4.2 Operational Experience with Modal Estimators

4.2.1 Primary Role of Modal Estimation in System Operation

Q1 The respondents state that continuous monitoring of system modes is performed to identify both inter-area and local-area modes. In the case of BPA, it is mainly the inter-area modes that are of interest. In some organizations, the relatively lightly damped modes associated with speed governors may be of concern.

Q2 The main purpose is to continuously assess the security of the system, but also for (i) plant performance assessment and commissioning; (ii) identifying the low-frequency, frequency-control modes of governors; (iii) small-signal model validation and calibration; and (iv) correlation of poor damping with potentially critical operating conditions.
Monitoring of the damping of inter-area modes is a prime concern of the Independent System Operators (ISOs). For Swissgrid it is the North-South and East-West modes within the Continental European system; for NG it is power flow across the England-Scotland border.

In Eastern Australia two independently developed continuous modal-estimation systems are in operation – (i) the OSM (Oscillatory System Monitor), developed by Queensland University of Technology (QUT) and TransGrid [28, 29, 30]; and (ii) the StormMinder system developed by Pysmetrix. These two systems are used to identify the inter-area modes of the system. The StormMinder system is also used to identify certain local modes. A third system, PhasorPoint (also developed by the same vendor and supercedes StormMinder) is under trial by AEMO. The two systems have complementary roles – (i) the OSM system, which requires relatively long window lengths, is used primarily for model calibration; and (ii) the StormMinder system is used primarily for system security assessment. Both systems are employed to validate the performance of damping controllers (e.g. PODs fitted to SVCs). The deployment of independently developed modal-estimation systems has value in validating the modal estimates. NTNU states that system security is the main purpose of modal-estimation in the Nordel power system. However, the modal estimation system is a prototype installation and presently its main purpose is still to get practical experience.

In BPA both security and situational awareness are immediate objectives in modal surveillance. Longer-term objectives include extending the knowledge base against which situational awareness tools compare present system behavior for evidence of abnormalities. Modal surveillance in the western interconnection is part of a general effort to enhance the use of measurement-based information in all aspects of grid management [33]. The contributions to system security and performance take many forms, and occur in many time frames. Much of this is done off line, e.g. by WECC technical groups. The BPA/DOE Dynamic System Identification toolset (DSI Toolbox or DSItools) was developed and widely distributed as the core resource for this general effort [39].

Q3 Various responses are offered to the question: ‘What is the threshold for unacceptable modal damping?’

Several organizations specify a damping ratio of 3%, and for some of these organizations, this level is adjustable according to certain rules.

The National Grid defines the following thresholds: “The ‘amber’ or ‘alert’ limit is an amplitude of 70MW for oscillations with decay time constant of 12 seconds and below. The amplitude limit reduces in proportion to increasing decay time constant up to a decay time constant of 40 seconds. The ‘red’ or ‘alarm’ limit is an amplitude of 100MW for oscillations with decay time constant of 12 seconds and below. The amplitude limit reduces in proportion to increasing decay time constant up to a decay time constant of 50 seconds.”

Swissgrid states that “On the Continental European grid, only significant frequency oscillations recorded at one or several substation(s) of the interconnected system will be considered for further offline analysis: the amplitude of the frequency oscillations must be at least 20 mHz peak to peak during at least 10 consecutive oscillation cycles. Since the period of the inter-area oscillations are in the range 3 to 7 seconds, the duration of the phenomenon should be at least several tens of seconds.”

Swissgrid advise that the concurrent availability of (i) time-series estimates of the damping factors, oscillation frequencies and amplitudes; and (ii) continuous recordings of power and frequency from a number of points in the system provides quite a complete view of the dominant system modes particularly during oscillatory events. The above combination of data provides the foundation of a reliable oscillation alarm. For Operations, Swissgrid is creating a ‘traffic light’ system which indicates the current status of system stability.
In Australia, the Australian National Electricity Rules Clause 5.1.8 [31] defines operations damping requirements as follows, “...To assess the damping of power system oscillations during operation, or when analyzing results of tests such as those carried out under clause 5.7.7 of the Rules, the Network Service Provider must take into account statistical effects. Therefore, the power system damping operational performance criterion is that at a given operating point, real-time monitoring or available test results show that [i] there is less than a 10 percent probability that the halving time of the least damped mode of oscillation will exceed ten seconds, and [ii] that the average halving time of the least damped mode of oscillation is not more than five seconds”

BPA state that a primary purpose of continuous modal estimation is to provide an alarm of at-risk conditions and provide guidance for operator action. Acceptable damping in BPA is largely judged by observed normal behavior for the mode in question —e.g., as encapsulated in the knowledge base for situational awareness. The WECC standard is that all modes should be positively damped. Historically, damping in the range of 5% to 10% had been normal for modes in the western interconnection. Some of these modes however have damping as high as 12% or more. Normal system damping should be such that the system will remain positively damped for the next outage.

BPA comments that appropriate response to low-damping alarms and oscillation alarms is a subject of continual study [44]. Both have many possible causes, and sharply focused responses require considerable information. Critical parameters for assessing the condition of an inter-area mode are strength, damping, frequency, and interaction pattern—in this order. A competent modal alarm system will include the first three parameters; interaction patterns are needed to associate the mode with specific facilities, and to determine whether a strong but otherwise normal mode reflects an “exogenous input” such as adverse controller action.

Q4 The actions taken if the modal damping threshold is breached vary between organizations. In some cases the organization that is monitoring damping is not directly responsible for system operation. Such organizations pass on information and advice to the ISO who is responsible both for system security and for directing any necessary evasive measures that are required to restore system security. Some organizations are now investigating what kind of actions can be taken. With interconnection across a number of countries in Europe, Swissgrid states actions are not easy to identify in a highly meshed network and are not easy to implement in a market-driven environment. The actions taken by AEMO are: “For inter-area modes, if the online monitoring shows an unacceptable damping, the relevant interconnector flows are reduced until the oscillatory behavior meets the 10 second halving time operation criterion. For flow reduction, specific interconnectors are defined for the various inter-area modes. For local mode oscillations operators may adjust system voltages or machine active or reactive power output in an attempt to isolate the cause of the problem. Operators will also examine other sources of information to identify areas where oscillations are more pronounced.”

National Grid provides the following comments. Firstly, in the case of inter-area oscillations (0.4-0.7 Hz) they reduce power flows by instructing generators to reduce output. This action is carried out in stages and the level of reduction depends upon whether the ‘amber’ or ‘red’ alarm has triggered. In each case, once the observed oscillations return to the ‘green’ region no further reduction in transfer is carried out. The cause of the oscillations is then investigated and offline studies are carried out to re-assess stability limits before transfers are allowed to increase. Secondly, for all other sustained alert or alarm status oscillations the first action would be to determine which generating stations are affected and to what extent. This may be done by contacting individual generators. Further investigation will include examining alarms from the Integrated Energy Management System, and noting other system effects such as significant voltage swings. Thirdly, when low frequency oscillations (< 0.4 Hz) occur they tend to be

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1 Numbering inserted for clarity.
caused by a malfunction in the speed control loop of the generator governor; such oscillations can be large and undamped. Depending on severity and location these oscillations could raise an alert or alarm mode in which case the Control Engineer may instruct suspect machine(s) to revert to a frequency insensitive mode. In BPA, a preferred solution is to reduce flows on one or more key paths, or to adjust controller status on some critical facility. In one case insertion of switched series capacitors is used. The final resort, probably reserved for actual oscillations, is to separate the interconnection into stable islands by cutting critical interaction paths.

Q5 Users of continuous modal estimation systems report that a variety of ‘other information’ (apart from estimates of modal parameters) is provided by the modal-estimator.

Continuous modal estimators used by BPA and BPA collaborators provide mode shape (right-eigenvector components) of the modes, various measures of the “strength” or “contribution” of identified modes in the signal(s), and time-domain/frequency-domain displays for identified parametric models. Non-parametric (e.g., Fourier) measurements of these same quantities are displayed for comparison, and/or as stand-alone indicators of oscillatory behavior. The technologies are well established, and various non-parametric displays of modal activity have been in common use since the 1970’s. At the operations level, the challenge has been to translate the associated information into appropriate guidelines for operator actions.

‘Other information’ provided by the Psymetrix estimators, includes:

(i) Mode Amplitude – corresponds to the RMS amplitude of the mode in the measured signal;
(ii) Relative Mode Phase – for a given mode, the phase angle between the modal contribution to signal i and the modal contribution to signal j
(iii) Mode Power Path – identification of the contributions to a mode from various regions of the grid [12 and 13]. (See section 4.2.4, Q2 (iv) for further information).

Various displays are provided by the Psymetrix estimators to present the above information in meaningful ways to the user. Accordingly the following displays are provided:

(i) Time series displays of Mode Amplitude, Mode Decay Time and Mode Frequency.
(ii) Mode Amplitude versus Mode Decay Time (scatterplot & locus) see, for example, Figure 4-1.
(iii) The Mode Shape display, as shown in Figure 4-2, compares for each monitored signal the relative magnitude and phase of a selected mode in the signals. The Mode Shape is the product of the output matrix (C) corresponding to the monitored signals and the right-eigenvector of the selected mode.
(iv) Histograms of occurrence of modes show, for a particular modal-estimator update rate of, say, 20 seconds, the number of times modes are identified within each user-specified frequency band within a user-specified period of time of, say, one month.
(v) Geographic displays of Mode Shape show the relative magnitude and phase of the selected mode at the geographical location of each signal. The signal sites are also color coded to indicate the alarm status of the selected mode.
(vi) Geographic displays of Mode Power Path indicate by means of markers on the transmission corridors on a geographical display those locations where opposing phase oscillations of a selected mode are occurring. The relative sizes of the markers indicate the relative amplitude of the frequency oscillations at the respective locations. The amplitudes of the frequency oscillations are indicative of the amplitudes of the associated power oscillations.
(vii) Geographic displays of Alarm Status show the alarm status for a user-selected mode on a color coded geographical display of signal locations. No status is shown at signal locations where the estimator is unable to identify the selected mode. Such displays are useful in identifying regions of the system where damping is poor.
(viii) Multiple simultaneous mode alarms/displays allow the user to simultaneously view the dominant mode frequency and decay time and associated alarm status for some or all of the several modes
which are identified by the modal-estimator at a given time. The user can then “drill down” to explore the behavior of any of these modes in terms of the above geographical displays and time-series plots, etc.

(ix) Long-term historic records. Analysis results for all modes are archived for extended periods of the order of a year or more.

(x) Aggregation of alarms for high-level annunciation. An overview-level alarm display allows the user to see any location where any problem has been identified. From this level the user can “drill down” to explore the reasons for each particular problem.

(xii) Fast response of results updated every 5 seconds (for alarming); and long-term results updated every 20 seconds (for analysis).

Figure 4-1: Mode Decay Time Constant –v- Mode Amplitude display produced by a Psymetrix modal-estimator. The plot compares the damping performance of a particular electromechanical mode with a PSS out-of-service (red) for a three week period; and then with the PSS in-service (blue) for a subsequent three week period. (Good damping performance is indicated by low Mode Amplitude and short Mode Decay Time Constant)
Q6 If 'other information’ is provided by the modal estimation system it is of interest to learn how is that information used.

A variety of applications is reported.

(i) Modal amplitude, or signal strength, is employed to determine the ‘credibility’ of an observed poorly damped mode, to investigate the need for urgent action and to identify an operational response (AEMO & NG).

(ii) Validation / calibration of small-signal models is a common current or future application (in AEMO, Eskom, PQld, TPNZ) and is examined in more detail in Section 4.2.3.

(iii) All of the information listed by BPA in Q5 above is necessary for comprehensive modal surveillance on a large power system, whether it be done on line or off line. At minimum, the results are used to refine the knowledge base for situational awareness. Additional uses include surveillance of staged system tests, characterizing and (hopefully) enhancing the validity of planning models, planning and certification of stability controls, siting and cross-validation of measurement equipment, refinement of identification tools, etc.

(iv) Psymetrix advise that by use of the Mode Shape and Mode Power Path information it is possible for the operator or analyst to devise measures to improve damping. For example, it is claimed that the Mode Power Path facility allows the operator or analyst to see the main route(s) of power oscillations of a lightly damped mode. By reducing the power flows in the associated transmission corridor(s) it is advised that generally the damping of the mode is improved. The Mode Shape can be used to determine if a mode is local and, if it is, the operator is able to concentrate efforts to remedy poor damping on generating plant that is mainly involved in the oscillation.

Q7 Some miscellaneous, relevant comments:

(i) Swissgrid monitors active power flow or voltage-angle difference signals between substations located in the center of the European system and one at the margin of the system. Voltage-angle difference signals reflect very satisfactorily the system behavior from both the steady-state and dynamic point of view.

(ii) NTNU refer to a comprehensive summary of the various modal estimation techniques investigated and used, together with the experience of wide-area monitoring in the Nordic power system [1, 2].

(iii) In order for NG & TPNZ to obtain more complete information on the system modes and understanding of oscillations, the number of sites being monitored by the modal estimation system is being increased. In this context AEMO state that many of the poor damping incidents observed on the power system using modal estimation relate to inadequately damped local modes or incorrect operation of controllers (particularly on some older generating plant). The experience of operators currently is that it is difficult to isolate these sorts of problems quickly and efficiently with the sparse monitoring that is currently installed on the system.

(iv) BPA advise that integration of advanced analysis tools into real-time grid management should end up at or adjacent to the operations floor, but operators want it to be well vetted before accepting it there. This includes clear instructions as to what actions should be taken when an oscillation or low damping alarm occurs.

Fully automated surveillance of power system modes requires supervisory (“expert system”) logic to balance prompt detection against false alarms originating under benign but unanticipated operating conditions. Existing supervisory logic covers just an average range of the many conditions that can abruptly arise in the western interconnection. For this and other reasons the most advanced use of modal
surveillance tools is NOT on the operations floor. Instead, numerous copies of the tools are distributed across BPA operations support staff, BPA planning staff, other collaborating utilities, and long-term contractors who function as BPA adjunct staff.

4.2.2 Use of Modal Estimation for Staged Testing/Tuning of PSSs and PODs; Coordinated Tuning of Stabilizer Controls; Associated Uses

The term PSS refers to the power system stabilizers fitted to synchronous generators; and POD refers to the power oscillation dampers fitted to SVCs, HVDC links and other FACTS devices.

Q1 Staged testing or tuning of PSSs. Conventionally, the tuning and testing of PSSs typically involves a number of short-term 'single-shot' tests. These tests include frequency response tests to verify the PSS transfer-function; AVR injection tests to estimate the GEP characteristic of the machine to perform - or validate - the tuning of the PSS; AVR injection tests (step, PRBS, etc.). In some cases transmission-line switching is conducted, with the PSS in and out-of-service, to verify that the measured improvement in damping of the dominant modes is consistent with that predicted from tuning studies, etc. In addition to these tests some respondents (PQld, TG, Psymetrix) report that continuous modal estimation based on the measurement of the ambient fluctuations in power system signals have been employed in practice to estimate the damping contribution made by specific PSSs. Swissgrid reports that PSS settings on generators in Turkey were improved prior to the recent interconnection of the Continental European and Turkish systems. System tests following the interconnection have included continuous monitoring and assessment of the damping of the new 0.15 Hz inter-area mode; the results have been very good. TPNZ have trialed the use of continuous modal estimation in tests at a recent commissioning of generating plant in which the PSS controls were switched on and off during the tests. However the PSS on/off periods used in these tests were only for transient response tests. Furthermore, only a small number of repetitions were carried out so satisfactory modal estimation results were not obtained. In [16 and 17] some experience with the application of continuous modal estimation methods to PSS testing is described. Such test information has facilitated benchmarking, the damping contribution derived by small-signal analysis being based on the relevant models of plant and controllers.

IREQ advised that data gathered from their wide-area monitoring system is used to identify relatively low-order small-signal models of their system for the purpose of tuning stabilizing controls. Further details of their approach are given in Section 4.7.

BPA report that the use of modal identification based on short-term or single-shot tests for model-level PSS evaluation and tuning is a fairly routine matter in the work of the Modeling and Validation Work Group. Important results can be found in [34 & 35]. Generally any single generator PSS does not sufficiently affect inter-area mode damping to be locally observable although modal identification can be used for testing the effect on local mode damping.

Q2 Where continuous modal estimation has been employed for staged testing of PSSs it has proved effective. It has been - and still is - a common practice during commissioning of new generating units to validate the PSS model and to measure the effectiveness of machine damping using small-signal disturbance tests (e.g. application of steps in reference voltage). However, employing modal estimation and switching the PSS in/out of service provides additional information not only on the PSS’s contribution to local- and inter-area mode damping, but also for small-signal model validation. Of concern in some cases, such tests based on continuous modal estimation have revealed that some PSSs are less effective than expected.
Q3 In Australia the OSM, a continuous modal-estimation system, has been employed to determine the effectiveness of the PODs installed in the Australian NEM (National Electricity Market). In one program of testing three PODs were switched on and off in 3-hour windows and the mode damping performance was measured by the modal estimation system over a 4-day period. The results were used to confirm the effectiveness of the controls, and to calibrate the small-signal models, see Figure 4-3.

BPA used the Pacific HVDC Intertie as a POD (the Celilo Damper) for about 12 years (1975-1989), but at present it has no involvement with operational POD units. While some of the technology, which is based on short-term or single-shot methods, is dated, BPA’s methodology for POD engineering is well represented by [36, 37, 38]. BPA’s preferred test method employs a low-level probing signal; details are provided in Section 4.6.1.

Figure 4-3: Calibration of small-signal models of PODs against modal estimates. Variation of the damping constant (Np/s) of an inter-area mode as PODs on 3 SVCs are switched in- and out-of-service in three hour increments over a 4-day period (Green curve – displayed when the corresponding POD is in-service; red curve – estimated damping-constant from the OSM system; black curve – damping-constant using an uncalibrated simulation model; blue curve – damping-constant using calibrated simulation model).

Q4 In the context of Figure 4-3 PQld comment that from modal estimation the damping contribution of one of the three PODs was found to be poorer than that found by small-signal analysis. Accordingly the model of the POD was calibrated so as to match the modal estimates.

BPA comment that their test methodology and analysis procedures for tuning the Celilo POD were effective, but their immediate value was limited by shortcomings in other information resources. The POD tests revealed serious modeling discrepancies that remain just partially resolved to this day. (E.g.,
see Figures 4 and 47 in [39].) The Celilo POD produced widespread interactions that the BPA planning models could neither predict nor explain. Compounding this, WSCC (the predecessor of the WECC) monitor facilities were unable to observe remote controller effects or to provide sufficient data for planning model correction. WAMS deployment in the western interconnection was and is a direct response to this situation [40 and 41].

4.2.3 Calibration of Small-Signal Dynamic Models of the System Used for Simulation Studies

Q1 AEMO, Eskom, PQld, Swissgrid and TPNZ state that modal estimation has been used to validate or calibrate small-signal models of their systems. Typically, system snapshots are taken by the state estimator periodically. These snapshots are used as a basis of simulation studies to compute the damping of system modes for comparison with corresponding modes estimated from system measurements.

In Australia there is a long-standing practice of validating the models of generating plant and their controls at the time of commissioning and when substantial plant modifications are made. This practice is enshrined in the NER [31]. Consequently, the dynamic models of the system are credible. AEMO has an Operation and Planning Data Management System (OPDMS) which generates a system snapshot (created by state estimator using Energy Management System data) every half hour. The snapshot includes options for load flow and large-signal dynamics models of the power system including a model for small-signal dynamic performance analysis. The associated recorded estimates of the inter-area modes are compared with the eigenvalues calculated from the small-signal analysis for the selected operating condition. Besides the modal parameters, the Mode Shapes derived from measurement and simulation are compared to assist in diagnosing the reasons for modeling discrepancies and to provide further confirmation of the accuracy of the system model.

BPA report that modal analysis, including modal identification, has been used regularly to assess model validity since 1975. The dominant results have been characterizations of planning model errors, plus determination of shortfalls in the range of simulation scenarios. Modal analysis has been useful for calibrating WECC models against inter-area modes below 0.5 Hz, however, model realism above 0.5 Hz is suspect.

BPA comment that the most formidable challenge seems to be that power system models are not sufficiently realistic with respect to oscillatory dynamics. Model validation and model calibration are technically distinct tasks that, in the WECC, are addressed by two or more different work groups. Model calibration, by far the more difficult task, requires a broad collaborative effort that is usually reserved for clearly perceived emergencies. Individual generator plant models have been validated using PMU data but not modal estimation techniques. Measurements taken on the generator high side bus have been compared to the same response that would be expected using the simulated model. Voltage measurements at the high side bus are used to drive the simulated model and comparison of simulated and actual current signals are made. Generators with good agreement may not be required to have detailed in-plant testing.

Psymetrix has direct experience of system model validation using continuous modal estimation on a number of power systems. The modal estimates obtained with longer analysis windows are employed due to the reduced variability in the modal estimates. They report that in one exercise [14], following some model tuning, there was agreement in the main inter-area modes obtained from measurement and

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2 Model validation is concerned with verifying that there is adequate agreement between measured and simulated behaviour. Model calibration involves adjustment of simulation models (including parameters) to realize better agreement between measured and simulated behaviour.
simulation. However, it was difficult to get alignment of some of the local modes. Similar to the approach taken by AEMO and TG in Australia, Psymetrix compare measured and simulated Mode Shapes as well as modal parameters. In another exercise [15] they advise that their objective was to explain the reason why poorly damped oscillations at 0.7 Hz that were observed in practice were well damped in the system model. Their investigation concluded that two potential contributing causes were (i) a wind farm with fixed-speed induction generators; and (ii) the gain setting in an AVR on a particular synchronous generator.

**Q2** Eskom, TPNZ, AEMO and Psymetrix have commented as follows on their experience and the nature and quality of the results that have been obtained to date.

Eskom advise they have found it very difficult to get good correlation between (i) modal estimates derived from the off-line analysis of the measured responses of certain system events; and (ii) the corresponding modal parameters derived from model-based simulation studies. This has led to a number of controller models having to be validated again.

TPNZ reports that the small number of monitoring sites (five) presently limits the ability to validate models. They have concerns with their models because in practice they have observed sustained low frequency oscillations that appear to be a limit-cycle phenomenon which, naturally, do not match with linear system models but may suggest a controller problem.

AEMO comments that, in general, for the derived-from-calculation (based on small-signal models) damping is better (more heavily damped) than the derived-from-measurement damping for the important inter-area modes. There are a number of possible reasons for the discrepancies. Among these are that some PSSs/PODs are less effective in practice due to:

(i) Quantization of PSS input signals: Modern PSSs use digital inputs and in some cases the quantization level of the input is so large that the PSS cannot respond to the signal variation within the level. For example, AEMO has found some PSSs with a quantization level of 10 mHz for the bus frequency input is less effective in damping the inter-area and local modes of oscillation.

(ii) Design of the PSS: Some manufacturers use “synthesized speed” signal as an input to the PSS. It appears the synthesized speed is not a true measure of the rotor speed signal; the fidelity of the synthesized signal depends on the fidelity of the conversion model and the selection of its parameters.

(iii) Operating status of PSSs and PODs: Currently, their operating status is not monitored; it is possible that some of these stabilizers are turned off.

BPA report that measurement facilities for dynamic behavior of the western interconnection are no longer a strategic impediment, though there are lingering problems with PMU technologies. Mathematical software for extracting information from measured data is generally satisfactory, though there is always room for refinement. At least some of the toolsets can be used in side-by-side comparisons of simulation results against actual measurements. The key problem is to better integrate dynamic information into grid management: for that, more staff skills in the area of system dynamics are necessary.

Psymetrix remark as follows:

(i) In general, model validation is a difficult and time-consuming process. In systems where there is a well-established compliance process over a long period of time, it is generally feasible to match an observed mode with a modeled mode, and show that the same behavior is observed in both systems. The model may be fine-tuned to match observations more closely, taking note of the long-term behavior of the mode.

(ii) In systems where there has not been a strong emphasis on model validation and compliance testing for generator dynamic performance, there can be a very poor initial match between the
model and the measurements. In such cases, modal identification is a key tool for embarking on a compliance program. It identifies which modes are the most significant, and focuses efforts in generator testing on units that contribute most significantly to these modes. It also identifies the system conditions that produce poor damping. Thus, such a program can yield quantifiable improvements in terms of system security earlier than would be possible without modal identification.

4.2.4 Use of Poor Damping Alarms with a Continuous Modal Estimation System

Q1 All respondents acknowledge that the continuous modal estimation system employed includes the ability to detect, in a timely manner, when system damping degrades below a preset threshold level.

BPA adds that under the Western Interconnection Synchrophasor Program (WISP) these will be included in the Wide Area Situational Awareness (WASA) display rather than in the mode estimation engine. Thresholds will depend on the oscillation monitoring tool used and mode under observation. WISP is using as a performance standard, detection of a significant change in damping within 60 seconds for ambient data. Faster detection is possible when disturbance events occur and/or a well-defined oscillation appears. WASA and other aspects of the ModeMeter effort make a distinction between oscillation alarms and low damping alarms.

Q2 (i) A difficult question, with a variety of responses, is what is the delay between the actual onset of inadequate damping and the raising of the damping alarm? In Swissgrid’s system the delay is a few seconds, for TPNZ it is within 2 minutes, usually within 30 seconds. Eskom states that the delay depends on the frequency of the oscillation, the damping level and the alarm hysteresis and resulting delay. Eskom employ a short 3-minute filter so from the onset of an inadequately damped oscillation the reporting delay would typically be 1 to 3 minutes. AEMO’s Psymetrix StormMinder system uses a 3 minute averaged window for alarm/alert trigger, updated every 4 seconds.

For the case of Mode Meter detection a performance standard of 60 seconds or less is used by WISP. Response for a Prony-based oscillation detector, useful for larger oscillations, can be much faster. Frequency determination requires about two swings of the slowest mode, and also a sufficient number of estimates to satisfy a persistence test. The persistence test must be adjusted to observed SNR conditions plus multi-site correlations.

Psymetrix provides the following comments: (a) The delay between the onset of the damping problem and raising the alarm depends on the level of damping, the threshold applied, and the analysis update rate. It also depends on the alarm configuration, in particular the number of consecutive crossings of the threshold before the alarm is triggered. The delay time also depends on the frequency of the oscillation - very low frequencies (i.e. under 0.1Hz) take longer than higher frequency modes because of the longer period. (b) In many observed cases of electromechanical oscillations, poorly damped oscillations start at a distinct point in time, which can be seen in the time domain signal. In such cases, an alarm can be raised around 15-30 seconds after the onset of the condition. On the other hand, where the underlying condition is close to the threshold, the hysteresis mechanism designed to avoid the alarms changing state too frequently will delay the onset of the alarm. Typically, it could take 1-3 minutes to detect a marginal alarm state.

3 The WISP project is funded 50% by the US Department of Energy and 50% by participating WECC utilities.

4 As defined in Chapter 2, a Mode Meter is an automated process that provides parametric estimates of modal parameters and related information continuously in real time.
Q2 (ii) A second difficult question asks: has the rate of false alarms been quantified and, if so, what is the rate?

AEMO respond: (a) The accuracy of the alarm systems has been benchmarked on a small-signal model of their system using a synthesized signal. In general the longer the window length the more accurate the results, but the poorer the alarm system is at rapidly detecting significant transitions in damping. (b) Analysis with short and longer data windows can be used in parallel to obtain a better gauge of the trend in damping in combination with a timely response. (c) The rate of false alarms on the actual power system has not been quantified.

NG’s existing system has identified about 25 alarm events per year. Unfortunately, it can be difficult for operators to determine whether alarms relate to a ‘real’ event cannot always be established. Others report that this is work in progress or that false alarms are rare.

At BPA work is in progress on the quantification of false and missed alarms.

A vendor, Psymetrix, has offered the following comments. “Assessments of false alarms are highly dependent on the testing process, and the rate cannot be accurately presented as a simple figure. If the underlying damping is good, there is a negligible occurrence of false alarms. In the case of the underlying damping approaching the threshold, but not crossing it, there is a higher occurrence of ‘false’ alarms, however the distinction between the damping being just under or just over a threshold is small, and not of great practical significance.” They have observed that the occurrences of poor damping are often very distinct from normal behavior. Figure 4-4 reproduces from [18] a three month recording of the damping of a 0.5 Hz mode observed throughout Scandinavia. Similar behavior is reported in [19] for a 0.7 Hz mode in the United Kingdom. In both cases there is a clear distinction between the well-damped and poorly-damped conditions. The vendor advises that it is not difficult to set a threshold that produces very few false alarms, but consistently produces alarms when the system is poorly damped. It is stated that this kind of pattern is consistently observed in power systems worldwide.

![Figure 4-4: Occurrence of poor damping of a 0.5 Hz mode on the Scandinavian power system during a three month period. Supplied by Psymetrix in their response to the Questionnaire.](image)

Q2 (iii) Not only are false alarms troublesome but missed alarms may be of concern. The latter are of interest but are found difficult to estimate. Use of simulation studies on a small-signal model of the power system can allow the user to investigate what are suitable parameter settings in the alarm system to reduce the occurrence of false and missed alarms.
Q2 (iv) If an alarm is raised, it is desirable that facilities are provided to assist the operator to (a) determine the reasons for poor damping; and/or (b) apply safe and effective corrective measures. In most organizations a process for implementing (a) & (b) are under development or are not their responsibility. In the case of AEMO & NG these issues are covered in section 4.2.1 paragraph Q4.

Several organizations state there is a need for a tool, available to operators, to identify sources of poor damping. Symetrixx offers a method of identifying contributions from regions of the grid. The method, known as “Mode Power Path” [12 and 13], calculates and displays the components of power oscillations that influence the rotor oscillations in generators. The pattern of these components of oscillation through the network shows the regions where there are the greatest contributions of energy to sustain the oscillation. Given a suitable network of monitors the operator can then drill down to identify the sources of such contributions within a region.

Organizations are interested to know if the rate of false alarms on an actual power system has been quantified - and how this has been done.

4.3 Evaluation of Which Continuous Modal Estimation Systems Might be Suitable for Future Practical Operation on the Operator’s Network

The following issues apply (i) to the above case in which a continuous modal estimation system is in practical operation; or (ii) the case when the performance of a potential modal estimation system is being - or has been - assessed but not yet operating. From the operator’s point of view such an evaluation is undertaken to establish the potential performance and benefits of a modal estimation system.

Q1 AEMO, Eskom, TPNZ have conducted an evaluation of modal estimation systems. Swissgrid has been in collaboration with the vendor in the development of the system, while NG has upgraded their system from earlier versions supplied by the same vendor. BPA is presently evaluating modal estimation tools and are expected to be operational by 2013.

Q2 All respondents report that active power flow perturbations on transmission corridors are monitored for modal estimation. Voltage-angle differences between monitors in four widely separated locations are employed by AEMO for their second estimation system, the multi-input QUT/TransGrid OSM estimation system. In addition to voltage-angle measurements, a number of organizations are planning to employ frequency (particularly at generating stations). It was noted that the quality of the signals can influence the performance of the estimator, e.g. gaps in the transmitted data, the dynamic response of the measurement transducers. (The measurement of dynamic phasors and the dynamic response of PMUs are covered in the new IEEE Standard for Synchrophasor Measurement for Power Systems, C37.118.1, now in draft form. The associated IEEE Standard C37.118.2 Standard for Synchrophasor Data Transfer for Power Systems (draft) deals with communication of synchrophasor data. These draft standards update the IEEE Synchrophasor Standard IEEE C37.118-2005 [32] – which did not include standardization of dynamic phasor measurement.)

Q3 The selection of the signal-types and of their locations in the network was based, firstly, on knowledge of the topology of the system and location of generation, performance of existing monitoring systems and availability of monitors or PMUs, secondly, on the cost and ease of deployment of additional monitoring facilities, etc.; and thirdly, on the recommendation of the vendor together with associated trial studies using already-monitored data.
Benchmarking the estimation system, independently of the vendor of the estimation system, was not undertaken by NG and Swissgrid as the new system was an upgrade of an earlier system. In the case of TPNZ, AEMO & PQld comprehensive sets of benchmarking studies were conducted on a small signal model of their systems (i) to assess the accuracy of the modal estimates under stationary conditions; and (ii) to assess the nature of the response of the estimator to changes in system damping under non-stationary conditions [6]. A range of system operating and generation dispatch conditions was examined with loads excited by uncorrelated random noise (standard models for the voltage and frequency characteristics of the loads may be employed, except for certain loads such as aluminum smelters). For the stationary system models the relative bias and variance of the modal estimates were the main criteria for the performance of the estimator. In the case of the non-stationary system tests, an assessment of the ability of the estimator to track changes in damping (ramp and step) was less formal and is based more on a comparison between the expected and actual responses of the modal estimator. The non-stationary system tests are also employed to evaluate the performance of alarms and to investigate their settings.

(Note: It is expected that developers / vendors of modal estimation systems will conduct their own extensive testing to verify the accuracy and robustness of their systems. The purpose of this question (Q4) is to ascertain what independent testing is conducted by the power system operator to benchmark the performance of modal estimation systems on their own power system.)

The respondents were invited to comment further on benchmarking issues or on any items in Section 4.2 and Section 4.3, or other related issues.

(i) NG plan to maintain a period of parallel operation of the previous and the upgraded estimation systems. This will enable operators to gain confidence in the information displayed on the upgraded system and provide a useful opportunity to compare data between the two systems for data validation.

(ii) Swissgrid state that their data concentrators receive power and frequency measurements from a number of PMUs across Continental Europe at the rate of 10 Hz. The time-series data is analyzed by their modal estimation system to produce estimates of oscillation damping, frequency and amplitude [4]. Off-line comparisons are made between (i) the recorded power and frequency time-series data from the PMUs; and (ii) the corresponding modal-estimates to verify the accuracy of the latter estimates.

(iii) AEMO comments that some research has been undertaken on methods for locating monitors for the best observation of particular inter-area modes of interest. This research (partly published [7]) suggests that monitors should be located, as expected, where there is best discrimination between modes (i.e. a dominant mode and other modes much weaker); importantly, however, that these locations can be determined by analysis (provided reasonable small-signal models are available). The analysis might need to be undertaken across a range of system conditions because the behavior of modes is dependent on the generation schedule and operating conditions.

(iv) Interestingly, TPNZ remarks that the modal estimator often reports different modal characteristics depending on the input signal analyzed. It appears from the monitoring that there is further information that could be gained from the combination of the analysis of different signals, but this is not a feature of the modal estimator in use.

(v) Swissgrid has two different methods in use for on-line modal parameter estimation, a fast one based on Kalman filtering gives results after a few seconds (time interval between two measurements is 100 milliseconds), and a second approach which is being tested is based on average calculations over minutes and therefore may deliver more accurate results.

A concluding comment was offered. In evaluating modal estimation systems consideration should be given to whether the process (i) is dependent on system topology (e.g. meshed, longitudinal), and (ii) requires tuning for the particular power-system.
4.4 No Continuous Modal Estimation System is in Operation nor Currently Being Planned

**Q1** Although IREQ has no continuous near real-time modal estimation system in operation at present they are now evaluating the possibilities of such a system offered by the installation of up-to-date PMUs in the near future. (As noted elsewhere, IREQ do currently perform off-line analysis of system measurements to identify system models (and therefore modes). The resulting models are used for several purposes including the tuning of PSSs).

**Q2** IREQ state that the primary purpose is the development of more robust and dynamic tuning of controllers including MF-MBPSS (Multi Functional − Multi Band PSS) in a decentralized/hierarchical configuration. The next step will include situational awareness purposes such as real-time monitoring security tool development, optimized with base-lining studies and data-mining features.

4.5 “Single-shot” Modal Estimation Techniques

4.5.1 Introduction

For the purpose of this survey “single-shot” modal estimation techniques include the following:

1. **Staged tests:**
   - (i) Injection of probing signals (e.g. band-limited noise, sinusoidal signals) into controllers (e.g. modulation of the power-reference of a HVDC link).
   - (ii) High power pulse inputs (e.g. energization of a braking resistor).
   - (iii) Network switching (e.g. switching of a transmission line).

2. **Non-staged events:**
   - (i) Scheduled network event (e.g. switching of a transmission line undertaken as part of normal system operation and not as a staged event for test purposes.)
   - (ii) Network disturbance (e.g. a transmission line fault, the loss of a generator.)

The single-shot methods are characterized as being based on discrete short duration tests or events. The staged tests are typically conducted at a scheduled time and place and typically occur when system loading is low to moderate for system security reasons. Staged tests may be repeated several times within a relatively short period to ensure consistent results. In the case of the injection of probing signals the injection period may be relatively long to ensure reliable estimates at the time of testing. In the case of non-staged events, repetition of the event to ensure the consistency of estimated modal parameters is generally not possible.

4.5.2 Responses

**Q1** The primary objectives of such single-shot methods are (i) model validation (small- and large-signal); (ii) determining the damping of power perturbations on major interconnectors; and/or (iii) assessing controller performance. However, it should be recognized that staged single-shot tests are invasive, disruptive and are not usually carried out at a time of high interconnector or system loading.

AEMO comment that, for generating unit controls, such as AVRs and PSSs, sinusoidal injection is common to ensure that control system settings have been applied correctly and to validate the small-
signal model. In the past, stand-still frequency response tests on synchronous generating units have been carried out at several locations to determine synchronous machine parameters. TransGrid add that, prior to the implementation of the continuous-monitoring modal-identification systems, staged single-shot ring-down tests were performed to try to identify the parameters of the inter-area oscillation modes in the eastern Australian system. In the past braking resistor switching and switching of loaded transmission lines were used to stimulate the oscillations. The main problem experienced with this type of testing was the interference produced by the background or ambient fluctuations during the ring-down period, i.e. fluctuations that were not initiated by the test switching, but by other random fluctuations in system load. This type of testing has not been used since the continuous-monitoring modal-identification systems have been implemented; it is not clear under what conditions these tests would be adopted again.

BPA state that single-shot tests, which the WECC refers to as probing tests, are a regular part of dynamics analysis in the western interconnection. Their immediate objectives are to directly examine system modes and, sometimes, to provide response data for candidate POD actuators. Brief pulses, with a resistor brake or other switched device, are simple and particularly convenient for comparative model simulations. Longer-term objectives are all of the usual ones: better understanding of power system behavior, validation & calibration of planning models, maintaining the capability to deploy POD technology as needed.

Q2 In Australia, both non-linear dynamic models and small-signal models of the power system are maintained and used for routine dynamic performance and control analysis. In New Zealand, in Europe and elsewhere non-linear dynamic models of the systems are presently employed for routine analysis of dynamic performance. In such cases, the identification of modes is conducted in time-domain simulations using a transient stability program [8]. Alternatively, the non-linear models of devices are automatically linearized about the steady-state operating point; for the controllers the conversions for standard models are automatically invoked by the small-signal analysis software [9].

At BPA small-signal models of the power system are employed but with some qualifications. Leading-edge planning activities employ a suite of reduced nonlinear models that reflect full system behavior. These are routinely linearized for such purposes as controller design or ModeMeter tuneup. The Department of Energy’s Prony tool automatically provides linear signal models that can be used in controller design.

4.6 Modal Estimation from Staged Single-Shot Tests

4.6.1 Probing Signal Injections

Prior to the availability of modal estimation, PQld used sinusoidal probing signal which modulated of the voltage reference on the AVR of a SVC. A comparison of the frequency responses with the POD in- and out-of-service was used to estimate the change in damping of the lightly-damped inter-area mode.

Psymetrix advise that they have injected pseudo-random noise into the summing junction of a generator AVR to provide accurate and relatively fast estimation of those modes that are controllable by the input. They claim such tests provide more accurate estimates of well-damped modes than continuous modal estimation. The tests are of relatively short duration and therefore provide information at a single operating condition. In their experience this type of test emphasizes the local mode and is not so effective for inter-area modes (although, they claim, further tuning may be possible to better identify the inter-area
modes). As with other types of invasive staged tests, signal injections are not normally permitted in poorly damped conditions which are of most interest.

BPA report on probing signal injection tests as follows:

(i) Probing signals have been applied to plant such as HVDC, SVC, TCSC, SMES, and perhaps generator controls. Switching plant such as resistor brakes, series capacitors, shunt capacitors and reactors in and out of service has been used to produce pulse inputs. Related tests have involved dropping of entire plants [42].

(ii) The types of probing signals employed for modulating HVDC have included synthetic noise, sine waves, pulse trains, other signals with special shapes. Similar modulation has been applied to SVC, TCSC, SMES, and perhaps generator controls.

(iii) Wide Area Measurements of the resulting transients are always conducted.

(iv) Other information derived from the tests includes mode shapes (interaction patterns); time domain displays for raw and filtered data (including histograms and activity measures [43]); Fourier spectra (autospectra, cross spectra, squared coherencies); Fourier transfer function estimates (including apparent transfer functions between response signals); transfer function models with confidence limits.

(v) A preferred test is sustained low-level probing with a synthetic noise signal, in part because it provides the all-important transfer function zeros. Ambient analysis and brake insertion tests provided independent checks on transfer function poles (system modes).

(vi) The main advantages and disadvantages of probing signals are: Low level probing (e.g., with synthetic noise) is minimally disruptive, but vulnerable to non-stationarities (powerflow shifts, topology changes, etc.). High level probing with a sharp pulse can be very brief. Thus it circumvents inherent non-stationarities, though it may sometimes trigger some protective control action.

(vii) A form of continuous synthetic-noise probing signal injection is being contemplated, however, the advantages over intermittent sustained probing are not well established.

(viii) Other relevant comments: Vulnerability of nearby plants and loads can be an important factor in probing signal design — e.g., a strong reactive power pulse may produce unacceptable voltage excursions. Probing with real power signals is usually several times more effective than probing with reactive power.

4.6.2 Braking Resistor Insertion and Transmission-line Switching Tests

Except for BPA, respondents report that staged braking resistor insertion tests have been used in the past to identify dominant system modes, however such tests are no longer employed. All but one respondent advised that transmission-line switching tests are no longer staged.

Psymetrix have advised that in some power systems, where they have direct experience, staged transmission line switching tests are employed to provide immediate indication of PSS performance and for comparison with time-domain simulations during the early stages of PSS performance verification ([16 & 17]).

Some comments on BPA’s resistor braking tests follow.

(i) The braking resistor is located at Chief Joseph substation and is rated 1400 MW.

(ii) The signal is 0.5 s pulse which produces a ringdown transient.

(iii) Wide Area Measurements of transients are always conducted.

(iv) Other information derived from the tests includes mode shapes (interaction patterns); time domain displays for raw and filtered data (including histograms and activity measures [43]); Fourier spectra (autospectra, cross spectra, squared coherencies); Fourier transfer function
estimates (including apparent transfer functions between response signals); transfer function models with confidence limits.

(v) The main advantages and disadvantages of braking resistor tests are:

(vi) Brake insertions are easy to execute, and they are safe under nearly all system conditions. Useful response signals are obtained throughout the interconnection, and comparative data are easily obtained from model simulations. Measured “signatures” for this event have shown the same general patterns for three decades; any strong departure from these patterns is cause for immediate investigation.

(vii) Other relevant information: Energizing a large braking resistor can be expected to produce some momentary saturation in nearby response signals (especially voltage). Ringdown analysis should avoid this initial interval and/or draw upon signals that are remote from the actuator.

### 4.7 Modal Estimation from Non-staged Events

Several respondents state that they utilize simultaneous recordings of transient disturbances made by their PMU network for a number of purposes. Commonly, the time recordings are employed in post-mortem analysis including review and assessment of controller performance and event simulation (based on system snapshots at the time of the incident). In some cases the transient recordings are utilized for modal-estimation; and in one case they are used to estimate a low-order state-space model of the system which forms the basis of stabilizer design. None of the respondents analyze transient events on-line.

The following summarizes responses received for questions Q1 to Q4

1. Transpower New Zealand
   - (i) On the TPNZ system the Psymetrix StormMinder records and stores data from the network of PMUs used for modal estimation continuously.
   - (ii) This same data base is used as a source of time-series data for transient events that can be analyzed off-line with facilities within the modal-estimation system; or exported for analysis with external tools.
   - (iii) Data stored from other PMUs on the TPNZ network which are not linked to the StormMinder system can also be accessed and analyzed.

2. Hydro-Quebec (HQ) (response via IREQ)
   - (i) An eight PMU-SCADA based wide-area monitoring system was commissioned on the HQ system in 2004 [25]. This monitoring network supplies a data concentrator with GPS-synchronized angles and frequencies from about 25% of the 735 kV buses on the HQ system. The associated SCADA is responsible for collecting (at a 60-Hz sampling rate) the wide-area phasors. (The monitoring network has additional facilities for magnetic storm detection which are not covered here). Processing is then performed on-line for time reconciliation and transmission/GPS error correction before transfer of the results to the appropriate applications.
   - (ii) Post-mortem modal analysis of individual signals from major events is undertaken by use of a reduced-order Eigensystem Realization Algorithm (ERA)-based method and Prony analysis [27].
   - (iii) The tuning of the decentralized/hierarchical multi-band PSS installed on the HQ system is optimized on the basis of a reduced-order small-signal system model which is identified from the transient event recordings. Mode shape information is obtained from the system model.
   - (iv) The signal analysis system is designed as a portable software tool that can be applied to signals derived from the PMU-SCADA wide-area monitoring system or from power system time-domain analysis software. The signal analysis software provides the
reduced-order system model and modal parameter information using seven user-defined estimation characteristics. It also reproduces the time-domain signals from the estimated parameters.

3. Statnet (response via NTNU)
   (i) Measurements from several PMUs have been used to analyze disturbances in the Norwegian system. The measurements have also been used to estimate the parameters of the dominant modes. The WAMS for the Norwegian 420 kV transmission grid is evaluated in [3] and potential applications of the system to (i) the real-time detection of poorly-damped oscillations; and (ii) provide remote inputs to PSSs are also highlighted.

4. BPA
   (i) The monitoring system is usually termed the WECC WAMS [38 and 39]. Backbone facilities consist of roughly 100 phasor measurement units, which at present are not fully networked into any one data collection point. There are extensive regional and local monitors, many of which are GPS synchronized. All but the most local devices record continuously, mostly at rates between 10 sps and 960 sps. Methods of analysis include all of those mentioned previously in the Questionnaire.
   (ii) Is the analysis conducted on-line in near real-time and/or off-line? Full analysis of a disruptive event is a lengthy process, and necessarily performed off-line. If the event is a progressive one then any problem with poor damping or actual oscillations should be evident in real-time displays, and operations engineers (BPA technical staff) may work with the operators to assess and/or mitigate the situation.
   (iii) Other information and results derived from non-staged events include all of those mentioned previously in the Questionnaire. Additional results include refinement of the event sequence, detailed time domain plots for critical conditions or events, and parameter histories for relevant modes. This work is prefaced by cross validation of multi-source data, and consummated by a detailed report that will usually include model studies.

5. Psymetrix
   (i) Use is made of the synchrophasor measurement system to collect transient responses for off-line model validation.
   (ii) An application to a governor tuning project for low frequency stability is described in [21]. The focus was on the Power / Frequency characteristics of key generators.

4.8 Other Single-shot Tests or Possible Future Developments of Single-shot Methods

Three respondents, IREQ, NTNU and BPA have provided information on possible future developments of single-shot methods.

1. Statnet (response via NTNU)
   (i) A proposed method to identify dominant modes and system transfer-functions by applying a probing signal to the control inputs of SVC units is described in [1]. It is proposed to employ subspace identification.

2. HQ (response via IREQ)
   (i) The possibility of applying numerical subspace state-space system identification (N4SID) to identify low-order models from system measurements is being investigated. A main purpose is to derive low-order models to facilitate tuning of the decentralized/hierarchical multi-band PSS installed on the HQ system. Early investigations based on signals derived from simulation models [27] revealed that, if application of N4SID is feasible, then better
models could be obtained than those obtained by application of ERA methods to pulse responses. IREQ are investigating the application of N4SID to either (i) ambient signals; or (ii) signals arising from probing of the inputs of appropriate controllers. IREQ expect that there may be the opportunity to compare low-order models derived from measurements made on the actual power system with corresponding models obtained from signals derived from detailed non-linear simulation models.

(ii) A Teiger-Keiser energy operator (TKEO) with linear filter decomposition method has been proposed for the robust and rapid detection of poor system damping [26]. An ERA based multi-band modal analysis (MBMA) optimized scheme is used to identify the modes in each of the filter output channels. The authors claim that the proposed method has the ability to discriminate between modes that are closely spaced in frequency. They also expect that the method can be applied to both ambient signals as well as ringdown signals recorded by the WAMS.

3. BPA
   (i) Mid-level probing on the HVDC line is another single-shot test employed and involves injecting selected frequencies at +/- 125 MW for four oscillation cycles (0.1, 0.3, 0.7 and 1.0 Hz). Generally these are evaluated by Prony analysis following completion of the probing signal injection.

4.9 Education and Training of Engineering Staff and Operators in the Application of Modal Estimators

Q1 The nature of training provided varies between organizations. Several organizations rely on training ‘on the job’ for engineering staff, such training being achieved by analyzing actual recordings and seeking correlation with system events, etc. Three-day training for operators or engineering staff is provided in some cases. For two organizations the vendor has supplied a short course on the use of the software and, in the case of the NG, the training of other engineers is provided by those who have attended the vendor course.

IREQ has provided a more formal program, a one week intra-organization course on modal analysis covering:
   LTI system and linear algebra, Power system modeling and dynamic stability, Modal analysis by means of direct eigenvalue calculation, Modal analysis by transient simulation, Modal analysis application and modal parameter interpretation.

Q2 IREQ provides training exercises involving the identification and simulation of the HQ network with multi-input-multi-output modal analysis in open and closed-loop, with the MB-PSS in- and out-of-service. These exercises are intended to emphasize the effects of stabilization and the importance of tuning stabilizer parameters.

Psymetrix advises that it typically provides two- to five-day training courses on the observation and management of dynamics issues and use of their software, including modal estimation and other WAMS applications. Sharing case studies and experience of related dynamics issues observed in different networks has been of interest. Examples of poorly-damped events and instability are replayed for training purposes. The use of simulated data playback is also used for training purposes.
4.10 Future Development of Modal Estimation Schemes

Some future developments listed by the respondents include the following features:

- Common requirements are the visualization of information for alarming and associated provision of advice on remedial actions, and development of guidelines and procedures, for control room staff to follow based on power system operating conditions. This is a particular challenge in meshed networks involving a number of ISOs.
- Using the playback facilities in the modal estimation system, a post-mortem analysis is conducted with the operators for training purposes.
- From a modal estimation system that provides system voltage magnitude and angle perturbations the ability to display power system voltage angles relative to a reference voltage node in real time could provide visualization of system separation points following significant system events. Such a feature could be of significant support to system operators when they are endeavoring to restore and re-synchronize the power system.
- More extensive monitoring coverage of the network based on optimum selection of sites and signals should not only permit the identification of poorly-damped oscillatory modes, but also provide information on the source of a problem to permit timely corrective actions.
- To allow development of damping controllers, it will be important to improve the accuracy of system dynamic models using the information provided by modal analysis such that the system model is able to closely replicate those critical oscillatory modes that arise in the actual network. This would expedite the development of centralized or decentralized controls to enhance system damping and stability. This is particularly critical for future systems where HVDC links are closely integrated into an AC system with series compensation.
- There is a need for portable modal estimation equipment that can be readily installed on plant fitted with system damping devices to provide on-line modal estimation to assess effectiveness of the damping controller.
- One respondent (IREQ) advised that HQ is developing a comprehensive proposal of a “smart-grid” vision for 2030. Among the objectives are the development of situational awareness facilities including modal estimation schemes.

4.11 Conclusions

In early 2011 continuous modal estimation systems in a number of organizations were under development, on trial, or in operational use on a day-to-day basis for a variety of purposes. Based on the responses by the contributors to the Questionnaire, the following set of conclusions is drawn on the current status of both continuous and non-continuous modal estimation.

- The primary purpose of continuous modal estimation is to continuously assess system security, to determine ‘at risk’ conditions, and to provide ‘situational awareness’ information for operations engineers and control room operators.
- The modes of concern are mainly the lightly-damped inter-area modes.
- A variety of approaches are offered to the question ‘what is the threshold for unacceptable modal damping?’ A damping ratio of 3% is used by several organisations. A traffic light system of alarm levels is employed by others specifying, for example, the amplitudes of power oscillations and associated decay times.
- There is a variety of approaches to the processing of poor-damping alarms with continuous modal estimation systems, for example, in the mechanism for triggering an alarm and in the time delay between the onset of poor damping and the raising of an alarm (i.e. alarm latency). One respondent (BPA) reports that separate alarm systems based on (i) ambient fluctuations and (ii) higher
amplitude transient disturbances are being developed. They expect that latency for the latter alarm system will be significantly less than for the former.

- As yet, little analysis has been conducted on rates of false and missed alarms. Apart from simulation studies to assess these rates, it appears no means of determining the rates based on historic records has been developed.

- In many of the organizations guidelines are under development for actions to be taken by control room operators when the thresholds for unacceptable modal damping are breached. Generally, experience is currently being gained by operations engineers and control room operators for the development of the guidelines for action; the necessary action is usually system specific. However, action such as reduction of generation in the appropriate areas to reduce power flows over interconnectors is now undertaken by several ISOs.

- To assist operations and planning staff, continuous modal estimation systems typically provide additional facilities such as Mode Shapes, time series displays of the characteristics of selected modes, and a variety of other information and diagnostic tools.

- Continuous modal estimation systems are employed for the staged tuning of PSSs and PODs and/or as a means of assessing their effectiveness on-line over a range of system operating conditions. Generally, a single generator PSS does not sufficiently affect inter-area modes, but the contribution to damping of PSSs in a generating station may do so. At the time of commissioning, the performance of a single PSS is typically assessed by staged tests such as responses to step changes for modelling validation, however, such tests are not usually conducted, say, at rated generator output at peak load conditions on the system. In Australia continuous modal estimation to assess the damping effectiveness of PODs fitted to SVCs in damping inter-area modes has proven to be useful.

- A number of organizations are employing continuous modal estimation both for validating system small-signal models (i.e. verifying adequate agreement between measured and simulated behaviour) and for their calibration (modifying simulation models and/or adjusting simulation results to account for important differences between measured and simulated behaviour). Some organizations find the results of validation are poor and are reviewing their models and undertaking further testing.

- Several organizations conducted benchmarking studies on one or more continuous modal estimation systems. Others are collaborating with vendors to develop a system appropriate to their requirements. In some cases independently developed modal estimation systems have been deployed which allow comparison of the results.

- Types of signals employed by continuous modal estimation systems are typically real power flows in interconnections, frequency, and voltage-angle difference between locations. The choice of locations of measurement monitors is based on the knowledge of the topology of the system and location of generation, the availability of monitors or PMUs, the cost of additional monitoring facilities, the analysis of already-monitored data, and on benchmarking studies.

- The use of 'single shot' modal estimation techniques covers tests such as braking resistor insertion, probing signals, and transmission-line switching. These are no longer used in most organizations covered by the Questionnaire as they are considered to be invasive, disruptive, and cannot carried out at a time of high interconnector or system loading.

- In the WECC, tests based on probing signals, resistor braking or other switched devices are considered simple and are particularly convenient for use in comparative model simulations. Probing signals have been applied to plant such HVDC links, SVCs, TCSC, etc., using signals such as synthetic noise, sine waves, pulse trains, and other signals with special shapes. A particular advantage is that the probing test provides the zeros of transfer function(s), which may be relevant to controller design.
• Simultaneous recordings from PMUs of transient disturbances in non-staged events are analyzed off-line by several organizations in post-mortem analysis including review and assessment of controller performance. In some cases the transient recordings are utilized for modal-estimation.
• The training of engineering staff and operators on a continuous modal estimation system and for analysis of single-event disturbances is mainly ‘on the job.’ In one case the vendor offers a two to five days training course on the use and applications of their continuous modal estimation system.

4.12 Chapter 4 References


G. Ledwich, D. Geddey., P.O'Shea. "Phasor Measurement Unit's for diagnosis and load identification in Australia-


[34] August 4th Oscillations: Analysis of Possible Effects of Control System Tunings, B. Corm, J. Jardim, and M. Kwok. BCH internal report provided to the Disturbance Monitoring Work Group, October 31, 2000. (available from BCH or DMWG)


APPENDIX 4

Summary of Questionnaire

The following is an abbreviated form of the Questionnaire in which some of the preamble has been deleted and the format compacted to reduce length. The questions themselves remain unaltered.

A4.1 Introduction

The questions in the Questionnaire cover some issues that may arise when a continuous modal estimation system is being proposed, evaluated prior to acquisition, or in practical operation. For example:

1. What are the intended purposes / applications of the (proposed) modal estimator?
2. Is the performance of a continuous modal estimator dependent on the nature of the system? (The system may heavily meshed or long and thin.)
3. Are the locations in the network at which the associated PMUs (or transducers) are sited suitable for use with the selected modal estimator? Are suitable, relevant signals available at these locations (i.e. with the relevant modal content and possessing sufficient amplitude and SNR)? By what method(s) can it be established if the location and its signal(s) are suitable for estimation of the modes of interest?
4. How can the accuracy of the modal estimates under steady-state conditions be ascertained? What are the variability and bias of these estimates. How long should the sample window be to ensure estimates of a desired accuracy, etc.?
5. How well does the estimator track step and/or ramp changes in the damping in the modes of interest?
6. Does the performance of the estimator improve with other signal types (e.g. power flow versus voltage-angle variation)?
7. How much do the modal estimates improve with a multi-input estimator, e.g. power flows from two or more separated locations, or from voltage-angle difference signals derived from such locations?
8. What are a suite of suitable tests to evaluate (or benchmark) the modal estimator over a range of stationary and non-stationary operating conditions?
9. What approaches are there to benchmark the estimator using dynamic models of the system over a range of operating conditions?
10. In order to produce the ambient noise which is observed in practice, how can we perturb a model of the system in a manner similar to that which occurs on the actual system?
11. Through simulation, can the basis for the settings of alarms be established to warn the operator that a system may be tending towards poor damping - and instability?
12. Can benchmarking/simulation reveal how long it may take to raise a reliable alarm after a selected trigger occurs?
13. Is the estimator suitable for staged testing (e.g. of stabilizer damping performance)? What other useful applications are there?

The questions which follow are intended to cover such issues in practice.

A4.2 Operational Experience with Modal Estimators

The following paragraphs are based on an assumption that a modal estimation system has been implemented on a network. It may be fully functional or under trial to assess its performance.
A4.2.1 Primary role of modal estimation in system operation

1. Is continuous monitoring of system modes performed for (a) inter-area (b) local area modes? e.g. Yes (for inter-area modes).
2. For what purpose? (e.g. System security).
3. What is the threshold for unacceptable modal damping? (e.g. The real part of the mode is greater than -0.05 Neper/s or the damping ratio is less than 3%).
4. What action is taken if the modal damping threshold is breached? (e.g. Reduce flows on the interconnector between X & Y to Z MW)
5. What other information (apart from estimates of modal parameters) is provided by the modal estimator? (e.g. right- eigenvector components of the modes, a measure of the “strength” or “contribution” of identified modes in the signal(s), frequency domain displays, time-domain displays, etc.)
6. If the answer to 5 is “Yes, other information is provided” then how is that information used? (e.g. model validation, characterization of modes, etc.)
7. Any other relevant comments? (e.g. The experience of system operators is that ....)

A4.2.2 Uses of modal estimation for staged testing/ tuning of PSSs and PODs; coordinated tuning of stabilizer controls; associated uses.

1. Has modal estimation been employed for staged testing of PSSs? If ‘yes’, how is the testing conducted? (e.g. Estimation of the local-area mode(s) associated with the station under test is conducted – for the selected PSS gain – with the PSS on for 2 hours, off for 2 hours, repeated for a period of 12 hours.)
2. If modal estimation has been employed for staged testing of PSSs, please comment on the effectiveness of the test. (e.g. The improvement in damping of the relevant mode is found to be consistent with the improvement predicted by small-signal analysis/simulation.)
3. Has modal estimation been employed for staged testing of PODs? If ‘yes’, how is the testing conducted? (e.g. Estimation of the inter-area mode(s) associated with the SVC under test is conducted – for the selected POD gain – with the POD on for 2 hours, off for 2 hours, repeated for a period of 12 hours.)
4. If modal estimation has been employed for staged testing of PODs, please comment on the effectiveness of the test. (e.g. It was found in one case that the POD was ineffective. Once the problem was corrected the damping of the relevant mode(s) was found to be improved to about 80% of the value predicted by small-signal analysis/simulation. This difference was attributed to inadequate modeling of system loads in the vicinity of the SVC.)

A4.2.3 Calibration of small-signal dynamic models of the system used for simulation studies

1. Has modal estimation been used to validate or calibrate small-signal models of the system? (e.g. Yes. Snapshots of the actual system operating conditions are taken every half-hour. The associated recorded estimates of the inter-area modes are compared with the eigenvalues of calculated from small-signal analysis for the selected operating condition. Then …. etc.)
2. Please comment on your experience and the nature and quality of the results that you have obtained to date.

**A4.2.4 Use of poor-damping alarms with a continuous modal estimation system**

1. Does the continuous modal estimation system include the ability to detect in a timely manner when system damping falls below a preset threshold level? (YES/NO):
2. If the response to 1 is “YES” please comment on the following aspects of the poor-damping alarm system:
   (i) What is the delay between the actual onset of inadequate damping and the raising of the damping alarm?
   (ii) Has the rate of false alarms been quantified and if so how what is the rate?
   (iii) Has the rate of missed alarms been quantified and if so how and what is the rate?
   (iv) What facilities are provided to assist the operator to (a) determine the reasons for poor damping; and/or (b) take apply safe and effective corrective measures?

If you have answered any of the above questions, please continue with the following set.

**A4.3 Evaluation of which Continuous Modal Estimation Systems Might be Suitable for Future Practical Operation on the Operator’s Network**

The following questions apply (i) to the above case in which a continuous modal estimation system is in practical operation; or (ii) when the performance of a potential modal estimation system is being - or has been - assessed but not yet operating.

Evaluation of future operation and benefits of a modal estimation system.

1. Was an evaluation of commercial/other estimator(s) conducted to establish its (their) performance on your own network? ‘YES’ or ‘NO’.
2. If response to 1 is ‘YES’: Which signals and at which locations in the system were used for modal estimation (e.g. active power flow perturbations on an interconnecting transmission line.)
3. If response to 1 is ‘YES’: On what basis was the selection of the signals and their locations decided? (e.g. On the recommendation of the supplier of the modal estimation system; by a process of benchmarking the estimator; etc.)
4. If benchmarking was employed to investigate the performance of the modal estimator on your system, please explain the methodology of benchmarking and the criteria employed to measure the accuracy/performance of the estimator (for examples of benchmarking methodologies see [6], Error! Reference source not found.). Among your comments, please include comments on the following if relevant: (i) Range of steady-state system operating conditions considered; (ii) tracking of modes under ramp/step/other changes in system damping; (iii) the criteria used to measure performance (e.g. bias and variance of estimates for steady-state operating conditions).
5. Are there any items in Sections 0 and 0, or any other related issues, for which you could offer some pertinent comments?

**A4.4 No Continuous Modal Estimation System is in Operation Nor Currently Being Planned**

1. May you consider the installation of a Continuous Modal Estimation System sometime in the future?
2. If ‘YES’, what are the primary purposes of the Continuous Modal Estimation System?
A4.5 “Single-Shot” Modal Estimation Techniques

A4.5.1 Introduction

For the purpose of this survey “single-shot” modal estimation techniques include the following:

1. Staged tests:
   (i) Injection of probing signals (e.g. band-limited noise, sinusoidal signals) into controllers (e.g. modulation of the power-reference of a HVDC link).
   (ii) High power pulse inputs (e.g. energization of a braking resistor).
   (ii) Network switching (e.g. switching of a transmission line).

2. Non-staged events:
   (i) Scheduled network event (e.g. switching of a transmission line undertaken as part of normal system operation and not as a staged event for test purposes.)
   (ii) Network disturbance (e.g. a transmission line fault, the loss of a generator.)

The single-shot methods are characterized as being based on discrete short duration tests or events. The staged tests are typically conducted at a scheduled time and place and typically occur when system loading is low to moderate for system security reasons. Staged tests may be repeated several times within a relatively short period to ensure consistent results. In the case of the injection of probing signals the injection period may be relatively long to ensure reliable estimates at the time of testing. In the case of non-staged events repetition of the event to ensure the consistency of estimated modal parameters is generally not possible.

A4.5.2 Responses

Q1. Please state the primary objectives of such the single-shot tests (e.g. Validation of system models)
Q2. Are small-signal models of the power system employed for routine dynamic performance and control analysis (e.g. No, a non-linear dynamic model of the system together with Prony analysis is used to determine the relevant modes and damping performance)?

A4.6 Staged Single-Shot Tests

A4.6.1 Probing signal injections. Please comment on:

- Device: (e.g. Modulation of HVDC link controls):
- Type of signal(s) (e.g. sinusoids, band-limited noise):
- Are Wide Area Measurements of the transients conducted?
- Apart from modal parameter estimates what other information is obtained (e.g. Mode-shape (i.e. right-eigenvector components), time-domain displays, etc.)?
- What are the main advantages and disadvantages of the probing signals?
- Is some form of probing signal injection on a continuous basis being contemplated?
- List of particularly pertinent publications or accessible reports in public domain?
- Other relevant information:

[A similar set of questions apply to braking resistor and transmission-line switching tests employed for modal estimation]
A4.7 Non-Staged Events

1. Are transients from non-staged system events recorded at various buses on the system and then analyzed to estimate the dominant modes?
2. If the response to 1 above is ‘YES’ please comment on the nature of the monitoring system and indicate the method(s) of analysis employed.
3. If the response to 1 above is ‘YES’, is the analysis conducted on-line in near real-time and/or off-line? If the analysis is conducted on-line is the operator alerted if the damping is worse than a preset threshold?
4. Apart from modal parameter estimates what other information is obtained (e.g. Mode-shape (i.e. right-eigenvector components), time-domain displays, etc.)?
5. If the response to 1 above is ‘YES’ please list particularly pertinent publications or accessible reports in public domain:

If none of the above single-shot methods are employed on your system, can you indicate if you use some other single-shot method - or plan to do so?

A4.8 Other Single-Shot Tests or Possible Future Developments of Single-Shot Methods

1. Are there other single-shot tests employed - other than those considered above? Please comment / explain, and indicate the method(s) of analysis employed.
2. At this time, is it intended to employ some other single-shot method(s) for modal estimation on your system?
3. Please list particularly pertinent publications or accessible reports in public domain.

Do you wish to comment on the education and training of staff in use of modal estimators?

A4.9 Education and Training of Engineering Staff and Operators in the Application of Modal Estimators

1. What was the nature of the training provided (e.g. a two-day course provided by the vendor of the modal estimation system)?
2. Were exercises provided by the vendor or another party to simulate the performance of the modal estimation system for lightly damped steady-state or dynamic conditions? (e.g. Simulation of our network under slowly degrading damping of an inter-area model is helpful in training operators in conditions which may lead to an alarm occurring.)

A4.10 Future Development of Modal Estimation Schemes

Based on your experience of the operation of modal-estimation schemes on your system please advise what you perceive as significant limitations of your existing scheme and/or desirable developments that would markedly improve the usefulness of your modal estimator. Categories for further development may be (i) visualization of information; (ii) the provision of reliable alerts in the event of inadequate damping; (iii) information on what action should be taken when damping falls below the alarm threshold; (iv) reliable damping estimates for the purpose of assessing the performance of damping controller.