Fundamental Parameters and Figures-of-Merit of Antennas

Constantine A. Balanis
Arizona State University
Tempe, AZ 85287 – 5706
balanis@asu.edu

Copyright © 2016 by Constantine A. Balanis
All rights reserved

IEEE EMC, Phoenix, AZ, Section
March 20, 2019

Antennas
Single Elements and Arrays

Antennas:
‘Electronic Eyes and Ears of the World’
John D. Kraus

Antenna As A Transition/Transducer Device

Copyright © 2016 by Constantine A. Balanis
All rights reserved

IEEE EMC, Phoenix, AZ, Section
March 20, 2019

TM Open Animation

Copyright © 2016 by Constantine A. Balanis
All rights reserved

IEEE EMC, Phoenix, AZ, Section
March 20, 2019

TM Box Animation

Copyright © 2016 by Constantine A. Balanis
All rights reserved

IEEE EMC, Phoenix, AZ, Section
March 20, 2019
Definition of Antenna

According to the IEEE Standard Definitions of Terms for Antennas:

"A means for radiating or receiving radio waves."

According to the Webster's Dictionary:

"A means for radiating or receiving radio waves."

In other words, the antenna is the transitional structure / transducer between free-space and a guiding device.

The guiding device or transmission line may take the form of a coaxial line or a hollow pipe (waveguide), and it is used to transport EM energy from the transmitting source to the antenna, or from the antenna to the receiver.

In the former case, we have a transmitting antenna and in the latter a receiving antenna.

Definition of Antenna

In addition to receiving or transmitting EM energy, an antenna in an advanced wireless communications system is usually required to optimize or accentuate the radiation energy in some directions and suppress it in others.

Thus the antenna must also serve as a directional device, in addition to a radiating device.

Thevenin Equivalent

In Transmission Mode

Horn Antenna

And Its Equivalent

Types of Antennas

1. Wire antennas
2. Aperture antennas
3. Microstrip antennas
4. Array antennas
5. Reflector antennas
6. Lens antennas
7. Other Antennas for Mobile Units
Coordinate System for Antennas

Coordinate System

Spherical Angular Limits

Area of Sphere = \( \int_0^{2\pi} \int_0^\pi r^2 \sin \theta \, d\theta \, d\phi \)

Area = \( \int_0^\pi r^2 \sin \theta \, d\theta \) \[d\phi \]

Area = \( (\pi)(2(1-1)) = 0 \) !!!

0 \leq \theta \leq 2\pi

0 \leq \phi \leq \pi

Spherical Angular Limits

Area of Sphere = \( \int_0^{2\pi} \int_0^\pi r^2 \sin \theta \, d\theta \, d\phi \)

Area = \( \int_0^\pi r^2 \sin \theta \, d\theta \) \[d\phi \]

Area = \( (2\pi)(2\pi) = 4\pi r^2 \)

0 \leq \theta \leq \pi

0 \leq \phi \leq 2\pi
Spherical Electric and Magnetic Field Components

\[ \hat{a}_r(E_\theta, H_\phi) \]

Amplitude Radiation Pattern

- **Field Pattern:**
  A plot of the field (either electric \( E \) or magnetic \( H \)) on a *linear* scale.

- **Power Pattern:**
  A plot of the power (proportional to either the electric \( E^2 \) or magnetic \( H^2 \) fields) on a *linear* or *decibel* (dB) scale.

2-D Normalized *Field* \( |E_r| \) Pattern of a Linear Array

- **Linear Scale**
  \( N = 10 \) elements
  \( d = \lambda / 4 \) spacing
  \( \text{HPBW} = 38.64^\circ \)

2-D Normalized *Power* \( |E_r|^2 \) Pattern of a Linear Array

- **Linear Scale**
  \( N = 10 \) elements
  \( d = \lambda / 4 \) spacing
  \( \text{HPBW} = 38.64^\circ \)

2-D Normalized *Power* \( |E_r|^2 \) Pattern of a Linear Array in dB

- **dB Scale**
  \( N = 10 \) element
  \( d = \lambda / 4 \) spacing
  \( \text{HPBW} = 38.64^\circ \)

Polar Pattern
Normalized Field Pattern (linear scale)

Normalized 3-D Amplitude Field Pattern of Linear Array

Linear Scale
\( N = 10, d = \lambda/4 \)

\[ E(r, \theta, \phi) = a_1 E_1(r, \theta, \phi) + a_2 E_2(r, \theta, \phi) + a_3 E_3(r, \theta, \phi) \]

\[ E = \sqrt{E_1^2 + E_2^2 + E_3^2} \]

Polar (dB Scale): Two-Dimensional Pattern

Directivity \( D \)

\[ D = \frac{U(\theta, \phi)}{U_0} = \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \]

\[ D_{\text{max}} = D_0 = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \]

\[ D(\text{dB}) = 10 \log_{10}[D(\text{dimensionless})] \]

\[ D_0(\text{dB}) = 10 \log_{10}[D_0(\text{dimensionless})] \]

Directivity

\[ D = \text{directivity (dimensionless)} \]

\[ D_0 = \text{maximum directivity (dimensionless)} \]

\[ U = \text{radiation intensity (W/unit solid angle)} \]

\[ U_{\text{max}} = \text{maximum radiation intensity} \]

\[ U_0 = \text{radiation intensity of isotropic} \]

\[ P_{\text{rad}} = \text{radiated power (W)} \]

Realized Gain

Gain

Directivity
Three-dimensional Directivity Pattern

Two-Dimensional Directivity Pattern

Example 1:

\[ W_{rad} = \hat{A} A_0 \sin \theta / r^2 \]

Solution:

\[ P_{rad} = \pi^2 A_0 \]
\[ U = rW_{rad} = A_0 \sin \theta \]
\[ U_{\text{max}} = U|_{\theta = \theta_{\text{max}}} = A_0 \sin \theta \]
\[ D_\theta = \frac{4\pi U_{\text{max}}}{P_{rad}} = \frac{4\pi (1) A_0}{\pi^2 A_0} = 1.27 \text{ dimensionless} \]
\[ D_\theta = 1.038 \text{ dB} \]
\[ D = D_\theta \sin \theta = 1.27 \sin \theta \]

Example 2:

\[ W_{rad} = \hat{A} A_0 \sin^3 \theta / r^2 \]

Solution:

\[ P_{rad} = \frac{8\pi A_0}{3} \]
\[ U = rW_{rad} = A_0 \sin^3 \theta \]
\[ U_{\text{max}} = U|_{\theta = \theta_{\text{max}}} = A_0 \sin^3 \theta \]
\[ D_\theta = \frac{4\pi U_{\text{max}}}{P_{rad}} = \frac{4\pi A_0}{8\pi/3} = 1.5 \text{ dimensionless} \]
\[ D_\theta = 1.761 \text{ dB} \]
\[ D = D_\theta \sin^3 \theta = 1.5 \sin^3 \theta \]
Approximate Formulas for Directivity

Kraus Formula

\[
\Omega_d = \int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin \theta \, d\theta \, d\phi \approx \Theta_{1r} \Theta_{2r}
\]

\[
D_0 = \frac{4\pi}{\Omega_d} \approx \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{41,253}{\Theta_{1d} \Theta_{2d}}
\]

\[
D_0 \approx \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi (180/\pi)^2}{\Theta_{1d} \Theta_{2d}} = 41,253
\]

Tai & Pereira Formula

\[
\frac{1}{D_0} = \frac{1}{2} \left( \frac{1}{D_1} + \frac{1}{D_2} \right)
\]

\[
D_0 = \frac{4\pi}{\Omega_d} \approx \frac{32 \ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1d}^2 + \Theta_{2d}^2}
\]

\[
D_0 = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2}
\]

N = 10 elements

d = \lambda/4 spacing

HPBW = 38.64°

FNBW = 73.8°

Kraus:

\[
D_0 = \frac{4\pi}{\Omega_d} = \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{41,253}{\Theta_{1d} \Theta_{2d}}
\]

\[
D_0 = \frac{41,253}{\Theta_{1d} \Theta_{2d}} = \frac{41,253}{(38.64)^2} = 27.63 = 14.51 \text{ dB}
\]

Tai & Pereira:

\[
D_0 = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{2(38.64)^2} = 24.38 = 13.87 \text{ dB}
\]

Using the computer program Directivity,

\[
D_0 = 10.1158 = 10.05 \text{ dB}
\]
Directional Patterns

U(\theta,\phi) = \begin{cases} 
B_o \cos^n(\theta) & 0 \leq \phi \leq \pi/2, 0 \leq \theta \leq \pi/2 \\
0 & \text{Elsewhere}
\end{cases}

n = 1, 2, 3 \ldots 10, 15, 20

Radiation Intensity Pattern

Exact & Approximate Directivities

Table 2.1

<table>
<thead>
<tr>
<th>n</th>
<th>Kraus' Error (%)</th>
<th>Tai and Pereira Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.86</td>
<td>2.33</td>
</tr>
<tr>
<td>2</td>
<td>5.09</td>
<td>4.99</td>
</tr>
<tr>
<td>3</td>
<td>7.35</td>
<td>6.48</td>
</tr>
<tr>
<td>4</td>
<td>9.61</td>
<td>8.18</td>
</tr>
<tr>
<td>5</td>
<td>11.473</td>
<td>10.14</td>
</tr>
<tr>
<td>6</td>
<td>14.17</td>
<td>11.03</td>
</tr>
<tr>
<td>7</td>
<td>16.39</td>
<td>12.69</td>
</tr>
<tr>
<td>8</td>
<td>18.66</td>
<td>16.47</td>
</tr>
<tr>
<td>9</td>
<td>20.93</td>
<td>18.47</td>
</tr>
<tr>
<td>10</td>
<td>23.19</td>
<td>20.47</td>
</tr>
</tbody>
</table>

| Kraus' Error = 0 |
| Kraus’ Error = T & P Error = 6.24% |
| n = 5.497 = 5.5 |
| n = 11.28 |
| HPBW = 56.35° |
| HPBW = 39.77° |

Omindirectional Patterns
Omnidirectional Patterns

\[ U = \left| \sin^n(\theta) \right| \]
\[ 0 \leq \theta \leq \pi \]
\[ 0 \leq \phi \leq 2\pi \]

Directivity:

- McDonald
  \[ D_0 = \frac{101}{\text{HPBW(degrees)} - 0.0027[\text{HPBW(degrees)}]^2} \]
- Pozar
  \[ D_0 = -172.4 + 191 \sqrt{0.818 + \frac{1}{\text{HPBW(degrees)}}} \]

Realized Gain

\[ P_{\text{in}} \]
\[ P'_{\text{in}} \]
\[ e_{\text{cd}} \]
\[ e_{\text{c}} \]
\[ e_{\text{d}} \]
\[ P_{\text{rad}} \]

Directivity

Gain

\[ G_0 = e_{\text{cd}} D_0 \]

(2-49a)

\[ e_{\text{cd}} = e_c e_d = \text{Radiation efficiency} \]
\[ e_c = \text{Conduction efficiency} \]
\[ e_d = \text{Dielectric efficiency} \]
Antenna Radiation Efficiency

\[ e_{rd} \]

Horn Antenna
And Its Circuit Equivalent

\[ G = e_{rd} D \]
\[ G_0 = e_{rd} D_0 \]

\[ G_{10}(\text{dB}) = 10 \log_{10} \left( e_{rd} D_0 \right) \]
\[ G_{01}(\text{dB}) = 10 \log_{10} (e_{rd}) + 10 \log_{10}(D_0) \]

Antenna Reference Terminals

Realized Gain

\[ e_{rd} = \text{Radiation Efficiency} \]
\[ e_{rd} = \frac{P_r}{P_{rd} + P_L} \]
\[ e_{rd} = \frac{R_r}{R_r + R_L} \]
Today antenna technology is Science Not Art
Very bright future and many challenges ahead.
We just need exercise creativity, imagination and science for its advancement.