Accelerating the Convergence of Stochastic Unit Commitment Problems by Using Tight and Compact MIP Formulations

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PES General Meeting
Denver-USA, July 2015
UC and MIP

- Unit Commitment (UC): essential tool for day(week)-ahead planning in the electricity sector
  - Decide on units’ physical operation (e.g., on-off) at minimum cost
  - UC is an integer computationally demanding problem
Introduction

UC and MIP

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- Significant breakthroughs in Mixed-Integer Programming (MIP)
  - Solving MIP 100 million times faster than 20 years ago\textsuperscript{1}

- The time to solve UC is still a critical limitation

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- How to reduce solving times?
  - Computer power (e.g., clusters)
  - Solving algorithms (e.g., solvers, decomposition techniques)

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  - Solving algorithms (e.g., solvers, decomposition techniques)
  - Improving the MIP-Based UC formulation ⇒ ↓ solving times

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Outline

1. Introduction

2. Good and Ideal MIP formulations

3. Tight & Compact (TC) UC Formulations

4. Case Studies
   - Deterministic Selft-UC
   - Stochastic UCs: Different Solvers
   - Stochastic UCs: IEEE-118 Bus System

5. Conclusions
An MIP Has Infinite LP Formulations
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LP1 and LP2 represent the same MIP problem.

Feasible solutions = •
An MIP Has Infinite LP Formulations

$\text{LP1}, \text{LP2}$ and $\text{CH}$ represent the same MIP problem

which one to choose?
Solving MIP Through The Powerful LP

Shaping the linear feasible region to arrive from vertex $Z_{LP}$ to $Z_{MIP}$

To prove optimality $Z_{MIP}$ must become a vertex by:

- Branch and bound (divide and conquer)
Solving MIP Through The Powerful LP

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To prove optimality $Z_{MIP}$ must become a vertex by:

- Branch and bound (divide and conquer)
- and/or by adding cuts
Convex Hull: The Tightest Formulation

Convex Hull (CH)
Smallest convex feasible region containing all feasible integer points

**Convex Hull: The Tightest Formulation**

**Convex Hull (CH)**
Smallest convex feasible region containing all feasible integer points

- The *convex hull* problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
  - So an LP optimum is also an MIP optimum

- Unfortunately,

---


Convex Hull (CH)

Smallest convex feasible region containing all feasible integer points

- The convex hull problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
  - So an LP optimum is also an MIP optimum
- Unfortunately, the convex hull is typically too difficult to obtain
  - To solve an MIP is usually easier than trying to find its convex hull

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Choosing The Best Formulation
Measuring The Tightness

Integrality Gap (IGap)
Relative distance between MIP and LP optima

\[
\text{IGap}_{LP1} = \frac{Z_{MIP} - Z_{LP1}}{Z_{MIP}} > \text{IGap}_{LP2} = \frac{Z_{MIP} - Z_{LP2}}{Z_{MIP}}
\]

\[\downarrow\]
As an MIP problem:
LP2 is expected to be solved faster than LP1
Choosing The Best Formulation
Measuring The Tightness

Integrality Gap (IGap)
Relative distance between MIP and LP optima

\[
\text{IGap}_{\text{LP1}} = \frac{Z_{\text{MIP}} - Z_{\text{LP1}}}{Z_{\text{MIP}}} > \text{IGap}_{\text{LP2}} = \frac{Z_{\text{MIP}} - Z_{\text{LP2}}}{Z_{\text{MIP}}} > \\
\text{IGap}_{\text{CH}} = \frac{Z_{\text{MIP}} - Z_{\text{CH}}}{Z_{\text{MIP}}} = 0
\]

\[\downarrow\]
As an MIP problem:
LP2 is expected to be solved faster than LP1
CH will be solved way faster than LP2
Concepts: Tightness and Compactness

- **Tightness**: defines the search space (relaxed feasible region) that the solver needs to explore to find the solution.
- **Compactness (problem size)**: defines the searching speed (data to process) that the solver takes to find the solution.
Concepts: Tightness and Compactness

- **Tightness**: defines the search space (relaxed feasible region) that the solver needs to explore to find the solution
- **Compactness (problem size)**: defines the searching speed (data to process) that the solver takes to find the solution
- **Convex hull**: The tightest formulation $\Rightarrow$ MIP solved as LP
Tightening an MIP Formulation

- The most common strategy is adding cuts
  - In fact, this is the most effective strategy of current MIP solvers\(^4\)

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Tightening an MIP Formulation

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  - They should be added as cuts in the B&B\(^5,6\) ⇒ ↓ Time
  - and not directly to the model, huge number of inequalities ⇒ ↑ Time

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Tightening an MIP Formulation

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- They should be added as cuts in the B&B\(^5,6\) ⇒ \(\downarrow\) Time
- and not directly to the model, huge number of inequalities ⇒ \(\uparrow\) Time
- Trade-off: Tightness vs. Compactness

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Tightening an MIP Formulation

- The most common strategy is adding cuts
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  - They should be added as cuts in the B&B\(^5,6\) ⇒ ↓ Time
  - and not directly to the model, huge number of inequalities ⇒ ↑ Time
  - Trade-off: Tightness vs. Compactness

- Improving the MIP formulation
  - Provide the *convex hull* for some set of constraints
  - If available, use the *convex hull* for some set of constraints

---


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3 Tight & Compact (TC) UC Formulations

4 Case Studies
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   - Stochastic UCs: Different Solvers
   - Stochastic UCs: IEEE-118 Bus System

5 Conclusions
Let’s focus on the core of UC formulations:

- Min/max outputs
- SU & SD ramps
- Minimum up/down ($TU/TD$) times
Tight and Compact (TC) Formulation

- Let’s focus on the core of UC formulations:
  - Min/max outputs
  - SU & SD ramps
  - Minimum up/down ($T_U/T_D$) times, convex hull already available\(^7\)

- The whole formulation can be found in the paper TC-UC\(^8\) and the convex hull proof in gentile et al.\(^9\)

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\(^7\) D. Rajan and S. Takriti, “Minimum up/down polytopes of the unit commitment problem with start-up costs,” IBM, Research Report RC23628, Jun. 2005


Formulation for a generating unit (I)

- Generation limits taking into account:

\[
pt \leq \left( P - \bar{P} \right) u_t - \left( P - SD \right) w_{t+1} - \max \left( SD - SU, 0 \right) v_t \quad \forall t
\]

\[
p_t \leq \left( P - \bar{P} \right) u_t - \left( P - SU \right) v_t - \max \left( SU - SD, 0 \right) w_{t+1} \quad \forall t
\]

Total generation = \( P \cdot u_t + p_t \).

<table>
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<th>Variables</th>
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Formulation for a generating unit (II)

- Logical relationship:

\[ u_t - u_{t-1} = v_t - w_t \quad \forall t \]  \hspace{1cm} (3)
\[ v_t \leq u_t \quad \forall t \]  \hspace{1cm} (4)
\[ w_t \leq 1 - u_t \quad \forall t \]  \hspace{1cm} (5)

where (4) and (5) avoid the simultaneous startup and shutdown.

- Variable bounds

\[ p_t \geq 0 \quad \forall t \]  \hspace{1cm} (6)
\[ 0 \leq u_t, v_t, w_t \leq 1 \quad \forall t \]  \hspace{1cm} (7)
Tightness of the Formulation

Let’s study the polytope (1)-(7) using PORTA\textsuperscript{10}:

- PORTA enumerates all vertices of a convex feasible region

\textsuperscript{10}T. Christof and A. Löbel, “PORTA: POlyhedron representation transformation algorithm, version 1.4.1,” Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany, 2009
Tight & Compact UCs

Tightness of the Formulation

Let’s study the polytope (1)-(7) using PORTA\textsuperscript{10}:

- PORTA enumerates all vertices of a convex feasible region
- Example: 3 periods and $\bar{P} = 200$, $P = SU = SD = 100$ for:
  - Case 1: $TU = TD = 1$
  - Case 2: $TU = TD = 2$

\textsuperscript{10} T. Christof and A. Löbel, “PORTA: POlyhedron representation transformation algorithm, version 1.4.1,” Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany, 2009
Case 1: Providing The Convex Hull

Formulation:

\[
p_t \leq \left( P - \bar{P} \right) u_t - \left( P - SD \right) w_{t+1} - \max (SD - SU, 0) v_t \tag{1}
\]

\[
p_t \leq \left( P - \bar{P} \right) u_t - \left( P - SU \right) v_t - \max (SU - SD, 0) w_{t+1} \tag{2}
\]

\[
u_t - u_{t-1} = v_t - w_t \tag{3}
\]

\[
v_t \leq u_t \tag{4}
\]

\[
w_t \leq 1 - u_t \tag{5}
\]

PORTA results for \((TU = TD = 1)\)
Case 1: Providing The Convex Hull

Formulation:

\[ p_t \leq (\overline{P} - P) u_t - (\overline{P} - SD) w_{t+1} \]
\[- \max (SD - SU, 0) v_t \]  (1)

\[ p_t \leq (\overline{P} - P) u_t - (\overline{P} - SU) v_t \]
\[- \max (SU - SD, 0) w_{t+1} \]  (2)

\[ u_t - u_{t-1} = v_t - w_t \]  (3)

\[ v_t \leq u_t \]  (4)

\[ w_t \leq 1 - u_t \]  (5)

PORTA results for \((TU = TD = 1)\):

\[ u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3:\]

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END
Case 1: Providing The Convex Hull

Formulation:

\[ p_t \leq (\overline{P} - \underline{P}) u_t - (\overline{P} - SD) w_{t+1} \]
\[ - \max (SD - SU, 0) v_t \quad (1) \]

\[ p_t \leq (\overline{P} - \underline{P}) u_t - (\overline{P} - SU) v_t \]
\[ - \max (SU - SD, 0) w_{t+1} \quad (2) \]

\[ u_t - u_{t-1} = v_t - w_t \quad (3) \]

\[ v_t \leq u_t \quad (4) \]

\[ w_t \leq 1 - u_t \quad (5) \]

All vertices are integer

\[ \downarrow \]

Convex Hull

PORTA results for \((TU = TD = 1)\)

\[ u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3:\]

DIM = 10

CONV_SECTION

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(14) 1 1 1 0 0 0 0 100 0 100
(15) 1 1 1 0 0 0 0 100 100 0
(16) 1 1 1 0 0 0 0 100 100 100
(17) 1 0 1 0 1 1 0 0 0 0

END
Case 2: Providing and Using Convex Hulls (I)

Formulation + \( TU/TD \) Convex hull:

\[
p_t \leq (\overline{P} - \underline{P}) u_t - (\overline{P} - SD) w_{t+1} \\
- \max (SD - SU, 0) v_t \tag{1}
\]

\[
p_t \leq (\overline{P} - \underline{P}) u_t - (\overline{P} - SU) v_t \\
- \max (SU - SD, 0) w_{t+1} \tag{2}
\]

\[
u_t - u_{t-1} = v_t - w_t \tag{3}
\]

\[
\sum_{i=t-TU+1}^{t} v_i \leq u_t \tag{4}
\]

\[
\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \tag{5}
\]
Case 2: Providing and Using Convex Hulls (I)

Formulation + \( TU/TD \) Convex hull:

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\[
u_t - u_{t-1} = v_t - w_t \quad (3)
\]

\[
\sum_{i=t-TU+1}^{t} v_i \leq u_t \quad (4)
\]

\[
\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \quad (5)
\]

How to remove the fractional vertices?

PORTA results for \( TU = TD = 2 \)

\begin{verbatim}
PORTA results for (TU = TD = 2)

u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3:

DIM = 10

CONV_SECTION

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( 2) 1/2 1 1/2 1/2 0 0 1/2 0 50 0
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( 4) 1/2 1 1/2 1/2 0 0 1/2 50 50 0
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( 6) 0 0 1 0 1 0 0 0 0 0
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END
\end{verbatim}
Case 2: Providing and Using Convex Hulls (II)

Reformulating (1) and (2) for $TU \geq 2$:

\[ p_t \leq (P - P) u_t - (P - SD) w_{t+1} \]
\[ \max (SD - SU, 0) v_t \]  

\[ p_t \leq (P - P) u_t - (P - SU) v_{t+1} \]
\[ \max (SU - SD, 0) v_{t+1} \]  

\[ p_t \leq (P - P) u_t - (P - SU) v_t \]
\[ - (P - SD) w_{t+1} \]  

\[ u_t - u_{t-1} = v_t - w_t \]

\[ \sum_{i=t-TU+1}^{t} v_i \leq u_t \]

\[ \sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \]

PORTA results for $(TU = TD = 2)$

\[ u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3: \]

\[ \text{DIM} = 10 \]

CONV_SECTION

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(14) 1 1 1 0 0 0 0 100 100 0  
(15) 1 1 1 0 0 0 0 100 100 100  

END
Case 2: Providing and Using Convex Hulls (II)

Reformulating (1) and (2) for $TU \geq 2$:

$\forall t \in [0, T_U - 1]$

$$p_t \leq (P - P) u_t - (P - P) w_{t+1}$$

$$\max(SD, SU, 0) v_t$$

(1)

$$p_t \leq (P - P) u_t - (P - P) w_{t+1}$$

$$\max(SU, SD, 0) v_t$$

(2)

$$p_t \leq (P - P) u_t - (P - P) w_{t+1}$$

$$- (P - SD) w_{t+1}$$

(8)

$$u_t - u_{t-1} = v_t - w_t$$

(3)

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t$$

(4)

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t$$

(5)

⇒ Convex Hull

PORTA results for $(TU = TD = 2)$

$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

DIM = 10

CONV_SECTION

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(2) 0 0 1 0 1 0 0 0 0 0
(3) 1 0 0 0 0 1 0 0 0 0
(4) 0 1 1 1 0 0 0 0 0 0
(5) 0 0 1 1 1 0 0 0 0 0 100
(6) 1 1 0 0 0 0 1 0 0 0
(7) 1 1 0 0 0 0 1 100 0 0
(8) 1 1 1 0 0 0 0 0 0 0
(9) 1 1 1 0 0 0 0 0 0 100
(10) 1 1 1 0 0 0 0 0 0 100 0
(11) 1 1 1 0 0 0 0 0 0 100 100
(12) 1 1 1 0 0 0 0 100 0 0
(13) 1 1 1 0 0 0 0 100 0 0
(14) 1 1 1 0 0 0 0 100 100 0
(15) 1 1 1 0 0 0 0 100 100 100

END
Outline

1. Introduction
2. Good and Ideal MIP formulations
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4. Case Studies
   - Deterministic Selft-UC
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5. Conclusions
Self-UC case Study

- **Case Study:** Self-UC for 10-units, for 32-512 days time span
- Basic constraints: max/min, SU/SD and TU/TD

All results are expressed as percentages of 1bin results
Case Studies

Determine Self-UC

Self-UC case Study

- **Case Study:** Self-UC for 10-units, for 32-512 days time span
  - Basic constraints: max/min, SU/SD and TU/TD
- Formulations tested – modeling the same MIP problem:
  - $TC^{11}$: Proposed Tight & Compact
  - $1bin^{12}$: 1-binary variable ($u$)
  - $3binTUTD^{13}$: 3-binary variable version ($u,v,w$) + $TU/TD$ convex hull

All results are expressed as percentages of $1bin$ results

---


Case Study: Self-UC (I)

Results presented as percentages of $1_{\text{bin}}$:

<table>
<thead>
<tr>
<th></th>
<th>3bin TUTD (%)</th>
<th>TC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>&lt;78</td>
<td>&lt;48</td>
</tr>
<tr>
<td>Nonzeros</td>
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</tr>
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<td>33.3</td>
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</table>

$\downarrow$

$TC$ is more Compact
Case Study: Self-UC (I)

Results presented as percentages of $1bin$:

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$TC$ is Tighter and Simultaneously more Compact
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<td>Integrality Gap</td>
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<td>=0</td>
</tr>
<tr>
<td>MIP runtime (speedup)</td>
<td>4.9 (20x)</td>
<td>0.107 (995x)</td>
</tr>
</tbody>
</table>

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\textit{TC} is Tighter \textbf{and Simultaneously} more Compact
Case Study: Self-UC (II)

Results presented as percentages of 1bin:

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The TC formulation describe the convex hull then solving MIP (non-convex) as LP (convex)
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5. Conclusions
Stochastic UC: Case Study

- 10 generating units for a time span of 2 days
- 10 to 200 scenarios in demand
Stochastic UC: Case Study

- 10 generating units for a time span of 2 days
- 10 to 200 scenarios in demand
- Comparing $TC$, $1bin$, $3binTUTD$ and Two additional formulations $Sh^{14}$ and $3bin^{15}$

---


Stochastic UC: Case Study

- 10 generating units for a time span of 2 days
- 10 to 200 scenarios in demand
- Comparing \( TC, 1\text{bin}, 3\text{bin TUTD} \) and Two additional formulations \( Sh^{14} \) and \( 3\text{bin}^{15} \)
- Different Solvers
  - Cplex 12.6.0
  - Gurobi 5.6.2
  - XPRESS 25.01.07
- Stop criteria:
  - Time limit: 5 hours or
  - Optimality tolerance: 0.1 %

---


Stochastic: Cplex
Stochastic: Cplex

TC deals with 200 scenarios within the time that others deal with 40
Stochastic: Gurobi

TC deals with 200 scenarios within the time that others deal with 50
Stochastic: XPRESS

$TC$ deals with 200 scenarios within the time that others deal with 80
Outline

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IEEE-118 Bus System

- 54 thermal units; 118 buses; 186 transmission lines; 91 loads
  - 24 hours time span
  - 3 wind farms, 20 wind power scenarios
  - Stop Criteria in Cplex 12.6.0
    - 0.05% opt. tolerance or 24h time limit
### UC performance comparisons (I)

<table>
<thead>
<tr>
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<th>Traditional Energy-Block Scheduling</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>3bin TUTD(^ {16})</td>
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<tr>
<td>o.f. [k$]</td>
<td>829.04</td>
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<td>opt.tol [%]</td>
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<td>0.023</td>
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<td>IntGap [%]</td>
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- Compared with 3bin TUTD, TC:
  - lowered IntGap by 53.3%

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Case Studies  Stochastic UCs: IEEE-118 Bus System

UC performance comparisons (I)

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Compared with 3binTUTD, TC:

- lowered IntGap by 53.3%
- is more than 420x faster

## UC performance comparisons (II)

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- TC solved the MIP before 3binTUTD solved the LP
- within the required opt. tolerance (0.05%)

## UC performance comparisons (III)

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<th>Power-Based UC</th>
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<tbody>
<tr>
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- **P-TC**\(^{18}\) has a more detailed and accurate UC representation

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- **P-TC**\(^{18}\) has a more detailed and accurate UC representation
  - it solved 100x faster than 3binTUTD
  - its UC core is also a convex hull\(^{19}\)

---


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Conclusions (I)

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  - **Tightness & Compactness**
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  - **Tightness & Compactness**
  - ↑ Binaries ⇒ ↑ Solving time **False myth**
- **Use the convex hull** of some set of constraints
  - Minimum up/down times\(^{20}\)
  - Unit operation in Energy-based UC\(^{21}\)
  - Unit operation in Power-based UC\(^{22}\)
  - ⇒ ↓ solving time by **simultaneously T&Cing** the final UCs\(^{23,24}\)

---


Conclusions (II)

- **Better UC core in stochastic UCs** ⇒
  - Critical solving time reductions
  - Even when the UC is modeled in more detail, i.e., P-UC
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  - Critical solving time reductions
  - even when the UC is modeled in more detail, i.e., P-UC

- **If convex hulls** are not available ⇒
  - Create simultaneously tight and compact models
    - by reformulating the problem, e.g., CCGTs\(^{25}\)
    - **Key hint**: start removing all big-M parameters

---

Conclusions (II)

■ **Better UC core** in stochastic UCs ⇒
  ■ Critical solving time reductions
  ■ even when the UC is modeled in more detail, i.e., P-UC

■ If *convex hulls* are not available ⇒
  ■ Create simultaneously tight and compact models
    ■ by reformulating the problem, e.g., CCGTs\(^{25}\)
    ■ **Key hint:** start removing all big-M parameters
  ■ Create tight cuts
    ■ **Don’t add them** directly to the model
    ■ Use them as **cuts in the B&B algorithm**\(^{26,27}\) ⇒ ↓ time

---


Questions

Thank you for your attention

Contact Information:
g.a.moralesespama@tudelft.nl
For Further Reading


For Further Reading (cont.)


For Further Reading (cont.)


For Further Reading (cont.)

Power-Based UC

Two main features are included:

- Schedules Power instead of Energy (for feasibility). **To avoid:**
  - Infeasible energy delivery\(^{28}\)
  - Overestimated ramp availability\(^{29}\)


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  - SU & SD ramps are deterministic events in day-ahead UCs
  - Ignoring them change commitment decisions and increase costs\(^\text{21,30}\)


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  - Infeasible energy delivery\textsuperscript{28}
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- Includes startup (SU) and shutdown (SD) power trajectories
  - SU & SD ramps are deterministic events in day-ahead UCs
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- The convex hull of the Power-Based UC for basic operating constraints is provided\textsuperscript{31}.


## Case Study: Self-UC (II)

Performance of the Energy-Based formulations:

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<th>$3binTUTD$ (%)</th>
<th>$TC$ (%)</th>
<th>$R-TC$ (%)</th>
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<tr>
<td>Constraints</td>
<td>&lt;78</td>
<td>&lt;48</td>
<td>&lt;56</td>
</tr>
<tr>
<td>Nonzeros</td>
<td>89</td>
<td>72</td>
<td>94</td>
</tr>
<tr>
<td>Real Vars</td>
<td>33.3</td>
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<td>66.7</td>
</tr>
<tr>
<td>Bin Vars</td>
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<td>=300</td>
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<tr>
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</tr>
<tr>
<td>MIP run (best-worst)</td>
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Performance of the Energy-Based formulations:

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The $T\text{C}$ formulations describe the *convex hull* then solving MIP (non-convex) as LP (convex)
Outline

- UC for 40 power system mixes
UC for 40 power system mixes

- **Case Study B**: UC for 40 power system mixes, from 28 to 1870 units\(^{32}\)
  - Including demand, ramps, reserves, variable SU costs

---

Integrality Gap: Small Cases 1-10
280-540 units x 1 day, OptTol: 0.001

Geometric Averages: 3bin 64%; TC 35%
**Conclusions**

UC for 40 power system mixes

**CPU Time: Small Cases 1-10**

280-540 units x 1 day, OptTol: 0.001

**Geometric Averages:** 3bin 36%; TC 12%
**Integrality Gap: Large Cases 11-20**

1320-1870 units x 1 day, OptTol: 0.01

**Geometric Averages:**
- 3bin 75%
- TC 43%
CPU Time: Large Cases 11-20
1320-1870 units x 1 day, OptTol: 0.01

Geometric Averages: 3bin 45%; TC 5%
## UC for 40 power system mixes

Results presented as percentages of $1bin$:

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<tbody>
<tr>
<td>Nonzeros</td>
<td>$\sim 100$</td>
<td>$&lt; 35$</td>
</tr>
<tr>
<td>Real Vars</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>Bin Vars</td>
<td>300</td>
<td>$&lt; 500$</td>
</tr>
<tr>
<td>Integrality Gap</td>
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<td>40</td>
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Conclusions

UC for 40 power system mixes

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TC is Tighter and Simultaneously more Compact
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<td>40</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>Average runtime</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>runtime (best-worst)</td>
<td>11 – 269</td>
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*TC is Tighter and Simultaneously more Compact*
### UC for 40 power system mixes

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<tr>
<td>runtime (best-worst)</td>
<td>11 – 269</td>
<td>2 – 57</td>
</tr>
<tr>
<td>Runtime Small Cases</td>
<td>67</td>
<td>11</td>
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<td>77</td>
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\textbf{TC is Tighter and Simultaneously more Compact}