Improving Search Space Exploration and Exploitation with the Cross-Entropy Method and the Evolutionary Particle Swarm Optimization

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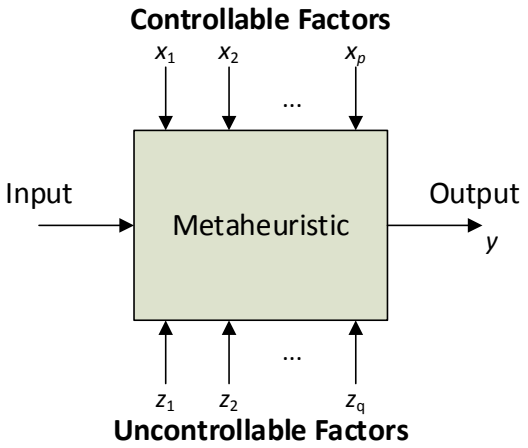
08-08-2018

Methodological Approach

- **Combination of two optimization methods**
  - **Cross-Entropy (CE) Method** for exploration
  - **Evolutionary Particle Swarm Optimization (EPSO)** for exploitation
    - *New recombination operator for multi-period constraints*

- **EPSO parameters** were **tuned** using an iterative optimization algorithm based on a **2^2 factorial design**
  - Mutation Rate $\boldsymbol{\tau}$
  - Communication Probability $\boldsymbol{P}$



CE Method for Optimization

- **Heuristic method** proposed by Reuven Rubinstein for combinatorial and continuous non-linear optimization based on the **Kullback–Leibler divergence**
  - Kullback–Leibler divergence (relative entropy) is a measure of how one probability distribution diverges from a target one

- It is based on the concept that **optimization problems can be defined as a rare-event estimation problem**
  - The probability of locating an optimal or near optimal solution using random search is a rare-event probability

CE Method for Optimization

- Instead of random search, the **CE Method** sets an **adaptive iterative importance sampling** process
  - The sampling distribution (Bernoulli, Binomial, Gaussian, Multivariate Bernoulli) is modified in each iteration so that the rare-event becomes more likely to occur

- The optimization process starts by defining a **sampling distribution** for the **variables** followed by an **iterated adjustment of the distribution parameters** (e.g. mean, standard deviation) according to the performance of an **elite subset of the samples**

CE Method for Optimization

Select μ_0 and σ_0^2, the number of samples per iteration N, the rarity parameter ρ, the smoothing parameter α, $k := 0$

Do

$k := k + 1$

Generate a sample of $\mathbf{X}_1, \ldots, \mathbf{X}_N$ from the sampling distribution $N(\mu_{k-1}, \sigma_{k-1}^2)$

Compute $S(\mathbf{X}_1), \ldots, S(\mathbf{X}_N)$ and order the samples from the worst to the best performing ones, i.e. $S(\mathbf{X}_1) < S(\mathbf{X}_2) < \ldots < S(\mathbf{X}_N)$

Compute γ_k as the ρth quantile of the performance values and select $N^{elite}=\rho N$; let ψ be the subset from the ordered set of samples that contains all the N^{elite} samples, i.e., the samples $S(\mathbf{X})<\gamma_k$

For $j = 1$ to n

$$\mu_{kj} := \sum_{i \in \Psi} \frac{X_{ij}}{N^{elite}} \qquad \sigma_{kj}^2 := \sum_{i \in \Psi} \frac{\left(X_{ij} - \mu_{kj}\right)^2}{N^{elite}}$$

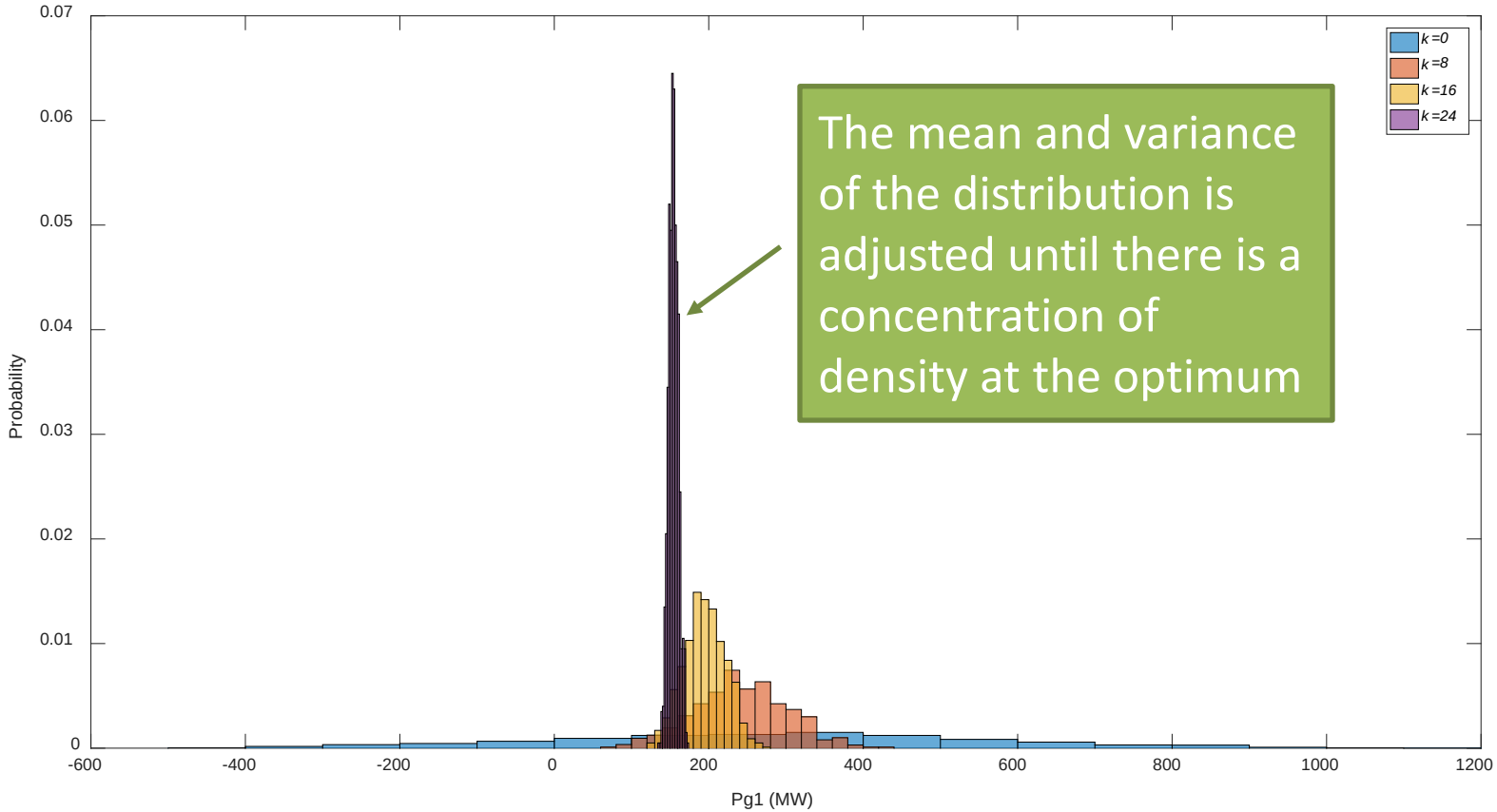
End For

Apply smoothing

$$\boldsymbol{\mu}_k := \alpha\, \boldsymbol{\mu}_k + (1-\alpha)\, \boldsymbol{\mu}_{k-1} \qquad \boldsymbol{\sigma}_k^2 := \alpha\, \boldsymbol{\sigma}_k^2 + (1-\alpha)\, \boldsymbol{\sigma}_{k-1}^2$$

Until $k < k^{MAX}$

CE Method for Optimization



The mean and variance of the distribution is adjusted until there is a concentration of density at the optimum

EPSO

- **Evolutionary Particle Swarm Optimization** (EPSO) is an hybrid between Evolutionary Strategies and Particle Swarm Optimization proposed by Vladimiro Miranda in 2002
  - **Replication**: each individual is replicated *r* times
  - **Mutation**: the *r* clones have their weights *w* mutated
  - **Recombination**: the *r*+1 individuals generate one offspring
  - **Evaluation**: each offspring has its fitness evaluated
  - **Selection**: the best particle out of the *r*+1 survives to be part of a new generation

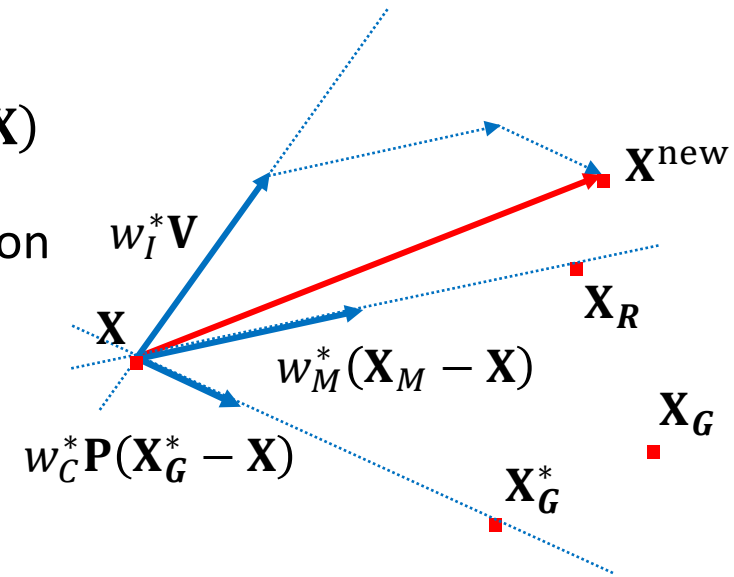
Recombination in EPSO

- Movement Rule

$$\mathbf{X}^{\text{new}} = \mathbf{X} + \mathbf{V}^{\text{new}}$$

$$\mathbf{V}^{\text{new}} = w_I^* \mathbf{V} + w_M^* (\mathbf{X}_M - \mathbf{X}) + w_C^* \mathbf{P}(\mathbf{X}_G^* - \mathbf{X})$$

  - **Inertia**: movement in the same direction
  - **Memory**: attraction towards the individual best solution
  - **Cooperation**: attraction to a region near the global best position



EPSO Particularities

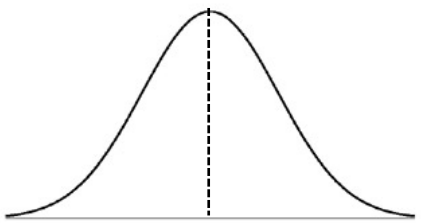
- Weights are mutated and subjected to selection

Mutation Rate

$$w^{*}=w[1+\sigma N(0,1)]$$

- The individuals are attracted to a region near the best solution found

$$\mathbf{X}_{\boldsymbol{G}}^{*}=\mathbf{X}_{\boldsymbol{G}}\,[1+w_{GB}^{*}N(0,1)]$$



$\mathbf{X}_{\boldsymbol{G}}$

- Matrix **P** acts as a communication barrier

$$\mathbf{V}^{\text{new}}=w_{I}^{*}\mathbf{V}+w_{M}^{*}(\mathbf{X}_{\boldsymbol{M}}-\mathbf{X})+w_{C}^{*}\mathbf{P}(\mathbf{X}_{\boldsymbol{G}}^{*}-\mathbf{X})$$

$$\mathbf{P}=\begin{bmatrix}1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & 0\end{bmatrix}$$

Communication Probability

If $i \neq j$ then $P_{ij} = 0$

ElseIf $U(0,1) > p$ then $P_{ij} = 0$

Else $P_{ij} = 1$

EPSO Recombination in the 2018 Competition

- **Test Bed B** consists of a **Dynamic OPF for 6 periods** that considers ramp constraints for generators (5 x 53 constraints)
  - Feasible solutions are difficult to obtain since the production in each period can be highly conditioned by the production in the adjacent periods

- **Idea**: to compute a new production for the current period based on the recombination of this production with the production of the next period

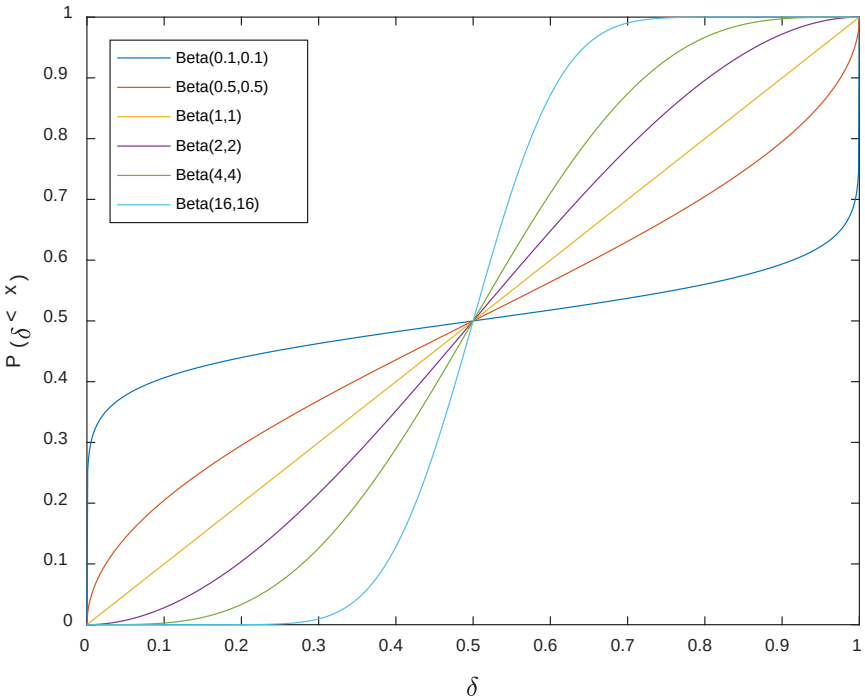
EPSO Recombination in the 2018 Competition

- Additional recombination rule for generators

$$X_{i,j}^{NEW} := X_{i,j}^{NEW}\delta + X_{i,j+1}^{NEW}(1-\delta)$$

- $0 \leq \delta \leq 1$ is random variable that follows a Beta(α,β) pdf

- Beta(0.5,0.5) was selected



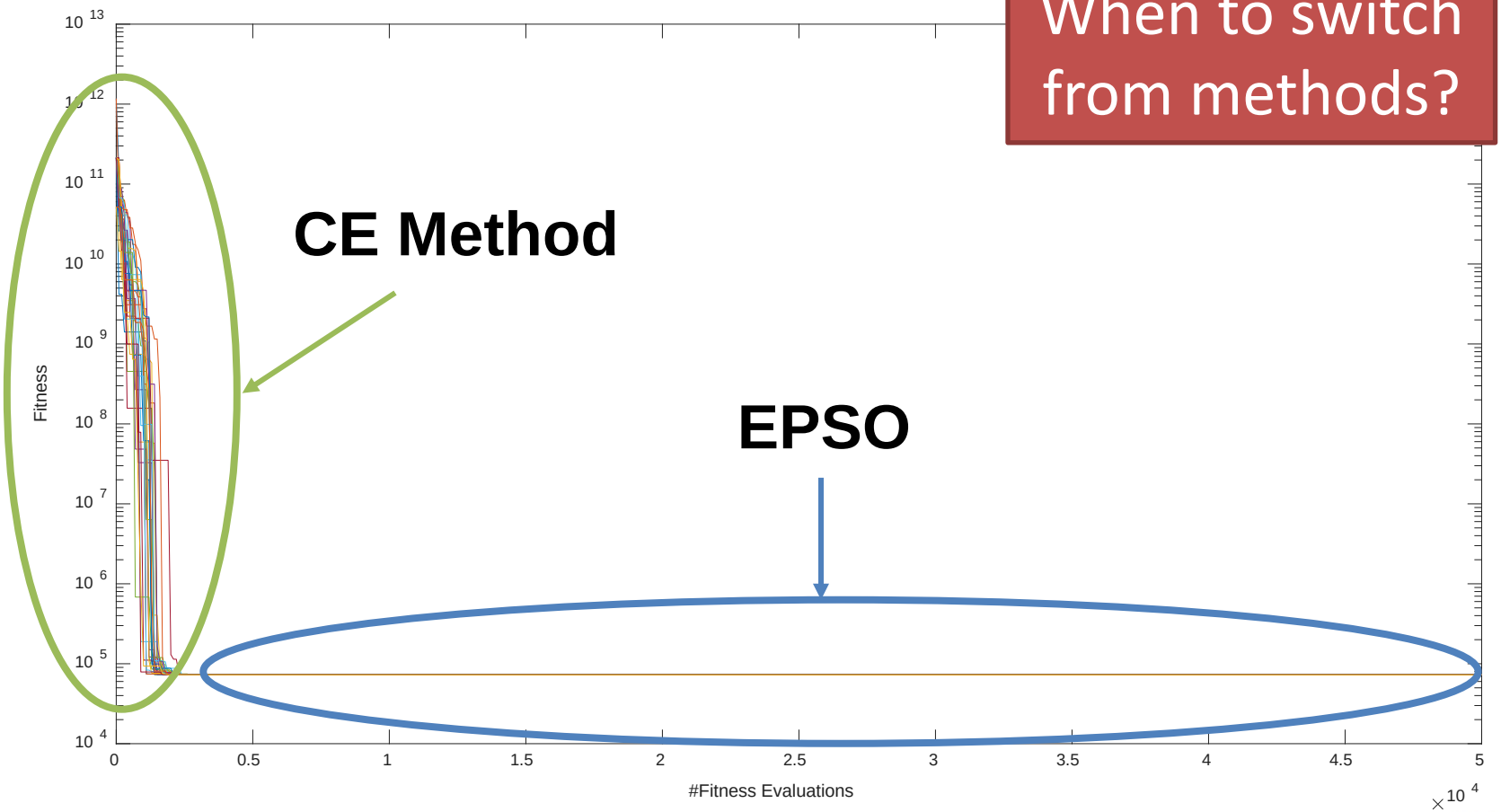
EPSO Recombination in the 2018 Competition

- In EPSO, **integer variables** are **modeled** as **real variables** and rounded for fitness computation

- **To force diversity**, after the computation of a new position, each integer in the new position is compared with that of its predecessor and, if all integers in the two positions are equal, then one integer in the new solution is randomly selected to be increased/decreased by the value of 1 with probability 0.5

Switch from CE Method to EPSO

When to switch from methods?



Switch from CE Method to EPSO

- **Trial & error**
  - Not very clever but a **practical approach** for the competition
- Track the **rate** of **improvement** of the best fitness over a number of iterations and switch when the rate becomes **inferior** a given **threshold**
  - A new parameter has to be defined
- **Monitor the variance of the CE Method sampling distributions**
  - The problem is that the variance can decrease very slowly for variables that can take a wide range of values without significantly affecting the fitness function

EPSO Parameter Tuning

- **Iterative method** based on **2^2 factorial design** and the **Two-way ANOVA** to guarantee the **best performance** for **EPSO**

  Define maximum allowable interval for τ and P (e.g. [0.2, 0.8])

  Run a 2^2 factorial design (**4 experiments**)

  Perform 4 × 31 runs of EPSO for the 4 combinations of the limit values of τ and P

  Do

  Run Two-way ANOVA and **select** the variable with the highest F-test statistic/the lowest *p*-value

  Compute the main effect to determine which limit should be updated

  Update the limit to the central value of the current interval (e.g. if the output decreases with the variable increase, then set the lower limit to the central value)

  Run a 2^2 factorial design with the updated limits (**+2 experiments**)

  While there is evidence that τ and P affect the output (*p*-value less than 0.05) or if the difference between interval limits is greater than a value (e.g. 0.1)

Parameter Tuning for Minimization

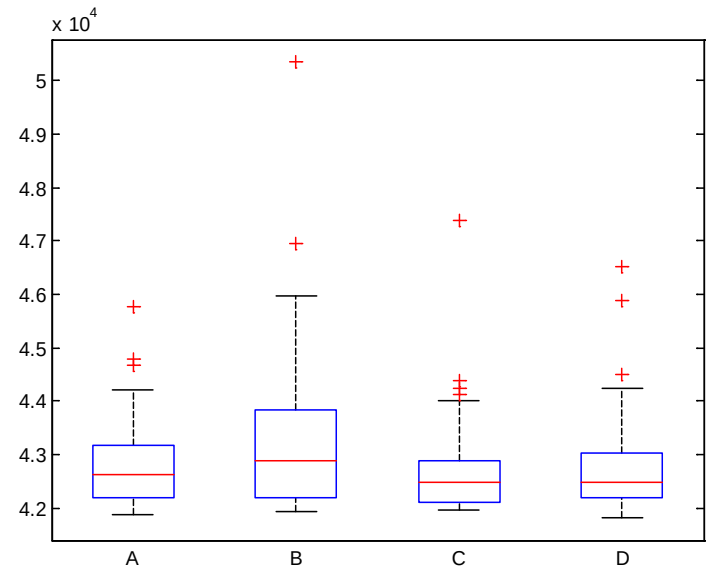
Two-way ANOVA results

Factor	F-test Statistic	P-value	Main Effect
	1st Iteration		
τ	3.474	0.064	-350.73
P	**6.675**	0.011	**486.14**
τ × P	0.225	0.636	89.17
	2nd Iteration		
τ	5.469	0.021	-548.35
P	**6.022**	0.015	**575.42**
τ × P	1.496	0.223	286.79
	3nd Iteration		
τ	**5.508**	0.020	**-448.7**
P	1.448	0.231	230.01
τ × P	0.958	0.329	187.14
	4th Iteration		
τ	2.849	0.093	-306.78
P	2.497	0.116	287.21
τ × P	1.807	0.181	244.34

Greater than the threshold level of 0.05

Setting values for *τ* and *P*

Factor	Iteration			
	1st	2nd	3rd	4th
τ (low)	0.2	0.2	0.2	**0.5**
τ (high)	0.8	0.8	0.8	0.8
P (low)	0.2	0.2	0.2	0.2
P (high)	0.8	**0.5**	**0.35**	0.35



Box-plots for the final range of *τ* and *P*

Results

- Test Bed A – Stochastic OPF: 12 independent runs

Fitness	Case 1	Case 2	Case 3	Case 4	Case 5
Best	80,732.46	67,709.06	55,245.86	84,382.21	71,044.22
Worst	81,547.19	68,923.91	56,683.60	84,880.76	71,128.74
Mean	81,077.07	68,473.43	55,935.62	84,442.94	71,065.91
Standard Deviation	240.41	306.05	418.30	138.71	22.97

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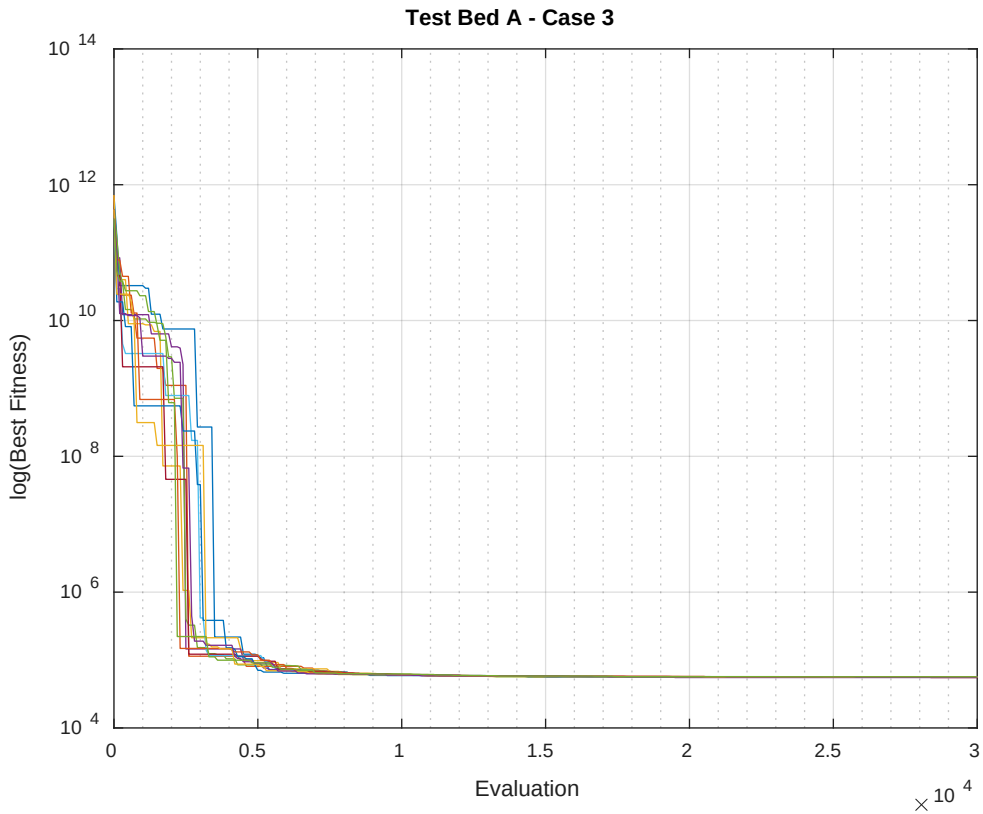

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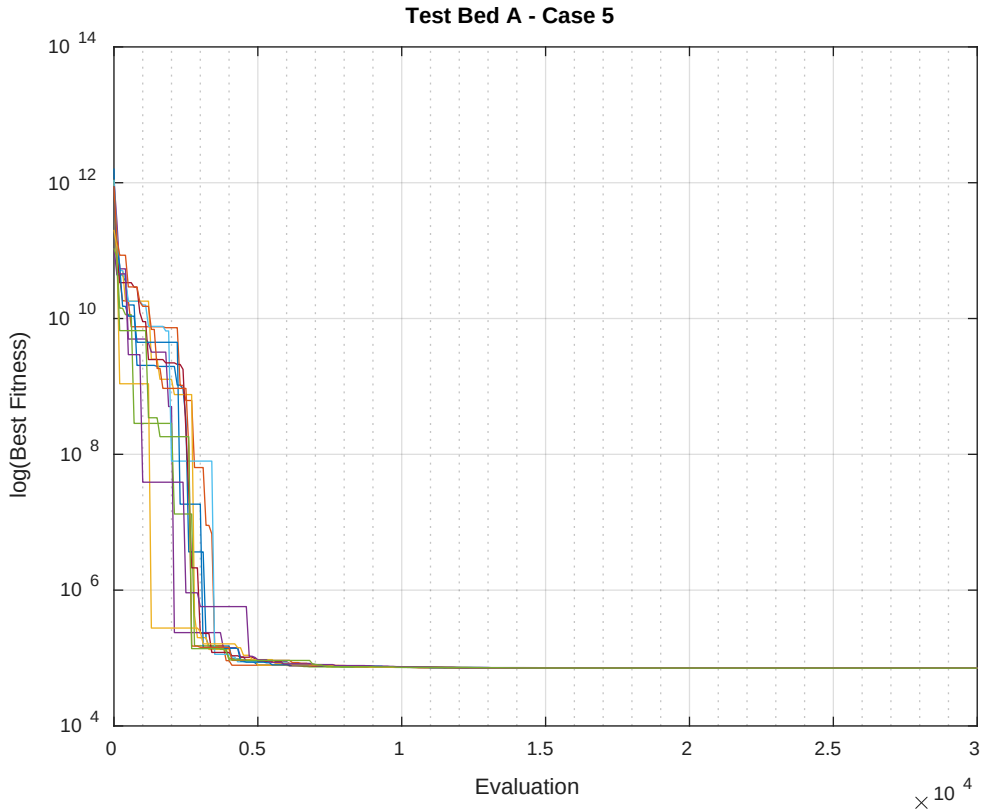

Results

- Test Bed B – Dynamic OPF: 10 independent runs

	Objective	Fitness
Best	753,071.99	773,193.77
Worst	797,029.49	823,684.44
Mean	766,841.73	789,719.58
Standard Deviation	15,175.79	15,618.73

Final Remarks

- **Combination of methods** to address different stages of the search can **greatly improve accuracy** and **robustness** of **heuristic methods**
- Systematic **parameter tuning is essential** to reduce the **information** required **from the user**
- Sometimes **tailor-made recombination rules to deal with specific constraints** of the optimization problems can lead to **better performances**

Thank you for your attention!

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