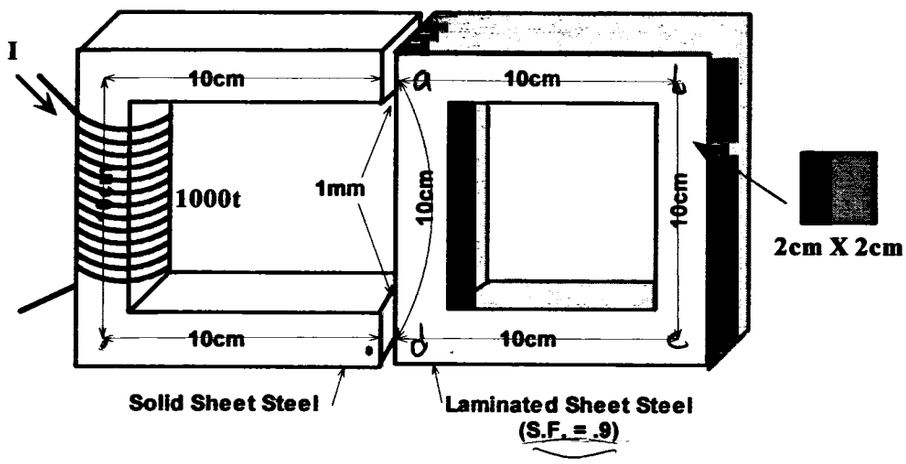
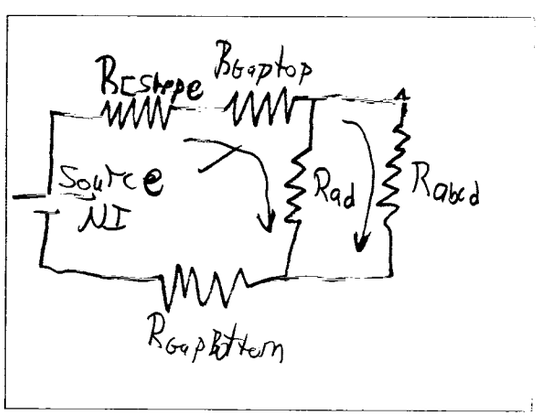
A magnetic circuit is shown in the accompanying figure. The 'C' section has a coil of 1000 turns and is made of solid sheet steel. It is positioned against a square section made of laminated sheet steel (stacking factor = 0.9), and is separated from the C section by 1mm air gaps as shown. The average path length of each straight section is 10cm, and all cross-sections are 2cm X 2cm. (Note: ignore fringing effects.)



a) Draw the 'electric equivalent' circuit. Label the elements including the source, NI, and the air gap(s).

[3]



b) What current is needed in the 1000 turn coil to establish a flux of 0.56 mWb in each of the air gaps?

I = 2.34 A

(Some B-H data for Sheet Steel can be found on the charts on the last page.)

[5]

Current

(Note: Use the following table to assist with your calculations)

Leg	Φ (Wb)	A (m^2)	B (T)	L (m)	H (At/m)	Hl (At)
Air Gap Top	0.56×10^{-3}	4.44×10^{-4}	1.27	0.001m	1010508	1011
Air Gap Bottom	0.56×10^{-3}	4.44×10^{-4}	1.27	0.001m	1010508	1011
C section	0.56×10^{-3}	4×10^{-4}	1.4	0.3m	1000	300
33% Rabcd ||	0.1848×10^{-3}	3.6×10^{-4}	0.513	0.3m	75	22.5
67% Rad	0.3752×10^{-3}	3.6×10^{-4}	1.042	0.1m	225	22.5

$\sum Hl = \sum NI$ $1011+1011+300+22.5 = 1000I$

$I = 2.34A$

c) Once this flux is established, how much force would be required to separate the 'C' section from the laminated square?

[3]

B = 1.27T at each connection point

$\mathcal{R} = \frac{l}{\mu A} = \frac{0.04m}{(4\pi\times10^{-7})(4\times10^{-4}\,m^2)}$ $\mathcal{R} = 7.96\times10^{7}$

$\mathcal{F} = \Phi\mathcal{R} = (0.56\times10^{-3}Wb)(7.96\times10^{7})$

$\mathcal{F} = 44.6kN$

d) What is the inductance, L, of the coil at this current and flux level?

[2]

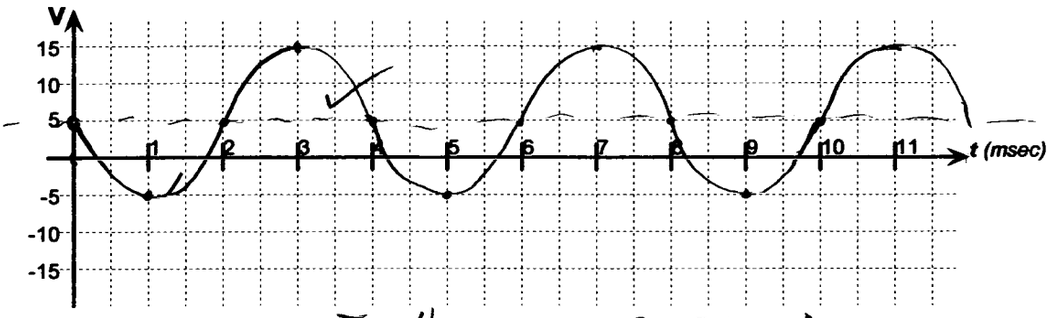
$B = \frac{N\Phi I}{L}$, $B = \mu H$

$\therefore L = \frac{N\Phi I}{H} = \frac{(1000)(2.34A)}{500}$

$L = 4.68H$

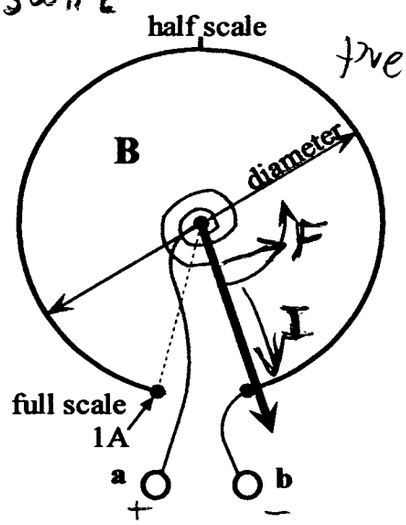
e) Sketch the waveform, $v(t) = 5 - 10\sin(1570.8t)$ *Volts* on the following graph

[2]



$T = 4msec$ $2\pi 250\,t$ $500\pi t$

f) A meter for measuring current is to be constructed using a movable indicator of rigid wire pivoted in the center and sliding on a frictionless, conducting ring as shown in the figure (Note: Top view.) There is a coil spring attached to the indicator which opposes any counter-clockwise rotation. This spring has a spring constant, $k = 0.75 \times 10^{-3}$ N-m/Radian. A permanent magnet provides a constant magnetic field with strength, B = .75T everywhere inside the ring. Electrical connections have been made so that the current to be measured can be routed through the moveable indicator via the stationary ring. (Note: The resistances of the ring, indicator and connections are negligible.)



+ve Z into board

i) If the terminal marked "a" in the diagram is to be the positive terminal (i.e. current "in"), which pole of the permanent magnet should be facing up inside the ring (North or South)? South from RHR

[2]

$N \cdot m = m$

j) If a full scale rotation of 286.5° corresponds to a current of 1A, what is the diameter of the ring?

$286.5° \cdot \frac{\pi}{180} = 5 rad$

$F_{sp} = 0.75 \times 10^{-3} \frac{Nm}{Rad} (5 rad) = 3.75 \times 10^{-3} Nm$

[4]

$F_{mag} = I\ell XB = (1A)r \times 0.75T = 0.75r$

At fullscale deflection these forces (torques) will be equal

$F_{sp} = F_{mag}$

$3.75 \times 10^{-3} N \cdot m = 0.75r\ N$

$r = 0.005m$

$\therefore d = 2r$

$d = 0.01m$

$d = 1cm$

k) What current is required for a half-scale deflection (rotation of 143.25°)?

[1]

$F_{sp} = (0.75 \times 10^{-3} \frac{N \cdot m}{Rad})(2.5 rad) = 1.875 \times 10^{-3} N \cdot m$

$1.875 \times 10^{-3} N \cdot m = I\ell XB$

$I = 0.5A$

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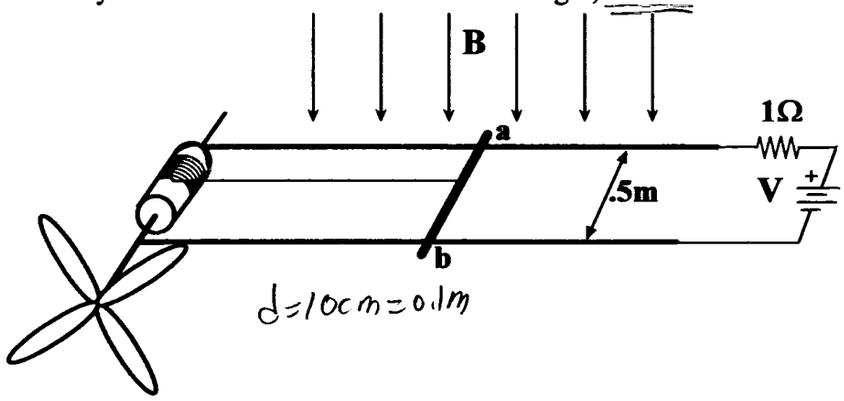
4) Write the expression for a sinusoidal waveform, $v(t)$, for which the peak to peak voltage, $V_{P\text{-}P} = 24$ Volts and the period, T = 20.0 msec. $\frac{2\pi}{T}t = \frac{2\pi}{20\times10^{-3}}t = 314.16t$

[1] $V(t) = 12\sin(314.16t)$ ✓ 1

5) A bar and rail are arranged to spin a small fan as shown in the figure. The fan requires 15W of power to turn it at 600rpm. The cable is attached to a drum that is 10cm in diameter. Assume that the spool of cable and the length of the rails are both long enough so that steady state can be reached. A field of strength, B = 1T exists in the area.



a) At what velocity must the bar move to spin the fan at the required speed?

[1] $600\text{rpm}\ \ 1\text{rev} = \text{String movement across C} = \pi d = 0.314\text{m/rev}$

$\frac{600\text{ rev}}{\text{min}} \cdot \frac{0.314\text{m}}{\text{min}} \cdot \frac{1\text{min}}{60\text{s}} = \boxed{3.14\text{ m/s}}$ ✓ 1

b) What force must be applied to the cable (by the bar) to deliver the required power?

[1] $P = FV$

$F = \frac{P}{V} = \frac{15\text{W}}{3.14\text{m/s}} = \boxed{4.77\text{N}}$ ✓ 1

c) How much current is required to supply the necessary force, and what is its direction through the bar (a⇒b or b⇒a)? direction is a⇒b

[2] $I = \frac{F}{lxB} = \frac{4.77\text{N}}{(0.5\text{m})\times 1\text{T}}$

$\boxed{I = 9.55\text{A}}$ ✓ 2

d) What is the motional electromotive force generated by the bar and what is its polarity (a or b positive)? a is +ve

[2] $V = l\,vel\,xB$

$V = (0.5\text{m})(3.14\text{m/s})\times 1\text{T}$

$\boxed{V = 1.57\text{V}}$ 2

e) If the bar also had an internal resistance of 1Ω, what supply voltage, V, would be required to provide the necessary current?

[2] $V = IR + 1.57\text{V}$

$V = (9.55\text{A})(2\Omega) + 1.57\text{V}$ ✓

$\boxed{V = 20.67\text{V}}$ 2

f) What is the overall efficiency of the system?

[2] $P_{out} = 15\text{W}$

$P_{in} = VI = (20.67\text{V})(9.55\text{A}) = 197\text{W}$ ✓

$\eta = \frac{P_{out}}{P_{in}}(100\%) = \frac{15\text{W}}{197\text{W}} = \boxed{7.61\%}$ 2

6) What is the frequency of a sinusoidal waveform that completes 42 cycles in 360? ms

[1] $f = \frac{1}{T} = \frac{1}{8.57} = \boxed{0.117\text{Hz}}$ $T = \frac{360\text{ms}}{42\text{cycles}} = 8.57\frac{\text{ms}}{\text{cycle}}$ 1

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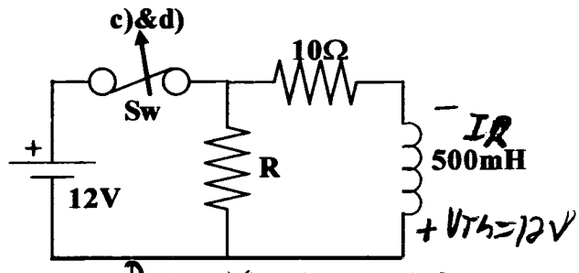
Consider the R – L circuit in the accompanying figure. The switch has been closed long enough so that steady state conditions have been reached.



a) What current is flowing through the inductor?

[2]

$I = \frac{V_{Th}}{R_{Th}} = \frac{12V}{\frac{10R}{R+10}} = \frac{12R+120}{10R}\ A$

$R_{Th} = 10\Omega \| R = \frac{10R}{R+10}$

$E_{Th} = 12V$

Steady state no voltage drop across inductor $\therefore I = \frac{12V}{10\Omega} = 1.2A$

b) How much energy is stored in the inductor?

[2]

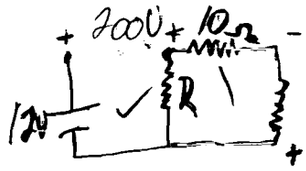
$W_L = \frac{1}{2}LI^2 = \frac{1}{2}500\times10^{-3}H\ \frac{12R+120}{10R}$

$W_L = \frac{3R+30}{2.5R}\ J$

$W_L = \frac{1}{2}LI^2 = \frac{1}{2}500mH(1.2A)^2 = 0.36J$

c) What is the maximum value of the resistor, R, that will ensure that the voltage across the switch contacts will not exceed 200V when the switch is opened?

[4]



As R gets bigger more and more current flows through the 10Ω resistor when $R \to \infty$ current flow through $10\Omega \to 1.2A$!

$V_L = \left[1+\frac{R_2}{R_1}\right]Ee^{-t/\tau'}$

$\tau' = \frac{L}{R}$

$188V = \left[1+\frac{R}{10\Omega}\right]12Ve^{\infty}$

$200V = 12V + 1.2AR$

$1+\frac{R}{10\Omega} = \frac{188}{12}$

$R = 146.7\Omega$

$\therefore$ Resistor must be greater than this to ensure no 200V so max val. $R \to \infty$

d) The manufacturer notes that the switch will not be damaged by arcing as long as the voltage across its open terminals does not exceed 500V for more than 500μsec. If a resistor, R = 1KΩ is used, will this requirement be met?

[4]

$488V \geq \left(1+\frac{1000\Omega}{10\Omega}\right)12Ve^{-\frac{500\times10^{-6}}{4.95\times10^{-4}}}$

$\tau' = \frac{L}{R_1+R_2}$

$\tau' = \frac{500\times10^{-3}H}{10\Omega+1000\Omega}$

$\tau' = 4.95\times10^{-4}s$

$488V \geq 441V$

$\therefore$ The requirement is met b/c the voltage would only read 441 V across terminals for that period of time not the max of 488 V.

2. 1 1 2 2

e) If $v(t) = 170\sin(377t)$ V,

[2]

a) what is the frequency, f? 60Hz

b) what is the period, T? 16.7ms

$V(t) = V_0 \sin 2\pi f t$ 2

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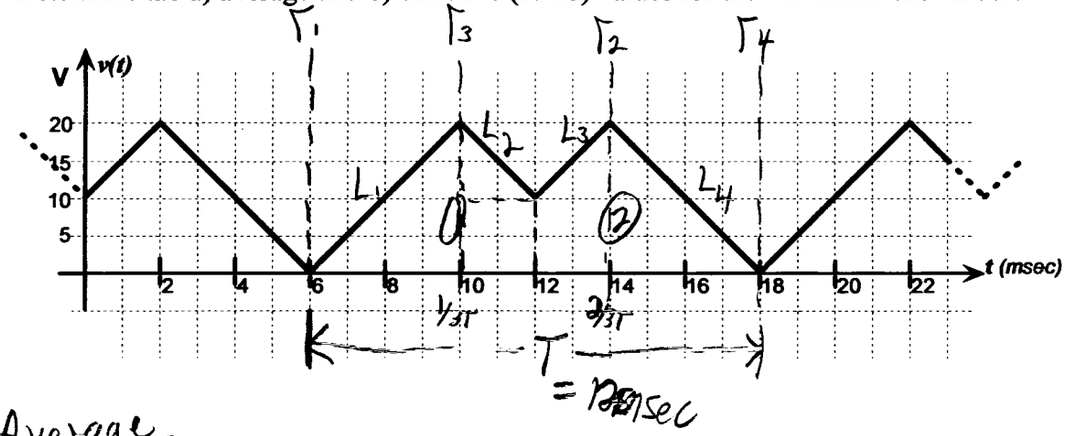
9) Determine the a) average and b) effective (RMS) values for the waveform shown below.



= 12 msec

[4] Average

Due to symmetry Area total = 2 × Area ①

$$\text{Area 1} = \frac{(4\text{ms})(20\text{V})}{2} + (2\text{ms})(10\text{V}) + (2\text{ms})(10\text{V}) = 0.07\text{ Vs}$$

$$\text{Average} = \frac{\text{Area}}{\text{period}} = \frac{0.14\text{ Vs}}{12\times10^{-3}\text{s}} = \boxed{11.67\text{ V}}$$

[6] Note: All reference axis moved so lines have y intercept = 0. (L1, L2, L3, L4) — X

$$I^2 = \int_0^4 \left(\frac{20\text{V}}{4\text{ms}}t\right)^2 dt + \int_{-4}^{-2} \left(\frac{-10\text{V}}{2\text{ms}}t\right)^2 dt + \int_2^4 \left(\frac{10\text{V}}{2\text{ms}}t\right)^2 dt + \int_{-4}^{0} \left(\frac{-20\text{V}}{4\text{ms}}t\right)^2 dt$$

$$I^2 = \int_0^4 25t^2\,dt + \int_{-4}^{-2} 25t^2\,dt + \int_2^4 25t^2\,dt + \int_{-4}^{0} 25t^2\,dt$$

$$I^2 = \frac{25}{3}t^3\Big|_0^4 + \frac{25}{3}t^3\Big|_{-4}^{-2} + \frac{25}{3}t^3\Big|_2^4 + \frac{25}{3}t^3\Big|_{-4}^{0}$$

$$I = \sqrt{\frac{(25)64}{3} + \frac{25(-8)}{3} - \frac{25(-64)}{3} + \frac{25(64)}{3} - \frac{25(8)}{3} - \frac{25(-64)}{3}}$$

$\boxed{I_{rms} = 44.7\text{ V}}$? V_{max} ??

10) If $I(t) = 15\sin(377t + \pi/6)$ A,

[1] a) what is the value of the current at t = 16.67 msec? 7.57 A

$\pi = 3.14159265358979323846264338327950288419716939937510$ (PI to 50 digits)

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