Some potentially useful constants/relationships:

Acceleration of gravity: $g = 9.81\ m/s^2$

Permeability of free space: $\mu_o = 4\pi \times 10^{-7}$ Wb/At-m

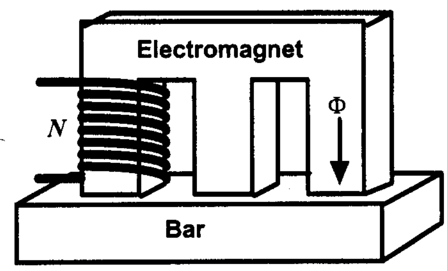
Magnetic field, B, a distance d from a wire carrying current I: $B = \frac{\mu_o I}{2\pi d}$

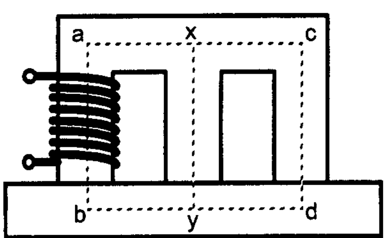
Magnetic force of attraction: $F_{att} = \frac{B^2 A}{2\mu_o}$

Description of an exponential change in value: $x(t) = X_f - (X_f - X_i)e^{-t/\tau}$

$P_{mechanical} = F \cdot vel$; $P_{Electical}$ = IV or I^2R or V^2/R

1) An electromagnet made of cast steel is used to hold up a cast steel bar. There is a winding made up of N turns around one of the outside legs as shown in the diagram. The flux in the right leg, Φ, is known to be 0.32mWb. The cross-sections in the electromagnet are all 4cm by 4cm. Although the bar is a different shape, the *effective* area is the same as in the electromagnet (i.e. you can assume it is also effectively 4cm X 4cm). Neglect fringing. The average path lengths through this magnetic circuit are given in following table (refer to the second sketch at right):





b-a-x	20cm
x-c-d	20cm
x-y	12cm
b-y	8cm
y-d	8cm

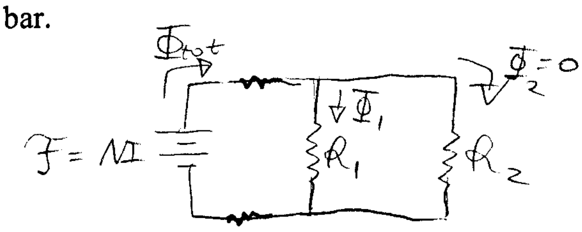
You can also assume that the electromagnet is in direct contact with the bar (i.e. no "gap")

If you need to calculate any of the magnetic circuit parameters to answer the questions that follow, please use the following table to organize your work. (See attached B-H curves if required)

$B = \frac{\Phi}{A}$

leg	Area	length	Φ	B	H	Hl
left	$(0.04\ m)^2$	0.28m	1.04mWb	0.65T	114.3	32 A-t
centre	$(0.04\ m)^2$	0.12m	0.72mWb	0.45T	266.67 A-t/m	32 A-t
right	$(0.04\ m)^2$	0.20m	0.32 mWb	0.2 T	160 A-t/m	32 A-t

[3] **a)** Draw the "electric equivalent" of the magnetic circuit formed by the coil, electromagnet and the bar.



$Hl_1 = Hl_2$ (parallel)

[5] **b)** What is the total flux produced by the coil?

$\boxed{1.04\ mWb\ \text{(see table)}}$

?? ∴ used Φ in right leg to find Hl, then knew Hl in centre leg = Hl in rt. leg, so used same Hl, found new flux

[2] **c)** If the winding is energized from a 12V battery through a 10Ω resistor, how many turns, N, are in the coil? (Assume the coil has negligible resistance.)

$I = \frac{12V}{10\Omega} = 1.2A$; $NI = Hl$

$N(1.2A) = 32\ A\text{-}t$

$\boxed{N = 26.7\ \text{turns}}$ ANS.

Student #: 10367898

[8] **d)** What is the maximum mass of the bar that can be supported by the electromagnet?

magnetizing force to hold up bar.

$F_{mag} = m \cdot g = \frac{B^2 A}{2\mu} \cdot 3$; 3 contact points

Since $B = \mu H$; $\mu = \frac{114.3\ A \cdot t}{0.65} = 175.85$

??

so, $m = \frac{1}{g} \cdot \frac{B^2 A}{2\mu} \cdot 3 = \frac{1}{(9.81 m/s^2)} \frac{(1.27)^2 (0.04 m)^2}{2(175.85)} \cdot 3 =$

other ways ??

$F_{mag} = \frac{A}{2\mu_0} (B_{LL}^2 + B_{CL}^2 + B_{RL}^2)$

773.6 N

[3] **e)** What is the inductance, L, of this coil under these conditions?

$L = .192 H$

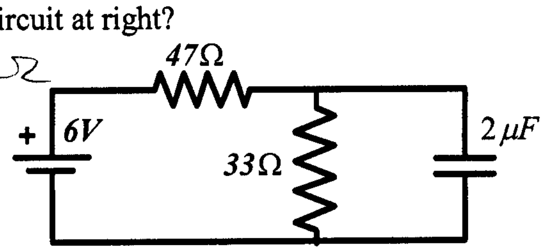
$L = \frac{N\Phi}{I} = \frac{(26.7)(1.04 mWb)}{1.2 A} \boxed{= 23.1 mH}$??

[3] **f)** If a thin film or paramagnetic material 1mm thick was inserted between the electromagnet and the bar, but the parameters were adjusted to keep the flux the same, what is the maximum mass the electromagnet could support?

$m = \frac{1}{g} \cdot \frac{B^2 A}{2\mu_0} = \frac{1}{(9.81 m/s^2)} \frac{(1.27)^2 (0.04 m)^2}{2 \cdot (4\pi \cdot 10^{-7} \frac{Wb}{A \cdot t})} \cdot 3 \boxed{= 280\ kg}$??

---

[2] **2)** What is the transient time constant, τ, for the circuit at right?



$R_{th} = 33\Omega \| 47\Omega = 19.3875\Omega$

$\tau = RC = (19.3875\Omega)(2\mu F)$

$\boxed{\tau = 38.8 \mu s}$

2

---

[2] **3)** What is expression that describes a sinusoidal waveform that completes 30 cycles in 12ms and has a peak-to-peak voltage of 3V?

$f = \frac{30 \text{ cycles}}{12 \text{ msec}} = 2.5 kHz$

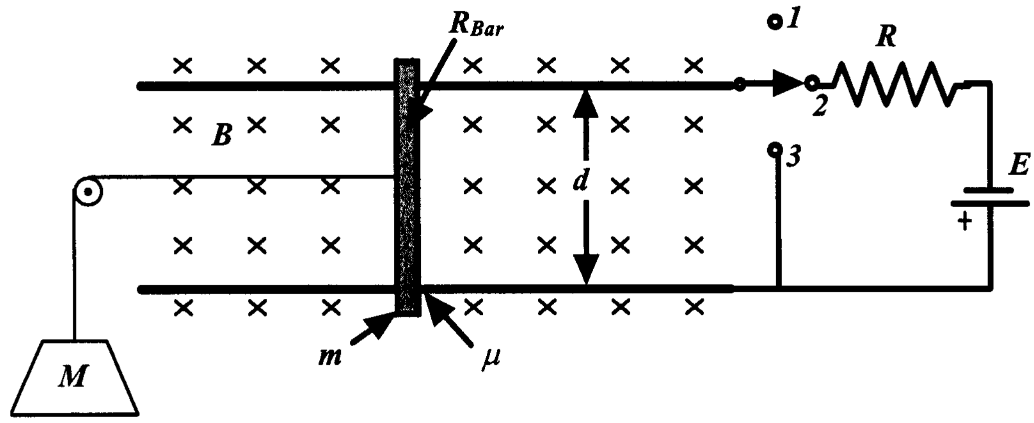
$\omega = 2\pi f = 2\pi(2.5 kHz) = 5000\pi$

So, $\boxed{e(t) = 3 \sin(5000\pi t)}$

$1.5 \sin(5000\pi t)$

1.5

4) Consider the bar & rail system shown in the diagram below and answer the related questions that follow. Assume the rails are long enough for the system to reach a steady state, and the magnetic field, B, exists uniformly everywhere in the area of interest.



Given: B = 0.7T, M = 1kg, m = 2kg, coefficient of friction, μ = 0.12 (both static and dynamic), d = 1.5m, E = 12V, R_{Bar} = 1.2Ω, R = 0.3Ω

[4] a) What is the magnitude and direction of an external force require to keep the bar from moving if the switch is in position 2 as shown? (Note: for this sub-question only, *neglect friction.*)

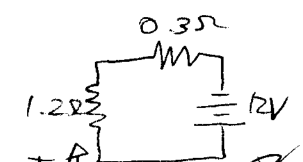
$m \cdot g$ $M \cdot g$ P(external force) F_{mag} (by RHR)

$P = M \cdot g - F_{mag} = M \cdot g - I\ell B$

$P = (1Kg)(9.81 m/s^2) + (8A)(1.5m)(0.7T)$

$\boxed{P = 1.\cancel{4}1\ N,\ \text{to the right}\ (\rightarrow)}$

ANS.



$I = \frac{12V}{(1.2+0.3)\Omega}$

$I = 8\ A$

[4] b) What is the magnitude and direction of the force require to keep the bar from moving if the switch is in position 1? In position 3? (Note: friction is back on.)

in pos'n 1: no current flows, so no F_{mag}:

$m \cdot g$ P $M \cdot g$ $F_{fr} = \mu \cdot m \cdot g$ $N = m \cdot g$

$P + F_{fr} = M \cdot g$; $P = (1Kg)(9.81m/s^2) - (.12)(2kg)(9.81m/s^2)$

$\boxed{P = 7.46N,\ \text{right}\ (\rightarrow)}$

4

in pos'n 3: Same situation as above, no current flows through the ckt., so $\boxed{P = 7.46\ N,\ \text{right}\ (\rightarrow)}$

If the bar is now released and allowed to reach a steady-state (still in position 2):

[2] c) Draw a simple Free Body Diagram showing the relevant forces acting on the bar. What is the magnitude of F_{mag}?

$m \cdot g$ $M \cdot g$ $F_{fr} = \mu \cdot m \cdot g$ F_{mag} $N = m \cdot g$

$F_{mag} + f_{fr} = M \cdot g$; $F_{mag} = (1Kg)(9.81 m/s^2) - (.12)(2kg)(9.81m/s^2)$

$\boxed{F_{mag} = 7.46\ N}$

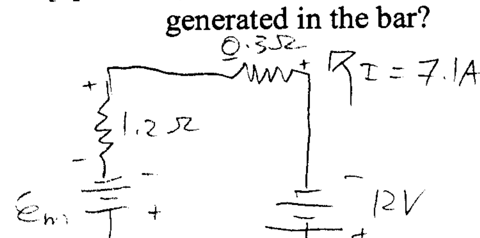
2

[3] d) What is the magnitude and direction of the current through the bar? (top ⇒ bottom, or bottom ⇒ top

$F_{mag} = I\ell B$; $I = \frac{F_{mag}}{\ell B} = \frac{7.46\ N}{(1.5m)(.7T)}$ $\boxed{= 7.1\ A,\ \text{top} \Rightarrow \text{bottom}}$

b/c F_{mag} is to the right, by RHR, I flows C.C.W.

[3] e) What is the magnitude and polarity (positive up, or down) of the motional emf generated in the bar?



KVL: $\mathcal{E}_m - 12V - 7.1A(0.3\Omega + 1.2\Omega)$

$\boxed{\mathcal{E}_m = 22.65V,\ \text{positive down}}$

3

Student #: 10367298

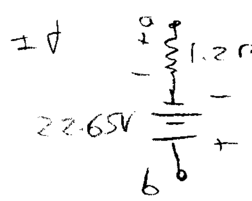
15

[3] **f)** What is the velocity of the bar, and its direction?

$\mathcal{E}_m = \ell \, vel \times B$; $vel = \frac{\mathcal{E}_m}{\ell B} = \frac{22.65V}{(1.5m)(0.7T)}$

$\boxed{vel = 21.57\ m/s \text{ left } (\leftarrow)}$ 3

[3] **g)** What is the voltage between the rails?



$\mathcal{E}_m - V_{resistor}$ = voltage between rails

$22.65V - (7.1A)(1.2\Omega)$ $\boxed{= -14.13V}$ 3

[2] **h)** What is main source of power going into the system, and how much power is it delivering? main source of power is the weight (M) pulling the bar 2

$P = Force \cdot vel = M \cdot g \cdot vel = (1Kg)(9.81\ m/s^2)(21.57\ m/s)$ $\boxed{= 211.6\ W}$

[2] **i)** What is the main sink for power ("useful power out") in this system? How much power is consumed? the battery is the main sink

$P = VI = (12\ V)(7.1A)$ $\boxed{= 85.2\ W}$ 2

[2] **j)** Is the system acting as a motor or generator? What is the efficiency, η?

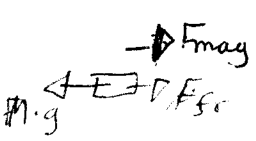
it is acting as a generator.

$\eta = \frac{\text{useful power out}}{\text{power in}} = \frac{85.2W}{211.6W} \times 100\%$ $\boxed{= 40.3\%}$ 2

The switch is now moved to position 3 and the system allowed to once again reach a steady state:

[4] **k)** What is the new velocity and direction of the bar?

direction is left (←) No current ~~flows~~ through.



$P = 211.6W$ (~~from~~ above), $F = M \cdot g = 9.81N$

so $vel = \frac{211.6W}{9.81N}$ $\boxed{= \cancel{21.57\ m/s \leftarrow}}$ 1

8.11 m/s ←

[2] **l)** What is the main sink of electric power in this case?

heat through the resistor in the bar. 2

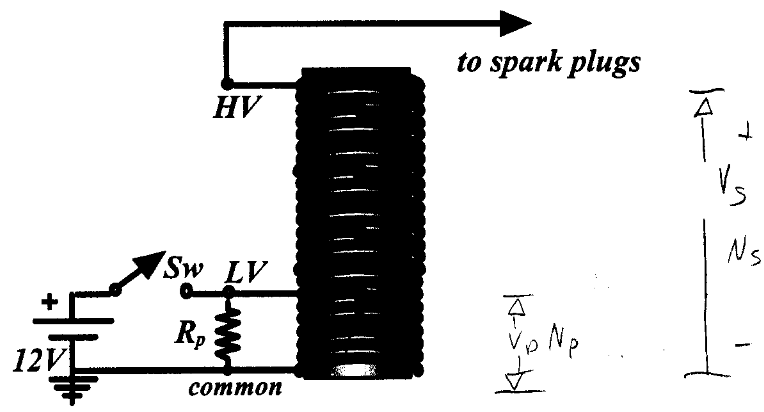
The switch is now moved to position 1, and M is replaced by a 200g mass:

[3] **m)** Assuming the *bar is at rest* to start with, write and expression for the velocity with respect to time of the bar (*vel(t)*)with this new mass attached?

vel(t)=0

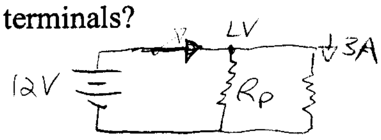
15

5) An arrangement *similar to* that shown in the diagram is used to create a spark in the gap in a spark plug in the cylinders of a gasoline engine. The *autotransformer* has a total of 600 turns between the *HV* and *common* terminals, all of the same wire, and uniformly wound on a sheet steel core. The low voltage tap (*LV*) is at 12 turns. The coil has an inductance of 650mH measured between the *HV* and *common* terminals. When the switch is closed, the current in the 12 turn (*LV*) portion of the coil reaches a steady-state value of 3A.



Answer the following questions about this circuit:

[2] a) What is the internal resistance of the transformer coil between the *LV* and *common* terminals?



$R = \frac{V}{I} = \frac{12V}{3A}$ $\boxed{= 4\Omega}$ ✓ 2

[4] b) What is the inductance of just the *LV-to-common* portion of the coil?

$V_1 = 3A(4\Omega) = 12V$

250 μH

don't know, so just assuming $\boxed{L = 650\ mH}$ ✗ 1

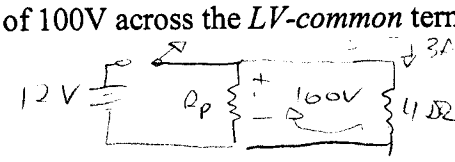
[2] c) What is the time constant, τ, of the L-R circuit formed with the battery, switch and *LV* portion of the transformer?

$\tau = \frac{L}{4\Omega} = \frac{650\ mH}{4\Omega}$ $\boxed{= 162.5\ ms}$ using assumed L above ✓ 2

[4] d) Write the expression that describes the current in the *LV* portion of the coil as a function of time after the switch is closed (assume $i_L = 0$ at $t = 0$).

$I_f = 3A$ $\boxed{i(t) = 3A\left(1 - e^{-t/162.5ms}\right)}$ ✓ 4

[3] e) Assuming the current reaches its steady-state value, what value of parallel resistor, R_p, will result in a voltage magnitude of 100V across the *LV-common* terminals the instant after the switch is opened?



$R_p = \frac{100V}{3A}$ $\boxed{= 33.3\Omega}$ ✓ 3

[2] f) What is the time constant of the L-R circuit with the switch opened?

$\tau = \frac{650mH}{33.3\Omega + 4\Omega} = 19.5\ ms$ 1.5

[4] g) Write the expression that describes the voltage across R_p as a function of time once the switch is opened.

$\boxed{V_R(t) = -100\left(e^{-t/19.5ms}\right)}$ 3

- decays to zero

[2] h) What maximum voltage would be measured at the HV terminal?

$e_s = e_p n = 100V(612/12) \boxed{\doteq 5.1kV}$

2

If the switch is repeatedly turned on for 35μs and then off for 35μs (you can assume the current, i_L, decays to 0A each cycle):

[5] i) what is the maximum voltage seen at the *HV* terminal?

at $t = 35\mu s$, $i(t) = 3A(1 - e^{-35\mu s/162.5ms}) = 0.646\,mA$

$v_1 = L\frac{di}{dt} = 650mH\,(0.646\,mA)$

$e_s = e_p n = (650mH)(0.646mA)(612/12) =$

2

[2] j) If the gap in the spark plugs is 0.44mm and the dielectric strength of the air-fuel mixture in the cylinder is 10MV/m ($10X10^6$V), will the voltage calculated in i) "jump the gap"?

$\frac{10\times10^6 V}{m} \; 0.44\times10^{-3}m = 4.4\,kV$

1

if v in i) is >4.4 kV, then, yes it will.

[3] k) What value of resistor (higher or lower) would be required instead of R_P to create *just enough* voltage across the secondary to jump the gap?

[2] l) What will the maximum voltage across the switch terminals be if the switch is "cycled" as described in i)?

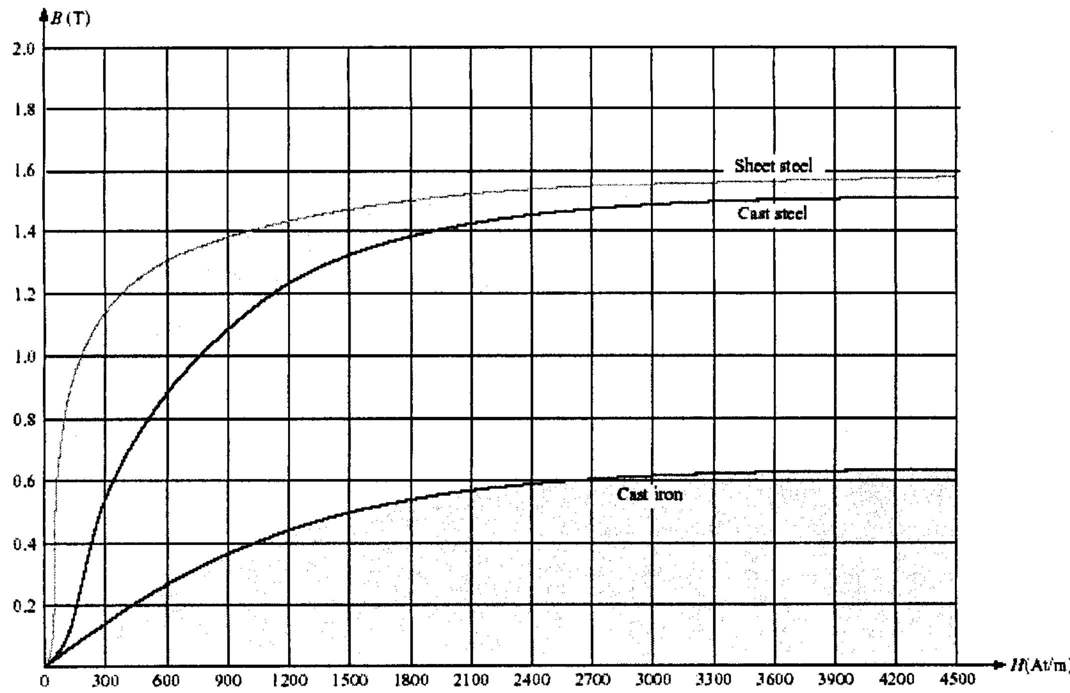


FIG. 11.23
Normal magnetization curve for three ferromagnetic materials.

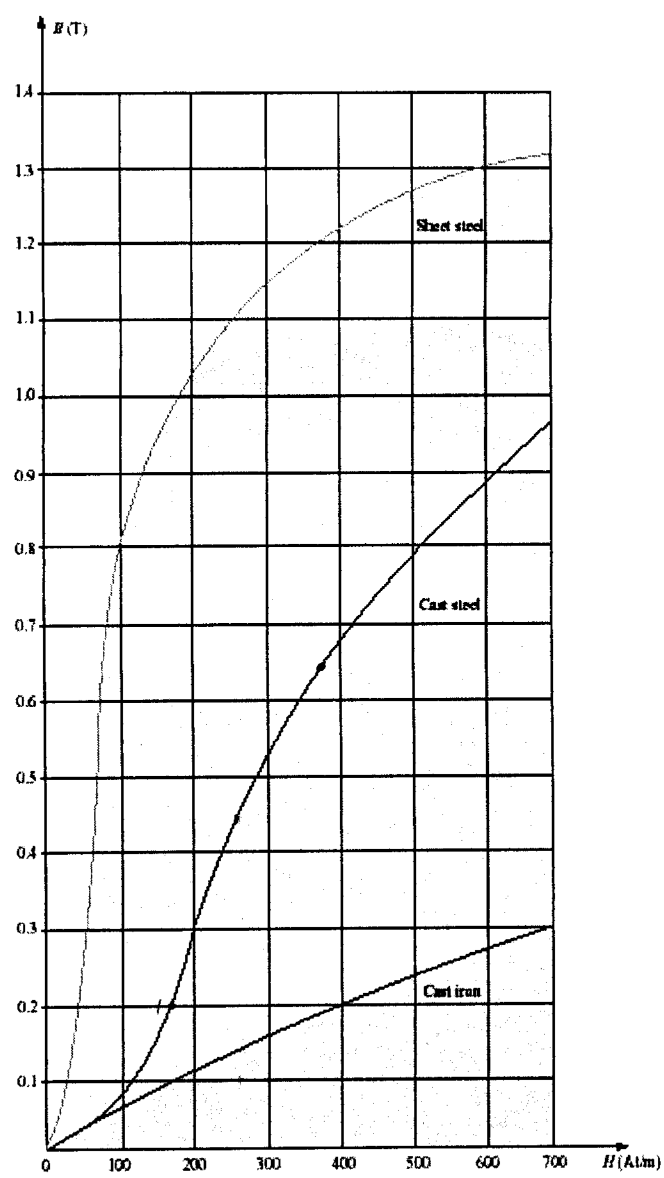


FIG. 11.24
Expanded view of Fig. 11.23 for the low magnetizing force region.

from: Boylestad's 2nd Canadian Edition