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University of Saskatchewan
Department of Electrical Engineering
EE 216.3 Probability, Statistics and Numerical Methods
Midterm Examination Part B

Instructor: N. Chowdhury November 21, 2006

Time: 1 hour & 20 minutes. Closed book Examination. Formula sheets are provided. Calculators are allowed.

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Marks

10 1. Cooking oil is produced in two main varieties: mono- and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

		Type of Oil	
		Canola	Corn
Type of unsaturation	mono	8	13
	poly	92	77

If two bottles of oil are selected at random without replacement:

(a) What is the probability that one is monounsaturated canola and the other is polyunsaturated corn oil?

(b) What is the probability that both bottles are polyunsaturated canola oil?

(c) What is the probability that the second bottle is monounsaturated canola oil given that the first one was polyunsaturated corn oil?

a) $\frac{8}{190} \times \frac{77}{189} = \boxed{0.017}$ X

b) $\frac{92}{190} \times \frac{91}{189} = \boxed{0.233}$ ✓

c) $\frac{8}{189} = \boxed{0.042}$ ✓

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Marks

10 2. The probability density function of the time to failure of an electronic component in a photocopier is:

$$f(x) = \frac{e^{-x/1000}}{1000} \text{ for } x > 0.$$

Determine the probability that

(a) The component lasts more than 3000 hrs before failure.
(b) The component fails in the interval from 1000 to 2000 hrs.
(c) The component fails before 1000 hrs.

a) $P_r[x \geq 3000 \text{hrs}] = 1 - \int_0^{3000} f(x)dx = 1 - \frac{1}{1000}\int_0^{3000} e^{-x/1000} dx$

$= 1 - \frac{1}{1000}\left[-1000\, e^{-x/1000}\right]_0^{3000} = 1 + \left[e^{-3} - 1\right] = \boxed{0.0498}$ ✓

b) $P_r[1000 \leq x \leq 2000 \text{ hrs}] = \int_{1000}^{2000} \frac{e^{-x/1000}}{1000} dx = -e^{-x/1000}\Big]_{1000}^{2000} = e^{-1} - e^{-2} = \boxed{0.2325}$ ✓

c) $P_r[1000 \geq x] = \int_0^{1000} \frac{e^{-x/1000}}{1000} dx = -e^{-x/1000}\Big]_0^{1000} = e^{0} - e^{-1} = \boxed{0.6321}$ ✓

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Marks

10 3. Based on recent records, the manager of a car painting shop has determined the following probability distribution for the number of customer per day, X.

x	0	1	2	3	4	5
$f(x)$	0.05	0.20	0.30	0.25	0.15	0.05

(a) If the shop has the capacity to serve two customers per day, what is the probability that one or more customers will be turned away on a given day?

(b) What is the probability that the shop's capacity will not be fully utilized on a day?

(c) By how much must the capacity be increased so the probability of turning a customer away is no more than 0.1?

a) Pr[more than two customers] = Pr[3 cust] + Pr[4 cust] + Pr[5 cust]
= 0.25 + 0.15 + 0.05
$\boxed{= 0.45}$ ✓

b) Pr[Less than two customers] = Pr[0 cust] + Pr[1 cust] = 0.05 + 0.20
$\boxed{= 0.25}$ ✓

c) Capacity must be increased by double to 4 customers per day to make the probability of turning someone away less than 0.1. ✓

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Marks

10 4. In a university scholarship program, anyone with a grade point average over 7.5 receives a $1,200 scholarship, anyone with an average between 7.0 and 7.5 receives $600, anyone with an average between 6.5 and 7.0 receives $ 200, and all others receive nothing. A particular class of 500 students has an overall average of 4.8 with a standard deviation of 1.2. Calculate the cost to the university of supplying scholarships for this class. Assume that the grade point average is normally distributed.

$z = \frac{x - \mu}{\sigma}$

7.5 $z_1 = \frac{7.5 - 4.8}{1.2} = 2.25$

7.0 $z_2 = \frac{7.0 - 4.8}{1.2} = 1.83\overline{3}$

6.5 $z_3 = \frac{6.5 - 4.8}{1.2} = \cancel{1.416}$ 1.42

$Pr[x \geq 7.5] = 1 - \phi(2.25) = 1 - 0.9878 = 0.0122$ => 1.22% of students

$Pr[7.0 \leq x \leq 7.5] = Pr[x \leq 7.5] - Pr[x \leq 7.0] = 0.9878 - 0.9664 = 0.0214$ => 2.14% of students

$Pr[6.5 \leq x \leq 7.0] = Pr[x \leq 7.0] - Pr[x \leq 6.5] = 0.9664 - 0.9222 = 0.0442$ => 4.42% of students

$500 \times 0.0122 = 6.1$ students => ~6
$500 \times 0.0214 = 10.7$ students => ~11
$500 \times 0.0442 = 22.1$ students => ~22

$6(\$1200) + 11(\$600) + 22(\$200) = \$18\,200$

$\boxed{\text{Total cost would be approximately \$18 200.}}$ ✓

10 ✓

Marks

10 5. Suppose 15% of the trees in a forest have severe leaf damage from air pollution. If 5 trees are selected at random, find the probability that

(a) Three of the selected trees have severe leaf damage.
(b) No more than two have severe leaf damage.
(c) None of them have severe leaf damage.

$p = 0.15$
$q = 0.85$

# damaged, r	Pr[x = damaged]
0	${}_5C_0 (0.15)^0 (0.85)^5 = 0.444$
1	${}_5C_1 (0.15)^1 (0.85)^4 = 0.392$
2	${}_5C_2 (0.15)^2 (0.85)^3 = 0.138$
3	${}_5C_3 (0.15)^3 (0.85)^2 = 0.024$
4	${}_5C_4 (0.15)^4 (0.85)^1 = 0.002$
5	${}_5C_5 (0.15)^5 (0.85)^0 = 0.00008$

a) $Pr[3 \text{ damaged}] = \boxed{0.024}$ ✓

b) $Pr[\leq 2 \text{ damaged}] = Pr[2 \text{ dam.}] + Pr[1 \text{ dam}] + Pr[0 \text{ dam}]$
$= 0.138 + 0.392 + 0.444 = \boxed{0.974}$ ✓

c) $Pr[0 \text{ damaged}] = \boxed{0.444}$ ✓

10

THE END