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University of Saskatchewan
Department of Electrical Engineering

EE301 Electricity, Magnetism and Fields
Final Examination
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Welcome to the EE301 Final. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **3** hours.

Answer **six** of the seven problems. Do not answer more than six problems or severe penalties will apply.

Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers but only if correct intermediate steps are shown. Each problem is weighted equally although subparts of a problem might be worth varying amounts depending on difficulty.

None of the problems require intricate mathematical manipulations. If you get stuck with an impossible integral or equation, you are likely approaching the problem incorrectly.

Problem 1

a) Two point charges of 100 nC and -50 nC are separated by 1 cm. What is the electric field vector at the midpoint between the two charges? What is the total electric flux passing through any infinite plane lying above the charges.

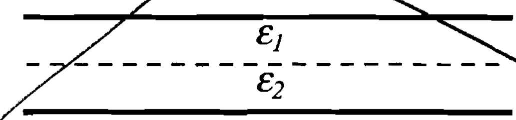
b) Space is filled with a charge density ρ_V that varies with the distance from the z axis according to the formula (in cylindrical coordinates)

$$\rho_V = Ae^{-\rho}$$

Determine the electric field everywhere.

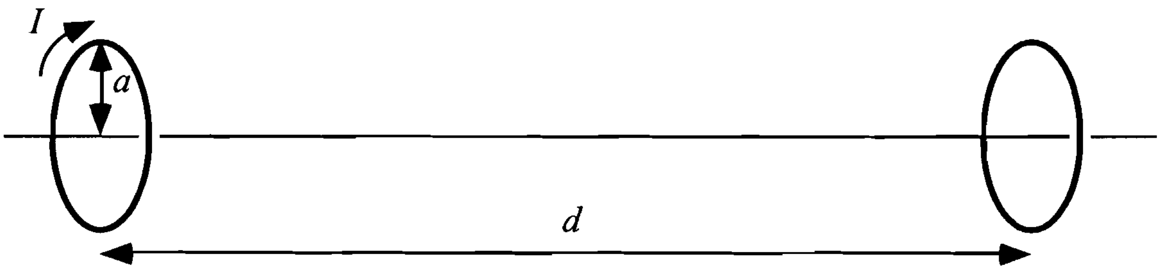
Problem 2

The parallel plate capacitor shown below has two dielectric layers between the plates with different dielectric constants ε_1 and ε_2. The boundary between the dielectrics lies at the center of the capacitor. The plate area is A and the separation between the plates is d. Determine a formula for the capacitance. If $\varepsilon_1 < \varepsilon_2$, which region has the larger electric field?



Problem 3

Two circular loops of wire each with radius a are separated by a distance d with their centers on a common axis with the plane of each loop perpendicular to the axis (as shown). The separation is much larger than the radii ($d >> a$). If one loop has an oscillating current $I = I_0 \cos(\omega t)$, calculate the emf induced in the other loop. Note: you should make a simplifying approximation valid for the conditions given.



Problem 4

a) An electromagnetic plane wave has an electric field given by

$$\vec{E} = 500(\vec{a}_x - \vec{a}_y)e^{j(\omega t - kz)} \text{ V/m}$$

Describe the polarization of the wave. Calculate the time-averaged power density (Poynting vector) of the electromagnetic wave.

b) A coaxial cable has an inner conductor of radius a and outer shield of radius b and is filled with a dielectric with dielectric constant ε_R. An oscillating signal is transmitted through the cable. The electric field between the two conductors is given by (in cylindrical coordinates)

$$\vec{E} = \frac{E_0}{\rho} e^{j(\omega t - kz)} \vec{a}_\rho \text{ for } a < \rho < b.$$

In which direction does the magnetic field point? Write down the equation for the magnetic field. Calculate the time-averaged power density (Poynting vector) of the electromagnetic wave. Where within the coaxial cable is the power flow greatest.

Problem 5

a) A two-wire transmission line consists of two parallel wires each with radius a and separated by distance d. The line has a capacitance per unit length of approximately

$$C \approx \pi\varepsilon_0 \left(\ln\left(\frac{d-a}{a} \right) \right)^{-1}$$

and an inductance per unit length of approximately

$$L \approx \frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a} \right)$$

A high-voltage, two-wire transmission line used to transmit 60 Hz power consists of 1.4 cm radius wires separated by 3 m. Calculate the propagation constant, characteristic impedance, velocity of propagation and wavelength. Assume the line is lossless.

b) An antenna has an impedance of $75 + j50\,\Omega$ at 1GHz. It is directly connected (i.e. no matching network) to a 1 m length of 50Ω transmission line. What is the reflection coefficient off the antenna? What is the input impedance of the transmission line? The velocity of propagation in the transmission line is 2×10^8 m/s.

Problem 6

a) On the Smith chart, clearly indicate the locations corresponding to a short, an open, a pure capacitance, and a pure inductance.

b) A 50Ω transmission line is terminated with a load impedance of $10-25j\Omega$. Use the Smith chart to determine the shortest length of transmission line such that the input impedance is purely real. What is the input impedance?

c) A cell-phone handset's antenna is designed to broadcast at 1.8 GHz. The power is delivered to the antenna by a micro-stripline (a type of transmission line formed on a printed circuit board) with a characteristic impedance of 50Ω. The velocity of propagation in the micro-stripline is 1×10^{8} m/s. The antenna has a complex impedance of $100+30j\Omega$. Design a matching scheme so that no power is reflected from the antenna. Clearly describe your matching scheme and work out the relevant parameters on the Smith chart.

(Hand in the Smith chart with your booklet).

Problem 7

A step generator with an internal impedance of 50Ω produces a 1 V step. It is connected to 2 m of 100Ω transmission line. The transmission line is terminated with a 60Ω load. The velocity of propagation in the transmission line is 1×10^{8} m/s.

a) What is the reflection coefficient at the load and at the generator?

b) Draw a bounce diagram for 70 ns after the step is applied and label each bounce line with the height of the step.

c) Graph the voltage at the midpoint of the transmission line and at generator end of the transmission line as a function of time for the first 70 ns.

Symbols and Constants

F	force	V	electric (scalar) potential
Q	charge	$\vec{A}$	magnetic (vector) potential
$\vec{E}$	electric field	ρ	charge density
$\vec{D}$	displacement field	I	current
$\vec{P}$	polarization field	$\vec{j}$	current density
$\vec{H}$	magnetic field	ε_R	relative permittivity
$\vec{B}$	magnetic flux density field	μ_R	relative permeability
$\vec{M}$	magnetization	$\varepsilon_0 \approx 8.85\times10^{-12}$ F/m	
Φ	magnetic flux	$\mu_0 = 4\pi\times10^{-7}$ N/A^2	

Vector Calculus

unit vector cross products:

Cartesian	$\vec{a}_x\times\vec{a}_y=\vec{a}_z$	$\vec{a}_y\times\vec{a}_z=\vec{a}_x$	$\vec{a}_z\times\vec{a}_x=\vec{a}_y$
cylindrical	$\vec{a}_\rho\times\vec{a}_\phi=\vec{a}_z$	$\vec{a}_\phi\times\vec{a}_z=\vec{a}_\rho$	$\vec{a}_z\times\vec{a}_\rho=\vec{a}_\phi$
spherical	$\vec{a}_r\times\vec{a}_\theta=\vec{a}_\phi$	$\vec{a}_\theta\times\vec{a}_\phi=\vec{a}_r$	$\vec{a}_\phi\times\vec{a}_r=\vec{a}_\theta$

differential volume elements:

Cartesian	$dx\,dy\,dz$
cylindrical	$\rho d\rho\, d\phi\, dz$
spherical	$r^2\sin\theta dr\, d\theta\, d\phi$

curl (cylindrical)

$$\vec{\nabla}\times\vec{A}=\left[\frac{1}{\rho}\frac{\partial \vec{A}_z}{\partial\phi}-\frac{\partial\vec{A}_\phi}{\partial z}\right]\vec{a}_\rho+\left[\frac{\partial\vec{A}_\rho}{\partial z}-\frac{\partial\vec{A}_z}{\partial\rho}\right]\vec{a}_\phi+\frac{1}{\rho}\left[\frac{\partial(\rho\vec{A}_\phi)}{\partial\rho}-\frac{\partial\vec{A}_\rho}{\partial\phi}\right]\vec{a}_z$$

curl (spherical)

$$\vec{\nabla}\times\vec{A}=\frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi\sin\theta)}{\partial\theta}-\frac{\partial A_\theta}{\partial\phi}\right]\vec{a}_r+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial(rA_\phi)}{\partial r}\right]\vec{a}_\theta+\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right]\vec{a}_\phi$$

Electrostatics

Coulomb's law $\quad \vec{F}_2=\dfrac{Q_1Q_2}{4\pi\varepsilon_0R^2}\vec{a}_{12}$

point charge $\quad$ field: $\vec{E}=\dfrac{Q\vec{a}_r}{4\pi\varepsilon_0r^2}$ $\quad$ potential: $V=\dfrac{Q}{4\pi\varepsilon_0r}$

charge distribution $\quad \vec{E}=\displaystyle\int_V\frac{\rho_V(\vec{r}')}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|^2}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}dV'$ $\quad V=\displaystyle\int_V\frac{\rho_V(\vec{r}')\,dV'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$

Gauss's law	$\varepsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{total}} = \int_V \rho \, dV$
	$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{free}} = \int_V \rho_{\text{free}} \, dV$
	$\oint_S \vec{P} \cdot d\vec{S} = -Q_{\text{enclosed}}^{\text{bound}} = -\int_V \rho_{\text{bound}} \, dV$
relating $\vec{E}$ and V	$\vec{E} = -\vec{\nabla} V \qquad V_{AB} = -\int_B^A \vec{E} \cdot d\vec{l}$
capacitance	$C = Q / V$
parallel plate capacitor	$C = \dfrac{\varepsilon_0 \varepsilon_R A}{d}$ of area A and plate separation d
Poisson's equation	$\nabla^2 V = -\rho / \varepsilon_0 \varepsilon_R$
Laplace's equation	$\nabla^2 V = 0$
linear dielectrics	$\vec{D} = \varepsilon_0 \varepsilon_R \vec{E}$
dielectric boundary	E_T and D_N continuous
energy	$W = \dfrac{1}{2} \int_V \vec{E} \cdot \vec{D} \, dV$

Magnetostatics

magnetic flux	$\Phi = \int_S \vec{B} \cdot d\vec{S}$
law of Biot-Savart	$\vec{H} = \oint \dfrac{I \, d\vec{l} \times \vec{a}_R}{4\pi R^2} \qquad \vec{H} = \int_V \dfrac{\vec{j} \times \vec{a}_R \, dV}{4\pi R^2}$
Ampere's law	$\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}} = \int_S \vec{j} \cdot d\vec{S}$
inductance	$L = N\Phi / I$
vector potential	$\vec{A} = \int_V \dfrac{\mu_0 \vec{j} \, dV}{4\pi R}$
relating $\vec{B}$ to $\vec{A}$	$\vec{B} = \vec{\nabla} \times \vec{A}$
linear materials	$\vec{B} = \mu_0 \mu_R \vec{H}$
boundary conditions	H_T and B_N continuous across boundary
energy	$W = \dfrac{1}{2} \int_V \vec{B} \cdot \vec{H} \, dV$

Electromagnetics

Maxwell's equations	$\vec{\nabla}\cdot\vec{D}=\rho$	$\vec{\nabla}\cdot\vec{B}=0$
	$\vec{\nabla}\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}$	$\vec{\nabla}\times\vec{H}=\vec{j}+\dfrac{\partial\vec{D}}{\partial t}$
Faraday's law	$\text{emf}=-d\Phi/dt$	
electromagnetic waves	$H_0=E_0/\eta$	$\eta=\sqrt{(\mu_0\mu_R)/(\varepsilon_0\varepsilon_R)}$
Poynting vector	$\vec{P}=\mathrm{Re}(\vec{E})\times\mathrm{Re}(\vec{H})$	
time-averaged $\vec{P}$	$\langle\vec{P}\rangle=\frac{1}{2}\vec{E}\times\vec{H}^*$	$\vec{H}^*$ is the complex conjugate of $\vec{H}$

Transmission lines

propagation constant	$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}=\alpha+j\beta$	
characteristic impedance	$Z_0=\sqrt{(R+j\omega L)/(G+j\omega C)}$	
reflection coeff.	$\Gamma=\dfrac{Z_L-Z_0}{Z_L+Z_0}$	
transmission coeff.	$T=\Gamma+1$	
input impedance	$Z_{in}=Z_0\dfrac{Z_L+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}$	$l\geq 0$
	$Z_{in}=Z_0\dfrac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}$	$l\geq 0$
	$\beta=2\pi/\lambda$	
propagation velocity	$v_p=\omega/\beta$	
standing wave ratio	$\mathrm{SWR}=(1+\lvert\Gamma\rvert)/(1-\lvert\Gamma\rvert)$	