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University of Saskatchewan
Department of Electrical Engineering

EE301 Electricity, Magnetism and Fields
Final Examination
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Welcome to the EE301 Final. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **3** hours.

Answer **five** of the seven problems. Do not answer more than five problems or severe penalties will apply.

Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers but only if correct intermediate steps are shown. Each problem is weighted equally, subparts of a problem are weighted as indicated

None of the problems require intricate mathematical manipulations. If you get stuck with an impossible integral or equation, you are likely approaching the problem incorrectly.

Problem 1

a) (6) Two point charges of 200 nC and -100 nC are separated by 2 cm. What is the electric field vector and the electric potential at the midpoint between the two charges? What is the electric flux through a closed surface that contains both charges?

b) (6) A charge density only depends on the variable r (spherical coordinates). What can you say about the electric field?

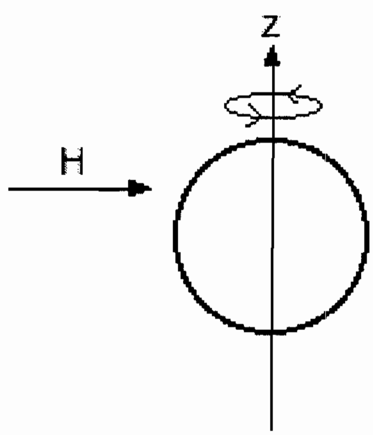
c) (8) An infinitely long, straight line of charge with linear charge density ρ_L is placed at the center of a metal pipe (also infinitely long) with inner radius a and outer radius b. The pipe is initially uncharged and is not connected to ground. Use Gauss's law to calculate the electric field in the interior of the pipe, inside the metal walls of the pipe and outside the pipe.

Problem 2

A current I flows through a straight, infinitely long metal wire with radius a. The current density is uniform inside the wire. Use Ampere's Law to determine the magnetic field both inside and outside the wire.

Problem 3

A circular loop of wire of area 1 m^2 rotates about its diameter, i.e. end over end, at an angular frequency ω_1 as shown in the illustration below. There is a uniform magnetic field perpendicular to the axis of rotation that has a magnitude that varies in time as $|\vec{H}| = H_0 \cos(\omega_2 t)$ A/m. Derive a formula for the emf induced in the wire as a function of time.



Problem 4

An electromagnetic plane wave has an electric field given by

$$\vec{E} = (5\vec{a}_x + 5e^{j\pi}\vec{a}_y)e^{j(\omega t + kz)} \text{ V/m}$$

The wave is propagating in a dielectric material with dielectric constant equal to 3.

a) (5) In what direction is the wave propagating? e.g. $\frac{\pi}{2}$ left $|E_{ox}| = |E_{oy}|$

b) (5) Describe the polarization of the wave. $-\frac{\pi}{2}$ right $|E_{ox}| = |E_{oy}|$

c) (5) Write down the equation for the magnetic field. $n\pi$ $|E_{ox}|$ [illegible]

otherwise elliptical

d) (5) Calculate the time-averaged power density (Poynting vector) of the wave.

Problem 5

The following three parts are independent of each other. Answer all three.

a) (6) A lossless transmission line has an inductance of 0.5 μH/m and a capacitance of 50 pF/m and is being used to carry signals with a frequency of 500 MHz. Calculate the propagation constant, characteristic impedance, and the velocity of propagation.

b) (6) A capacitor with impedance $-25j\,\Omega$ is attached as the load to a quarter-wave transformer with characteristic impedance $Z_Q = 50\Omega$. What is the input impedance of the quarter-wave transformer?

c) (8) An antenna has an impedance of $(25 + j25)\,\Omega$ at 1GHz. It is directly connected (i.e. no matching network) to a 3 cm length of 50Ω transmission line. What is the reflection coefficient at the antenna? What is the input impedance of the transmission line? The velocity of propagation in the transmission line is 1×10^8 m/s.

Problem 6

a) (3) On the Smith chart, clearly label the following locations:

zero reflection ($\Gamma = 0$)

a shorted line ($Z_L = 0$)

a normalized load impedance of $0.2 - 0.4j$ ✓ Find ℓ.

c) (17) A transmission line with a characteristic impedance of 50Ω is connected to an antenna with a complex impedance of $(75 - 20j)\Omega$. The antenna is designed to broadcast at a frequency of 1 GHz. Design a matching scheme so that no power is reflected. You may use any matching scheme but you must clearly describe it in your booklet and determine the relevant parameters on the Smith chart. Be sure to label the Smith chart clearly. $v =$

(Put your name and student number on the Smith chart and hand in the Smith chart with your booklet.)

Problem 7

A transmission line with characteristic impedance $Z_0 = 50\Omega$ is used to supply power to two identical loads; each load has a purely real impedance $Z_L = 200\Omega$. The transmission line is split as shown below. Each arm is a piece of the same transmission line that is exactly $\lambda/4$ long. Calculate the total reflection coefficient.