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December 7, 2007

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EE301 Electricity, Magnetism and Fields
Final Examination
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Welcome to the EE301 Final. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **3** hours.

Answer all five of the problems.

Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers but only if correct intermediate steps are shown. Each problem is weighted equally, subparts of a problem are weighted as indicated

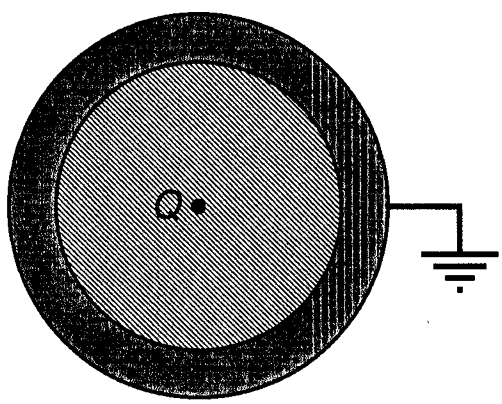
None of the problems require intricate mathematical manipulations. If you get stuck with an impossible integral or equation, you are likely approaching the problem incorrectly.

Problem 1

a) (7) Three point charges are arranged as follows: 50 nC is at (0,0.5), -5 nC is at (0,0.1), and 15 nC is at (0.2,0.2). The locations are in Cartesian coordinates (x, y) and the distances are in meters. Calculate electric field vector and the electric potential at the origin, and the electric flux through a spherical surface with radius 0.4 m centered on the origin.

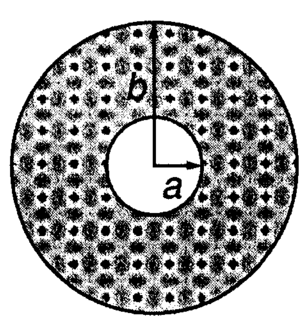
b) (6) A charge density only depends on the variable ρ (cylindrical coordinates). From symmetry, what can you conclude about the electric field?

c) (7) A hollow metal sphere is filled with a dielectric with relative dielectric constant ε_R. A point charge Q is located in the center of the dielectric. The metal is grounded. Determine the electric field inside the dielectric, inside the metal and outside the metal sphere. Indicate with a drawing (using +, –) where there are bound charges in the dielectric and free charges in the metal.



Problem 2

A current I flows through a straight, infinitely long, hollow metal wire. The hollow inside the wire has radius a and the wire has outer radius b. The current density is uniform inside the metal of the wire. Use Ampere's Law to determine the magnetic field inside the hollow, inside the metal and outside the wire.



Problem 3

An electromagnetic plane wave in free space has an electric field given by

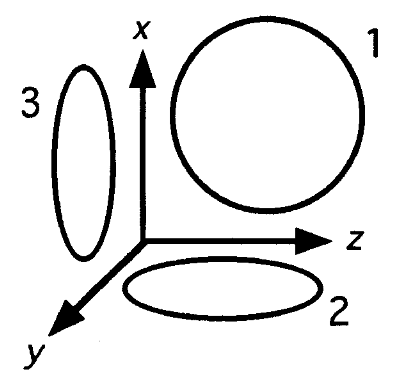
$$\vec{E} = (100\vec{a}_x + 100e^{j\pi/2}\vec{a}_y)e^{j(\omega t+kz)} \text{ V/m}$$

a) (6) Describe the polarization of the wave.

b) (7) Calculate the time-averaged power density (Poynting vector) of the wave.

c) (7) A loop antenna consists of a circular loop of wire. A changing magnetic flux through the loop from a passing electromagnetic wave induces an emf via Faraday's law. For the above EM wave, which of the orientations (1, 2, 3) shown will generate an emf around the loop? Estimate the emf induced in the loop for those orientations that produce an emf assuming the radius of the loop a is very much smaller than the wavelength of the EM wave.

Loop 1 is in the x-z plane, loop 2 is in the y-z plane, and loop 3 is in the x-y plane.



Problem 4

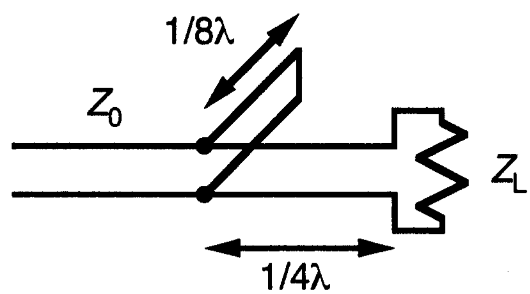
a) (5) A lossless transmission line has an inductance of 0.64 μH/m and a capacitance of 400 pF/m and is used to carry signals with a frequency of 500 MHz. Calculate the propagation constant, wavelength, characteristic impedance, and the velocity of propagation.

b) (7) An antenna has a purely real impedance of 75 Ω at 300 MHz. It is directly connected (i.e. no matching network) to a 0.2 m length of 50Ω transmission line. The velocity of propagation in the transmission line is 1.5×10^8 m/s. Calculate:

the reflection coefficient off the antenna

the input impedance of the transmission line

c) (8) A transmission line with characteristic impedance $Z_0 = 50\Omega$ is terminated with a load with impedance $Z_L = (25 + j50)\Omega$. Exactly $\lambda/4$ from the load, a shorted stub is inserted across the transmission line. The stub is made from transmission line with the same Z_0 and is exactly $\lambda/8$ long (see diagram). Calculate the reflection coefficient.



Problem 5

a) (3) On the Smith chart, clearly label the following locations:

the reflection coefficients $\Gamma = 1$ and $\Gamma = j$

a capacitance of 10pF used with a 50Ω coaxial cable at 1 GHz

a normalized load impedance of $5 + j5$

b) (17) A transmission line with a characteristic impedance of 80Ω is connected to an antenna with a complex impedance of $(240 - j112)\Omega$. The antenna is designed to broadcast at a frequency of 500 MHz. Design a matching scheme so that no power is reflected. You may use any matching scheme but you must clearly describe it in your booklet and determine the relevant parameters on the Smith chart. Be sure to label the Smith chart clearly.

(Put your name and student number on the Smith chart and put the Smith chart inside your booklet.)

Symbols and Constants

F	force	V	electric (scalar) potential
Q	charge	$\vec{A}$	magnetic (vector) potential
$\vec{E}$	electric field	ρ	charge density
$\vec{D}$	displacement field	I	current
$\vec{P}$	polarization field	$\vec{j}$	current density
$\vec{H}$	magnetic field	ε_R	dielectric constant
$\vec{B}$	magnetic flux density field	μ_R	relative permeability
$\vec{M}$	magnetization	$\varepsilon_0 \approx 8.85 \times 10^{-12}$ F/m	
Φ	electric or magnetic flux	$\mu_0 = 4\pi \times 10^{-7}$ N/A^2	

Vector Calculus

unit vector cross products:

Cartesian $\quad \vec{a}_x \times \vec{a}_y = \vec{a}_z \quad \vec{a}_y \times \vec{a}_z = \vec{a}_x \quad \vec{a}_z \times \vec{a}_x = \vec{a}_y$

cylindrical $\quad \vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \quad \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho \quad \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$

spherical $\quad \vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi \quad \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \quad \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$

cylinders

$$Vol = \pi a^2 L \quad Area = 2\pi a^2 + 2\pi a L$$
$$dV = \rho\, d\rho d\varphi dz$$

spheres

$$Vol = (4/3)\pi a^3 \quad Area = 4\pi a^2$$
$$dV = r^2 \sin\theta\, dr d\theta d\varphi \quad dS = r^2 \sin\theta\, d\theta d\varphi$$

curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{a}_z$$

Electrostatics

Coulomb's law

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\vec{a}_{12} \qquad \vec{F} = Q\vec{E}$$

point charge

field: $\vec{E} = \dfrac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^3}$ potential: $V = \dfrac{Q}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|}$

point charge at origin

$$\vec{E} = \frac{Q\vec{a}_r}{4\pi\varepsilon_0 r^2} \qquad V = \frac{Q}{4\pi\varepsilon_0 r}$$

charge distribution

$$\vec{E} = \int_V \frac{\rho_V(\vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^2} \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|} dV' \qquad V = \int_V \frac{\rho_V(\vec{r}\,')\, dV'}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|}$$

Electric flux

$$\Phi = \varepsilon_0 \oint_S \vec{E} \cdot d\vec{S}$$

Gauss's law	$\varepsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{total}} = \int_V \rho \, dV$
	$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{free}} = \int_V \rho_{\text{free}} \, dV$
	$\oint_S \vec{P} \cdot d\vec{S} = -Q_{\text{enclosed}}^{\text{bound}} = -\int_V \rho_{\text{bound}} \, dV$
relating $\vec{E}$ and V	$\vec{E} = -\vec{\nabla} V \qquad V_{AB} = -\int_B^A \vec{E} \cdot d\vec{l}$
capacitance	$C = Q / V$
parallel plate capacitor	$C = \dfrac{\varepsilon_0 \varepsilon_R A}{d}$ of area A and plate separation d
Poisson's equation	$\nabla^2 V = -\rho / \varepsilon_0 \varepsilon_R$
Laplace's equation	$\nabla^2 V = 0$
linear dielectrics	$\vec{D} = \varepsilon_0 \varepsilon_R \vec{E}$
dielectric boundary	E_T and D_N continuous
energy	$W = \dfrac{1}{2} \int_V \vec{E} \cdot \vec{D} \, dV$

Magnetostatics

magnetic flux	$\Phi = \mu_0 \int_S \vec{H} \cdot d\vec{S}$
law of Biot-Savart	$\vec{H} = \oint \dfrac{I \, d\vec{l} \times \vec{a}_R}{4\pi R^2} \qquad \vec{H} = \int_V \dfrac{\vec{j} \times \vec{a}_R \, dV}{4\pi R^2}$
Ampere's law	$\oint_{\text{loop}} \vec{H} \cdot d\vec{l} = I_{\text{enclosed}} = \int_S \vec{j} \cdot d\vec{S}$
inductance	$L = N\Phi / I$
vector potential	$\vec{A} = \int_V \dfrac{\mu_0 \vec{j} \, dV}{4\pi R}$
relating $\vec{B}$ to $\vec{A}$	$\vec{B} = \vec{\nabla} \times \vec{A}$
linear materials	$\vec{B} = \mu_0 \mu_R \vec{H}$
boundary conditions	H_T and B_N continuous across boundary
energy	$W = \dfrac{1}{2} \int_V \vec{B} \cdot \vec{H} \, dV$

Electromagnetics

Maxwell's equations $\vec{\nabla} \cdot \vec{D} = \rho$ $\qquad$ $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

Faraday's law $\quad \text{emf} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S}$

electromagnetic waves $\quad |H_0| = |E_0|/\eta \qquad \eta = \sqrt{(\mu_0 \mu_R)/(\varepsilon_0 \varepsilon_R)}$

Poynting vector $\quad \vec{P} = \text{Re}(\vec{E}) \times \text{Re}(\vec{H})$

time-averaged $\vec{P}$ $\quad \langle P \rangle = \frac{1}{2\eta}\left|\vec{E}_0\right|^2$ (valid when $\vec{E}$ and $\vec{H}$ are in phase)

Transmission lines

propagation constant $\quad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

$\beta = 2\pi / \lambda$

propagation velocity $\quad v_p = \omega / \beta$

characteristic impedance $\quad Z_0 = \sqrt{(R + j\omega L)/(G + j\omega C)}$

reflection coeff. $\quad \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

transmission coeff. $\quad T = \Gamma + 1$

standing wave ratio $\quad \text{SWR} = (1 + |\Gamma|)/(1 - |\Gamma|)$

input impedance $\quad Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \qquad l \geq 0$

$$Z_{in} = Z_0 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \qquad l \geq 0$$

quarter wave transformer $\quad Z_{in} = Z_Q^2 / Z_L$