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University of Saskatchewan
Department of Electrical and Computer Engineering

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EE301 Electricity, Magnetism and Fields
Midterm Examination II
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Welcome to the EE301 Midterm II. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **1:30** hours.

Each problem is worth the same; subparts of a problem might be weighted differently. Show your work and briefly explain what you are doing if appropriate; credit will be given only if the steps leading to the answer are clearly shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers. Be reasonably neat; credit will not be given for illegible answers. If you do not understand any aspect of a problem, ask about it!

Answer problems 1 and 2 and one of problems 3 and 4.

None of the problems require intricate mathematical manipulations.

1. Law of Biot-Savart

Two concentric circular loops of wire with different radii a and b are centred in the x-y plane. Each wire carries current I but the currents flow in opposite directions. Use the law of Biot-Savart to determine the formula for the magnetic field along the z-axis. How does the magnitude of the magnetic field decrease with z for z much greater than a and b?

2. Ampere's law

A triaxial cable consists of a central wire with radius a and two concentric shields, an inner shield with radius b, and an outer shield with radius c. The central wire carries current I and the inner and outer shields each return half the current. Use Ampere's law to determine the magnetic field in all regions. Assume the current is uniformly distributed within the central wire.

Answer only one of the following two problems.

3. EM waves

a) An electromagnetic plane wave has an electric field given by

$$\vec{E} = (5e^{j\pi/8}\vec{a}_x - 5e^{-j\pi/8}\vec{a}_y)e^{j(\omega t - kz)} \text{ V/m}$$

Describe the polarization of the wave. Calculate the time-averaged power density (Poynting vector) of the electromagnetic wave.

b) A plane electromagnetic wave with frequency 1 GHz is traveling inside a material with conductivity $\sigma = 1$ S/m. Can the material be treated as a "good metal" for this wave (assume $\epsilon' = \epsilon_0$)? Calculate the wavelength, the skin depth, and the amplitude of the magnetic field. Finally, determine the distance the wave has to travel for the amplitude to decrease by a factor of 100.

4. EM waves and a dielectric boundary

Two dielectrics meet at $z = 0$. The dielectric for $z < 0$ is characterized by η_1 and the dielectric for $z > 0$ by η_2. An electromagnetic plane wave linearly polarized in the x direction is traveling from left to right (for $z < 0$) and is incident normally on the boundary; the electric field has an amplitude E_0. Another electromagnetic plane wave linearly polarized in the x direction is traveling from right to left (for $z > 0$) and is incident normally on the boundary; the electric field has the same amplitude E_0. At the boundary, the two waves are in phase.

There will be two plane electromagnetic waves traveling away from the boundary, one for $z < 0$ and the other for $z > 0$.

a) Write down the equations for the electric and magnetic fields of all four waves.

b) Determine the amplitudes of the electric fields for the outgoing waves in terms of E_0, η_1, and η_2.