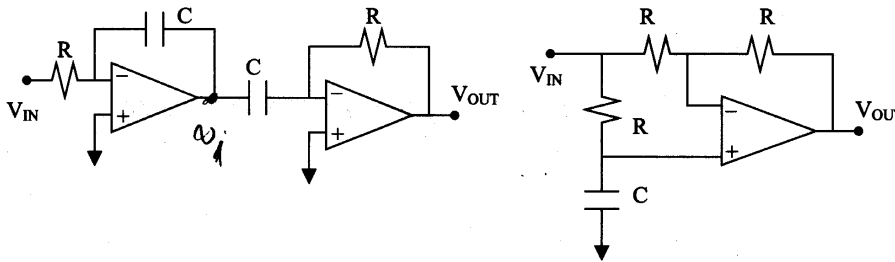


Solution
Open books, open notes.
Good luck and have a Merry Christmas.

1. Question 1: (20 marks)

For the circuits (a) and (b) below, derive transfer functions V_{OUT}/V_{IN} as a function of frequency. For $R=10K$ and $C=15.9nF$, sketch amplitude and phase response of V_{OUT}/V_{IN} .

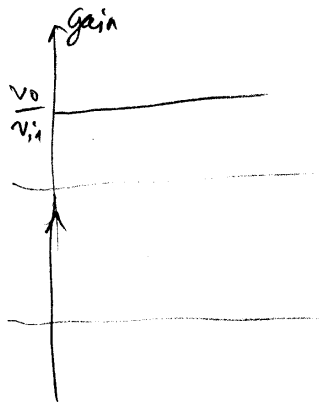


(a)

$$v_1 = -\frac{Z_c}{R} v_{in}, \quad v_o = -\frac{R}{Z_c} v_1$$

$$\Rightarrow v_o = \left(-\frac{R}{Z_c}\right) \left(-\frac{Z_c}{R}\right) v_{in}$$

$$v_o = v_{in} \Rightarrow T(s) = \frac{v_o}{v_i} = 1$$



(b)

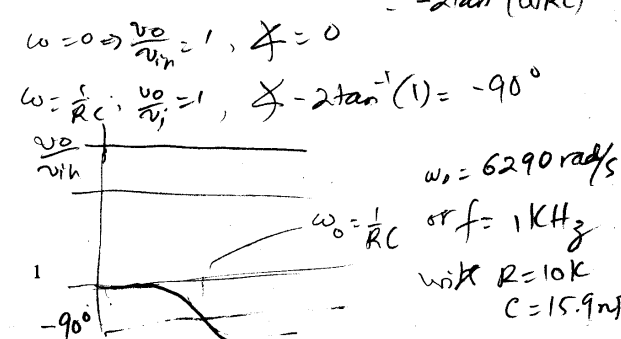
$$v_+ = v_- \Rightarrow \frac{Z_c}{Z_c + R} v_{in} = \frac{1}{2} v_o + \frac{1}{2} v_{in} = \frac{1}{2} (v_o + v_{in})$$

$$\frac{1}{2} v_o = -\frac{1}{2} v_{in} + \frac{\frac{1}{sC}}{\frac{1}{sC} + R} v_{in} = v_{in} \left(\frac{1}{1 + j\omega RC} - \frac{1}{2} \right)$$

$$v_o = 2 v_{in} \left(\frac{1}{1 + j\omega RC} - \frac{1}{2} \right) = 2 v_{in} \left(\frac{2 - 1 - j\omega RC}{2(1 + j\omega RC)} \right)$$

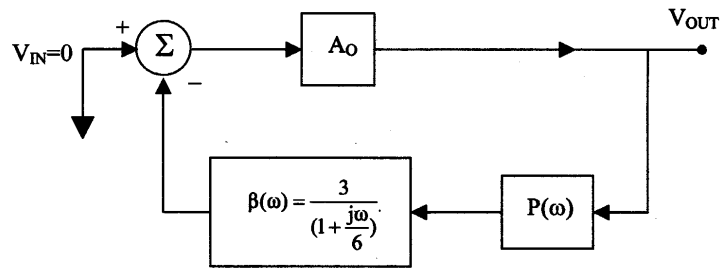
$$\frac{v_o}{v_{in}} = \frac{1 - j\omega RC}{1 + j\omega RC} \quad \phi = \tan^{-1} \frac{-\omega RC}{1} - \tan^{-1} \left(\frac{\omega R}{1} \right)$$

$$= -2 \tan^{-1}(\omega RC)$$



2. Question 2: (20 marks)

The feedback diagram shown below describes an oscillator circuit. In this case, $|P(\omega)|=0.1$ and $\angle P(\omega) = -135^\circ$ for all ω .



- Find the frequency of oscillation.
- Find the minimum value of A_0 needed to maintain oscillation.

⊛ to get -180° , $\angle \beta$ must be -45° since $\angle P(\omega) = -135^\circ$
 $-45^\circ = -\tan^{-1}\left(\frac{\omega}{6}\right)$ or $\frac{\omega}{6} = 1$ $\omega = 6 \text{ rad/s}$

$$f = \frac{6}{2\pi} = \underline{\underline{0.95 \text{ Hz}}}$$

⊛ Magnitude = 1

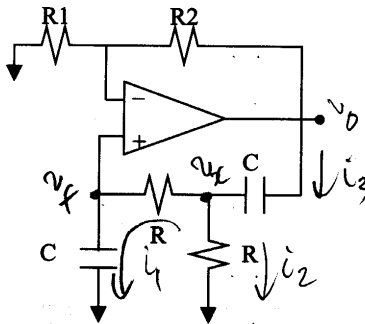
$$A\beta P = 1 \Rightarrow A = \frac{1}{\beta P} = \frac{1}{|B_{\omega}| \cdot 0.1}$$

$$|\beta(\omega)| = \frac{3}{|1+j|} = \frac{3}{\sqrt{2}}$$

$$A = \frac{1}{\frac{3}{\sqrt{2}} \cdot (0.1)} = \frac{\sqrt{2}}{0.3} = \underline{\underline{4.7}}$$

3. Question 3: (20 marks)

For the circuit below, find the loop gain $L(s)$, $L(j\omega)$, the frequency for zero loop-phase. Find R_2/R_1 for oscillation.



Many way to solve:

use KCL at nodes, find v_f vs v_o

$$\frac{v_f}{v_o} = \frac{1}{sRC + 3 + \frac{1}{sRC}} = \frac{1}{3 + j(\omega RC - \frac{1}{\omega RC})}$$

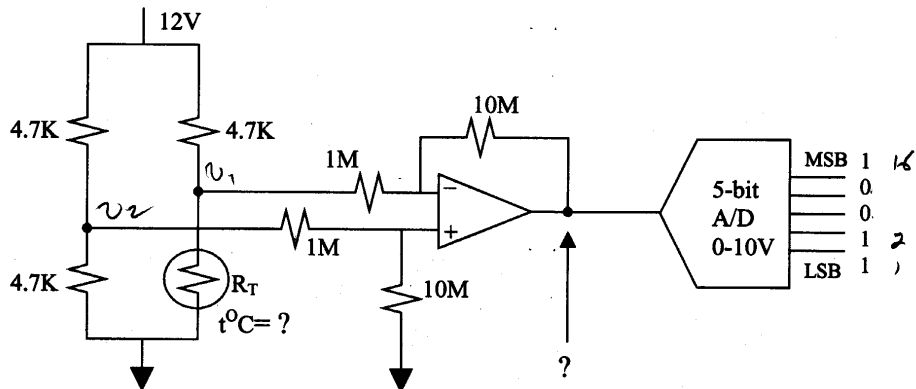
phase = 0 $\Rightarrow \omega RC = \frac{1}{\omega RC}$ or $\boxed{\omega = \frac{1}{RC}}$

then at $\omega = \frac{1}{RC}$, $\frac{v_f}{v_o} = \frac{1}{3}$

or $\frac{v_o}{v_f} = 3 = 1 + \frac{R_2}{R_1}$ or $\boxed{\frac{R_2}{R_1} = 2}$

4. Question 4: (20 marks)

Consider the circuit in a temperature measurement below. The A/D is a 5-bit successive-approximation A/D converter type with an analog span of 0 to 10V, find the input voltage of the A/D converter. The thermistor, R_T , has a resistance of 2K at 20°C and the coefficient β is assumed to be constant at 3650, find temperature of the thermistor.



A/D 5bit $\rightarrow 2^5 = 32$ steps or $\frac{10}{2^5} = 0.3125V$ each step.

Total 19 steps \Rightarrow input = $(19)(0.3125V) = 5.9375V$

The amplifier has a gain of:

$$v_o = -10v_1 + \frac{10}{11}v_2(1+10) = 10(v_2 - v_1)$$

$$\therefore v_2 - v_1 = \frac{5.9375V}{10} = 0.59375V$$

$$v_1 = v_2 - 0.59375V = 6V - 0.59375V = 5.40625V$$

$$v_1 = v_{RT} = 5.40625V = \frac{R_T}{R_T + 4K} \cdot 12V \Rightarrow R_T = 3.85K\Omega$$

$$R_T = R_0 e^{\beta(\frac{1}{T} - \frac{1}{T_0})}$$

$$e^{\beta(\frac{1}{T} - \frac{1}{T_0})} = \frac{R_T}{R_0} = 1.9267 \Rightarrow \beta(\frac{1}{T} - \frac{1}{T_0}) = \ln(1.9267)$$

$$\frac{1}{T} = \frac{\ln(1.9267)}{\beta} + \frac{1}{T_0}$$

$$T = 19.9^\circ C$$

5. Question 5: (20 marks)

In a digital instrumentation system to measure velocity of a fluid pipe, the A/D converter has a sampling rate of 20Ksample/second. Find the Nyquist frequency of the analog signal from the transducer. Design an active filter for anti-aliasing purpose in front of the A/D converter. The filter should have a cut off frequency at Nyquist frequency with a selection of F_{50}/F_3 is at least 3. Since the output signal of the transducer has a wide range of frequency, no ripple is allowed in the filter passband and only 10K resistors are available to realize the filter.

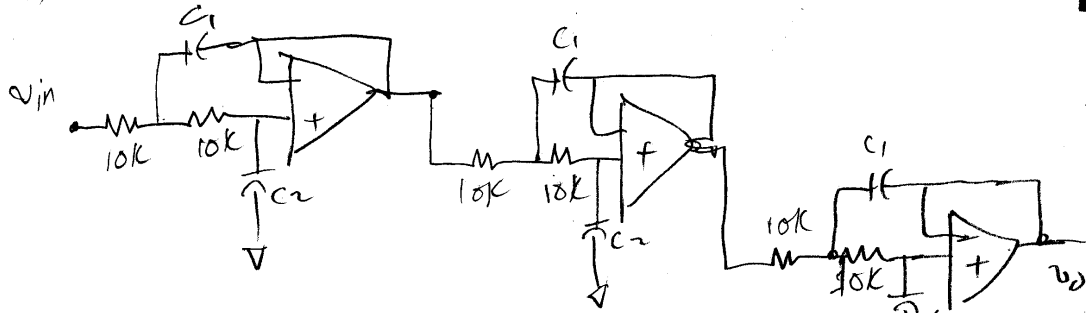
Table 12-1. Design Data for Chebyshev Filters

Ripple = 0 dB (Butterworth)		Cutoff frequency = section frequency = 1.0								
Number of sections	$\frac{F_{50}}{F_3}$	Q Sct 1	Q Sct 2	Q Sct 3	Q Sct 4	Q Sct 5	Q Sct 6	Q Sct 7	Q Sct 8	
	1	17.79	0.7071							
2	4.22	0.5411	1.305							
3	2.61	0.5176	0.7071	1.932						
4	2.05	0.5098	0.6014	0.8999	2.563					
5	1.78	0.5062	0.5812	0.7071	1.101	3.198				
6	1.61	0.5043	0.5412	0.6902	0.8213	1.307	3.831			
7	1.51	0.5032	0.5237	0.5905	0.7071	0.9401	1.514	4.468		
8	1.43	0.5024	0.5225	0.5889	0.5486	0.7882	1.081	1.722	5.101	

Ripple = 0.1 dB		Cutoff frequency = 1.0									
Number of sections	$\frac{F_{50}}{F_3}$	F Sct 1	F Sct 2	F Sct 3	F Sct 4	F Sct 5	F Sct 6	F Sct 7	F Sct 8		
		Q Sct 1	Q Sct 2	Q Sct 3	Q Sct 4	Q Sct 5	Q Sct 6	Q Sct 7	Q Sct 8		
1	16.59	0.9221									
2	3.36	0.8481	0.9481								
3	1.96	0.8180	2.185								
4	1.52	0.7923	0.8129	0.8483	0.9628						
5	1.32	0.7640	0.8065	0.7282	0.8984	0.9867					
6	1.22	0.7469	0.4296	0.6314	0.8038	0.9275	0.9820				
7	1.16	0.7326	0.3723	0.5839	0.7187	0.8823	0.9469	0.9941			
8	1.12	0.7266	0.3380	0.4820	0.6483	0.7786	0.8882	0.9882	0.9965		

$$f_N = \frac{f_s}{2} = \frac{20\text{kHz}}{2} = 10\text{kHz}$$

$$\frac{f_{50}}{f_3} = 3 \Rightarrow 3 \text{ section Using Sallen Key (Ripple = 0dB)}$$



$$f_1 = 10\text{kHz}$$

$$Q_1 = 0.5176$$

$$f_2 = 10\text{kHz}$$

$$Q_2 = 0.7071$$

$$f_3 = 10\text{kHz}$$

$$Q_3 = 1.932$$

$$C_1 = \frac{Q}{\pi f R}$$

$$C_2 = \frac{C_1}{4Q^2}$$

$$C_1 = 1647\text{nF}$$

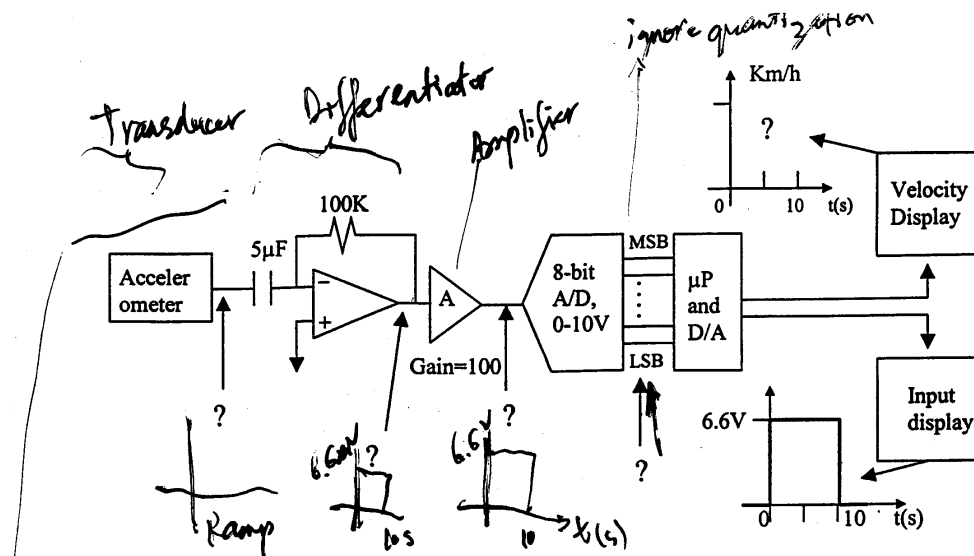
$$C_2 = 1.537\text{nF}$$

$$C_1 = 2.251\text{nF}$$

$$C_2 = 1.125\text{nF}$$

$$C_1 = 6.1497\text{nF}$$

$$C_2 = 0.419\text{nF}$$

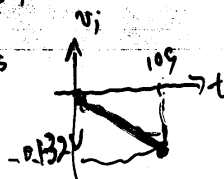


The above arrangement is used to measure velocity of a vehicle (not a good design). The waveform shown at the input display is the output of the D/A converter (data from A/D connects directly to D/A). Ignore quantization error, find the A/D output word. Sketch analog input voltage waveform at the A/D converter, the amplifier A input and the accelerometer output. The accelerometer has an inversion factor of $0.25V/m/s^2$ (i.e., $250mV$ corresponds to $1m/s^2$), find final velocity of the vehicle if its initial velocity is $100Km/h$ and sketch the vehicle velocity.

$$v_0 = -\frac{1}{RC} \frac{dv_i}{dt} \quad \text{or} \quad v_i = -\int RC v_0 dt$$

$$v_i = \frac{1}{(100K)(5\mu F)} \int_0^{10s} (6.6mV) dt$$

$$= -2 (0.066t) \Big|_0^{10s}$$



$$v = at + v_0$$

$$= (-0.132)(10s) + \frac{100K}{hr}$$

convert into m/s^2

$$= 0.526 \frac{m}{s^2}$$

$$\frac{10}{24} = \frac{10}{24} \text{ steps} = 0.0391 \text{ V/step}$$

$$\frac{6.6V}{0.0391 \text{ V/step}} = 169 \text{ steps}$$

45A 25B
10101001

