EE 323
Analog circuits and Instrumentation

Midterm Examination

November 3th, 2006

Time: 2:30pm-3:50pm

Room: 1B79 Eng

84/100

5% Curve
= 89%

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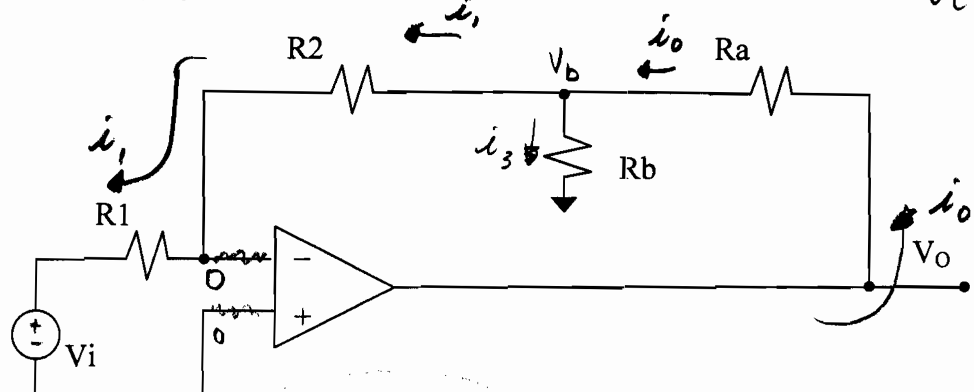
Answer all questions.
State your assumptions if required.
Calculator and 2 pages of formula are allowed.

$\frac{\text{product}}{\text{sum}}$

Question 1:

The inverting amplifier below uses a feedback network in form of a "T-bridge" of Ra, Rb, and R2. Derive the transfer function of the amplifier Vo/Vi as a function of resistors R1, R2, Ra, and Rb. Assuming the op-amp is ideal. $\frac{V_o}{V_i}$

Simplify Vo/Vi in the limits R2>>Rb and R2>>Ra.



$\underline{i_1}$ $\quad i_1 = i_1$

$$\frac{V_b}{R_2} = \frac{-V_i}{R_1}$$

$$V_i = -\frac{R_1}{R_2} V_b$$

$$V_b = -\frac{R_2}{R_1} V_i$$

$\underline{i_o}$

$$\frac{V_o - V_b}{R_a} = \frac{V_b}{R_b} + i_1$$

$$\frac{V_o}{R_a} = \frac{V_b}{R_b} + \frac{V_b}{R_a} + \frac{V_b}{R_2}$$

$$V_o = V_b\left(\frac{1}{R_b} + \frac{1}{R_a} + \frac{1}{R_2}\right) R_a$$

$$V_o = -\frac{R_2}{R_1} V_i \left(\frac{R_a}{R_b} + 1 + \frac{R_a}{R_2}\right)$$

$$\frac{V_o}{V_i} = \frac{-\cancel{R_2}\,\cancel{R_a}}{R_1}\left(\frac{R_a R_2 + R_b R_2 + R_b R_a}{R_b \cancel{R_a}\,\cancel{R_2}}\right)$$

$$\boxed{\frac{V_o}{V_i} = -\left(\frac{R_a R_2 + R_b R_2 + R_b R_a}{R_1 R_b}\right)}$$

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$$\lim_{R_2 \gg R_b} \frac{V_o}{V_i} = -\left(\frac{R_a R_2 + R_b R_2}{R_1 R_b}\right) = -R_2\left(\frac{R_a - R_b}{R_1 R_b}\right)$$

$$\lim_{R_2 \gg R_a} \frac{V_o}{V_i} = -\left(\frac{R_2 R_a + R_2 R_b}{R_1 R_b}\right) = -R_2\left(\frac{R_a + R_b}{R_1 R_b}\right)$$

simply neglect RaRb

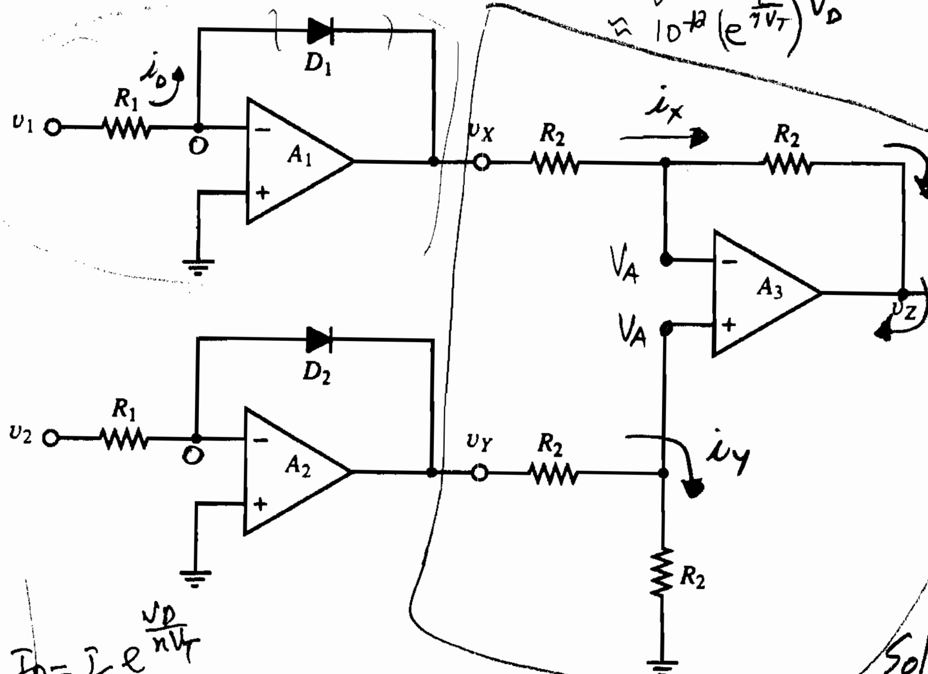
Question 2:

Find the expression of the output voltage Vz with respect to the input voltages V_1 and V_2. It has been known that the current through a diode is: $I_D \approx I_S \exp(V_D/\eta V_T)$ where V_D = diode voltage, $I_S = 10^{-12}$A, V_T= 25mV, η=1.5.

$I_D \approx I_S\, e^{\left(\frac{V_D}{\eta V_T}\right)}$

$\approx 10^{-12}\left(e^{\frac{1}{\eta V_T}}\right)^{V_D}$

$\boxed{I_D \approx 10^{-12}(381\cdot 10^{9})^{V_D}}$



$a \ln\left(\frac{V_1}{V_2}\right)$

Find $V_Z(V_1, V_2)$

$\boxed{\text{Find } \frac{V_X}{V_1} = G_d}$

$V_X = -V_D$

$\frac{V_1}{R_1} = I_D$ ✓

then $I_D = I_S e^{\frac{V_D}{\eta V_T}}$

$\frac{V_1}{R_1} = 10^{-12}(381\cdot 10^{9})^{-V_X}$

$\frac{V_1 10^{12}}{R_1} = 381\cdot 10^{9\,-V_X}$

couldn't Find

assume $G_d = \frac{V_X}{V_1} = \frac{V_Y}{V_2}$

$V_X = G_d V_1$

$V_Y = G_d V_2$

$\boxed{V_Z = G_d(V_2 - V_1)}$

Where $G_d = \frac{V_X}{V_1} = \frac{V_Y}{V_2}$

Solving this:

$V_A = \frac{1}{2} V_Y$

$\frac{V_X - V_A}{R_2} = \frac{V_A - V_Z}{R_2}$

$V_X = 2V_A - V_Z$

$V_X = 2\cdot\frac{1}{2}V_Y - V_Z$

$V_X = V_Y - V_Z$

$\boxed{V_Z = V_Y - V_X}$ ✓

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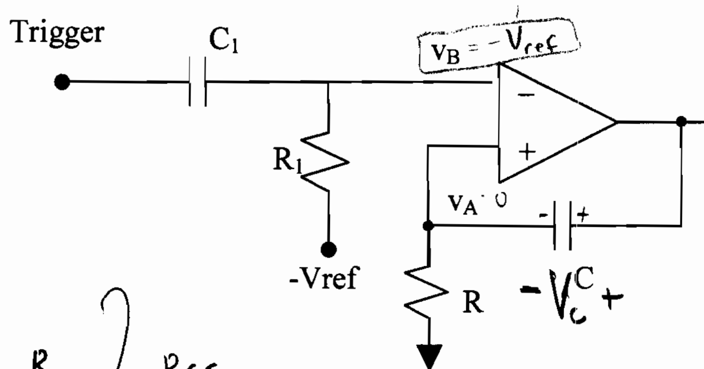
looks like a differential amplisier. ✗

Question 3:

The circuit below is a monostable multivibrator. In the stable state, the output V_{OUT}=L+, v_A=0 and v_B=-Vref. The circuit can be triggered by applying a positive input pulse of height greater than Vref. For normal operation, $C_1R_1 << CR$. Show the resulting waveforms of V_{OUT} and v_A. Find the width of the output pulse T. (i.e., T is a function of C, R, L+, L-, and Vref).

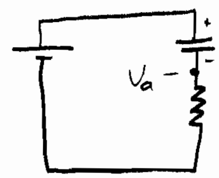
$V_{out} = L_+$
$V_A = 0$
$V_B = -V_{ref}$



initally, C is charged to L_+

V_A is intially 0 b/c no current through R

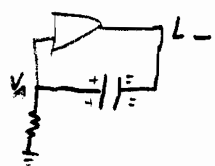
right after trigger:



$$\beta(s) = \frac{V_A}{V_{out}} = \frac{R}{R+\frac{1}{Cs}} = \frac{RCs}{RCs+1}$$

$V_A = \beta V_{out}$

After trigger:



$$V_C = L_- - (L_- - L_+)e^{-t/RC}$$

$$V_A = V_{out} - V_C$$

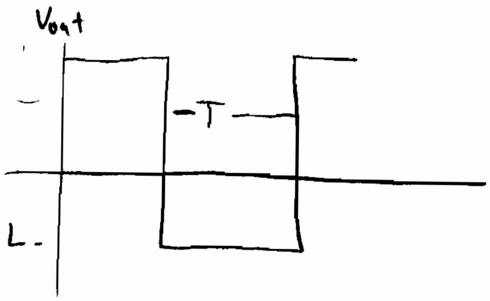
-15-15

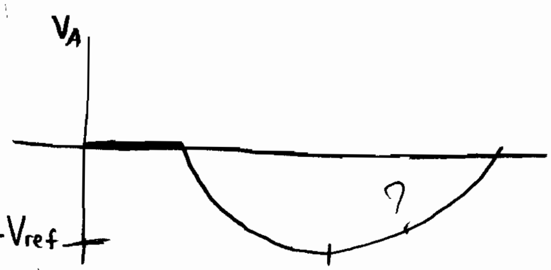
After trigger (sends V_{out} to L_-)

$$V_A = L_- - V_C$$

$$V_A = (L_- - L_+)e^{-t/RC}$$

-30





The opamp will flip again to L_+ when $V_A > -V_{ref}$. This is the period, t = T

$$-V_{ref} = (L_- - L_+)e^{-T/RC}$$

$$\frac{-V_{ref}}{L_- + L_+} = e^{-T/RC}$$

$$\ln\left(\frac{-V_{ref}}{L_- + L_+}\right) = \frac{-T}{RC}$$

$$\boxed{T = RC\ln\left(\frac{-V_{ref}}{L_- + L_+}\right)}$$

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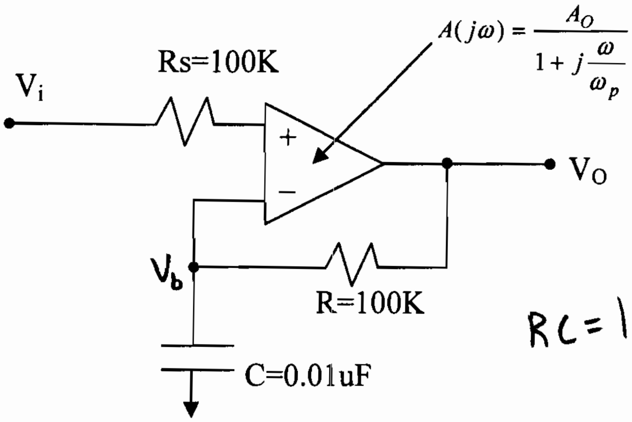
Question 4:

$A_{OL} = A = 10^5$

The op-amp in the circuit below has an open-loop gain of 10^5 and a single pole with ω_p=10rad/s.



$RC = 1 \cdot 10^{-3}$ $\left(1 + j\frac{\omega}{\omega_p}\right)$

a. Sketch the open-loop Bode plot.

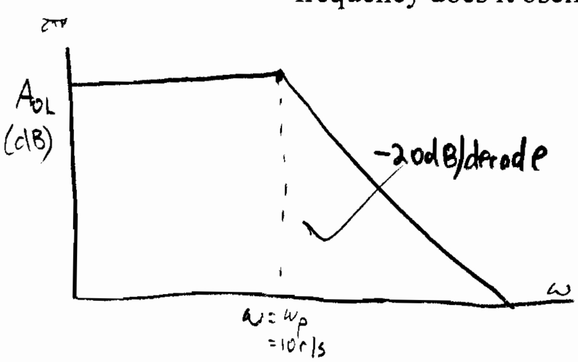
b. The gain equation for this non-inverting amplifier with negative feedback is in the form: $\frac{Vo}{Vi} = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$ where $\beta(j\omega)$ is the negative feedback factor. Find this closed-loop transfer function, including its poles and zero.

c. Sketch the magnitude of the transfer function (gain) versus frequency and label important parameters on your sketch.

d. Find the frequency at which $|A\beta|=1$.

e. Is this network stable or it will oscillate at certain frequency? If it is unstable, what frequency does it oscillate?

a)



$$\beta(j\omega) = \frac{V_b}{V_o} = \frac{\frac{1}{cs}}{\frac{1}{cs} + R} = \frac{1}{1 + RCs} = \frac{1}{1 + RCj\omega}$$

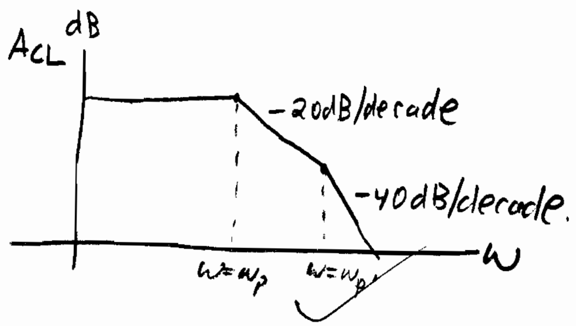
$$A_{CL}(j\omega) = \frac{\frac{A_o}{1 + j\frac{\omega}{10}}}{1 + \frac{A_o}{1 + j\frac{\omega}{10}} \cdot \frac{1}{1 + RCj\omega}} = \frac{A_o}{1 + j\frac{\omega}{10} + \frac{A_o}{1 + RCj\omega}}$$

$$= \frac{A_o(1 + RCj\omega)}{1 + RCj\omega + j\frac{\omega}{10}(1 + RCj\omega) + A_o} =$$

$$= A_o \frac{(1 + RCj\omega)}{1 + RCj\omega + j\frac{\omega}{10} - \frac{\omega^2}{10}RC + A_o}$$

$$= \frac{A_o + A_o RCj\omega}{1 - \frac{\omega^2}{10}RC + A_o + j\left(RC\omega + \frac{\omega}{10}\right)}$$

c)



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$L(s) = 1$ $\omega = 2\pi f$

d) Find f when $A\beta = 1$

$$|L(s)| = 1 = \left| \frac{10^5}{1 + j\frac{\omega}{10}} \cdot \frac{1}{1 + RCj\omega} \right|$$

$$1 = \left| 10^5 \cdot \frac{1}{(1 + j\frac{\omega}{10})(1 + 10^{-3} j\omega)} \right|$$

Solve via HP

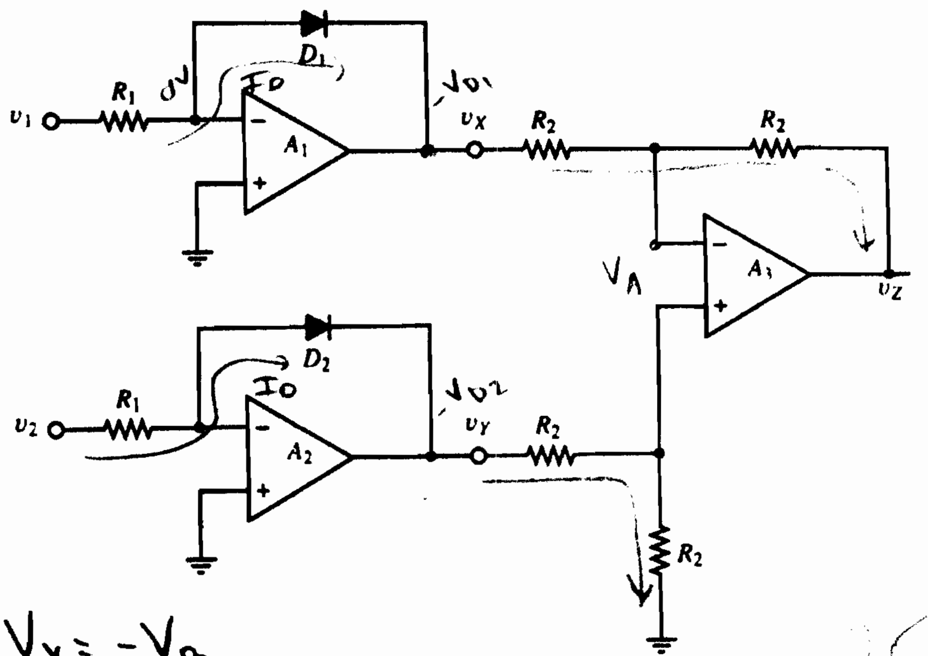
$\omega = 31\,615$ r/s $\quad f = \frac{\omega}{2\pi}$

$\boxed{f = 5.03\text{ kHz}}$

e) It is unstable (oscillator) with an oscillating frequency of 5.03 kHz.

Question 2:

Find the expression of the output voltage Vz with respect to the input voltages V_1 and V_2. It has been known that the current through a diode is: $I_D \approx I_S \exp(V_D/\eta V_T)$ where V_D = diode voltage. $I_S = 10^{-12}$A, V_T= 25mV, η=1.5.



$V_x = -V_{D_1}$ $\quad V_y = -V_{D_2}$

$\frac{V_x - V_A}{R_2} = \frac{V_A - V_Z}{R_2} \Rightarrow 2V_A = V_x + V_Z$

$V_A = \frac{V_x + V_Z}{2}$

$\frac{V_y - V_A}{R_2} = \frac{V_A}{R_2} \Rightarrow V_A = \frac{V_y}{2}$

$V_x + V_Z = V_Y$

$V_x - V_Y = -V_Z$ ① $\Rightarrow V_Z = V_Y - V_X$

$I_D = \frac{V_1}{R_1} \Rightarrow \frac{V_1}{R_1} = I_s \exp(V_D/\eta V_T) \Rightarrow$ Solve for $V_D \Rightarrow \ln\frac{V_1}{R_1 I_s} = \frac{V_D}{\eta V_T}$

$V_{D_1} = \eta V_T \ln\left(\frac{V_1}{R_1 I_s}\right)$ ② $\quad V_{D_2} = \eta V_T \ln\left(\frac{V_2}{R_1 I_s}\right)$ ②

Using ①,②,③

$\ldots \eta V_T \ln(\ldots) - \eta V_T \ln\left(\frac{V_1}{R_1 I_s}\right) = \eta V_T (\ln V_1 - \ln R_1 I_s - \ln V_2 + \ldots$