

**University of Saskatchewan  
College of Engineering  
EE 342: Power Systems I  
Mid-term Examination  
A formula sheet is allowed**

**Instructor: S.O. Faried  
Duration: 2 hours**

**October 2008**

1. A 60-Hz, 500-kV, 3-phase transmission line is 320 km long. The distributed line parameters are:  $r = 0.01864 \Omega/km$ ,  $x_l = j0.3728 \Omega/km$ ,  $y = j4.4739 \mu S/km$ . The transmission line delivers 700 MW at 500 kV with 90% power factor lagging. Find:
  - (a) The sending-end voltage and current.
  - (b) The transmission line efficiency.
  - (c) The voltage regulation.
  - (d) It is required to compensate the line by a series capacitor bank. The degree of series compensation is 70%. Find the value of the capacitance per phase of the series capacitor bank.
  
2. The configuration of a double circuit, bundle conductors, three-phase, 60-Hz, 500 kV, 400 km, transposed line is shown in Fig. 1. The bundle spacing is 45 cm.

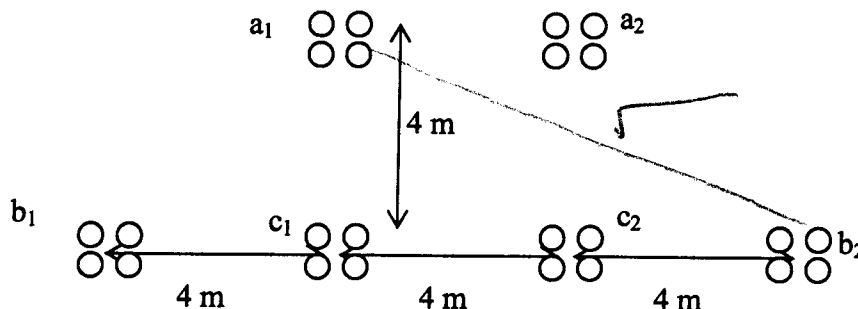


Fig. 1

Each conductor of the bundle is Bluebird with an outside diameter of 1.762 in, a GMR of 0.0586 ft and a resistance of 0.034 ohm/km.

Find:

- a) The inductive reactance and shunt admittance of the line per phase per km.
- b) The surge impedance loading.
- c) The wavelength of the line.
- d) The disruptive corona voltage in kV for air temperature of  $27^\circ \text{C}$ , a barometric pressure of 740 mm and irregularity factor of 0.85.

$$\begin{aligned} \text{II } \ell &= 320 \text{ km} \\ R &= .01864 \Omega/\text{km} \\ X_L &= j \cdot 3728 \Omega/\text{km} \\ Y &= j4.4739 \mu\text{S}/\text{km} \end{aligned}$$

delivers: 700 MW  
500 kV  
.9 (lagging)

FIND ABCD PARAMETERS:

$$\begin{aligned} \text{a) } Z_{\text{series}} &= (R + jX_L)\ell \\ &= (.01864 \Omega/\text{km} + j \cdot 3728 \Omega/\text{km})(320 \text{ km}) \\ Z_{\text{series}} &= 122.1231 \angle 77.6477^\circ \Omega \end{aligned}$$

$$\begin{aligned} Y &= (j(4.4739 \times 10^{-6} \text{ S}/\text{km}))(320 \text{ km}) \\ Y &= .001432 \angle 90^\circ \text{ S} \end{aligned}$$

$$Z_c = \sqrt{\frac{Z}{Y}} = 292.0300 \angle -6.1762^\circ \quad \delta = \sqrt{ZY} = .4182 \angle 83.3239^\circ$$

$$\begin{aligned} A = D &= \cosh(\delta) = .9159 \angle 1.1372^\circ \\ B &= Z_c \sinh \delta = 118.6801 \angle 78.0091^\circ \\ C &= \sinh(\delta) / Z_c = .001392 \angle 90^\circ \end{aligned}$$

$$V_{\text{Rph}} = \frac{500 \text{ kV}}{\sqrt{3}} \angle 0^\circ$$

$$\begin{aligned} P &= \sqrt{3} |V_L| |I_L| \cos \phi \\ |I_R| &= \frac{700 \times 10^6 \text{ W}}{\sqrt{3} (500 \times 10^3 \text{ V})(.9)} \end{aligned}$$

$$\begin{aligned} \angle I_R &= -\cos^{-1}(.9) \\ &= -25.8419^\circ \end{aligned}$$

$$|I_R| = 895.1004 \text{ A}$$

$$I_R = 895.1004 \angle -25.8419^\circ \text{ A}$$

$$\begin{aligned} \begin{bmatrix} 4 & 19 \\ 7 & 43 \end{bmatrix} \begin{bmatrix} 23 \\ 59 \end{bmatrix} \\ = 1213 \\ 2695 \end{aligned}$$

$$V_{sph} = AV_{Rph} + BI_R$$

$$V_{sph} = (.9159 \angle 1.1572^\circ) \left( \frac{500 \times 10^3 V}{\sqrt{3}} \angle 0^\circ \right) + (118.6801 \angle 78.0091^\circ) (898.1004 \angle -25.8417^\circ)$$

$$V_{sph} = 341.634 \angle 15.1752^\circ \text{ kV}$$

$$\boxed{|V_s| = 591.728 \text{ kV}}$$

$$I_s = (V_{Rph} + DI_R)$$

$$I_s = (1.001392 \angle 90.3615^\circ) \left( \frac{500 \times 10^3 V}{\sqrt{3}} \angle 0^\circ \right) + (.9159 \angle 1.1372^\circ) (898.1004 \angle -25.8417^\circ)$$

$$\boxed{I_s = 747.0065 \angle 4.4563^\circ \text{ A}}$$

$$b) \gamma = \frac{700 \times 10^6 \text{ W}}{3(341.634 \times 10^3 V)(747.0065 \text{ A})(\cos(15.1752^\circ - 4.4563^\circ))} \times 100\%$$

$$\boxed{\gamma = 93.05\% \text{ or } .9305} \quad \times$$

$$c) \epsilon = \frac{591.728 \times 10^3 V}{.9159} \frac{500 \times 10^3 V}{500 \times 10^3 V} \times 100\%$$

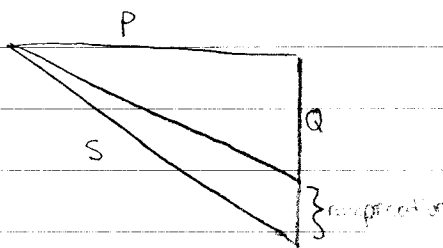
$$\epsilon = 29.21\%$$

$$d) \begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} 1 & -jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A - (jX_c)C & B - D(jX_c) \\ C & D \end{bmatrix}$$

$\downarrow$   $\text{Port A}$                        $\downarrow$   $\text{Port B}$

$$.7 \times 100 = 70\%$$

$$= 7\%$$



$$\text{old: } I_S = 747.0065 \angle 4.4563^\circ \text{ A}$$

$$V_{Rph} = \frac{500 \times 10^3 \text{ V} \angle 0^\circ}{\sqrt{3}}$$

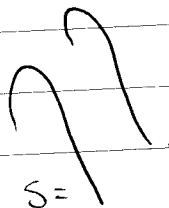
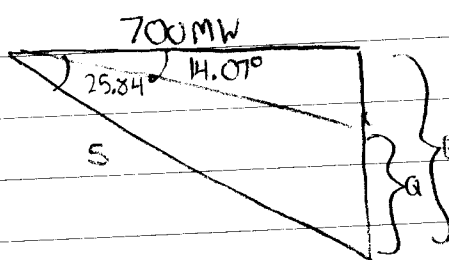
$$V_S = (A - C j Y_c) V_{Rph} + (B - D j Y_c) I_R$$

$$I_R = 898.1004 \angle -105^{-1} (.97)$$

$$I_R = 898.1004 \angle -14.0699^\circ$$

$$I_S = (V_{Rph} + D I_R)$$

$$I_S = 828.2985 \angle 15.2402^\circ \text{ A}$$



$$Q = Q_{wh} - Q_1$$

$$Q = 305.10 \text{ VAR} - 170.18 \text{ VAR}$$

$$Q = 134.92 \text{ VAR}$$

$$Q = V I$$

$$Q = \frac{V^2}{X_c}$$

$$Y_c = \frac{(500 \text{ kV})^2}{134.92 \text{ VAR}}$$

$$134.92 \text{ VAR}$$

$$V_s = AV_{Rph} - C_j Y_c V_{Rph} + B I_R - D_j Y_c I_R$$

$$j X_c (C V_{Rph} + D I_R) = AV_{Rph} + B I_R - V_s$$

$$Y_c = \frac{AV_{Rph} + B I_R - V_s}{C V_{Rph} + D I_R}$$

$$X_c = 394.5964 \Omega$$

$$X_c = \frac{1}{2\pi f C}$$

$$C_{ph} = \frac{1}{2\pi f X_c}$$

$$C_{ph} = 6.7229 \times 10^{-6} \text{ F}$$

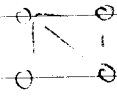
$$45\text{cm} = 1.4764\text{ft}$$

$$D_s = .0586\text{ft} \quad D_{\text{bundle}} = \sqrt[4]{(.0586\text{ft})(1.4764\text{ft})^3 \sqrt{2}}$$

$$D_{\text{bundle}} = .7186\text{ft}$$

$$D_{ab} = \sqrt[4]{(\sqrt{2} \cdot 4\text{m}) \sqrt{4^2 + 8^2} \sqrt{4^2 + 8^2} \sqrt{2} \cdot 4\text{m}}$$

$$D_{ab} = 7$$



$$\boxed{2} \quad D_s = .0586\text{ft} = .01786\text{m}$$

$$a) \quad D_{\text{bundle}} = \sqrt[4]{(.01786\text{m})(.45\text{m})^3 \sqrt{2}}$$

$$= .2190\text{m}$$

$$D_{ab} = \sqrt[4]{(\sqrt{2} \cdot 4\text{m})^2 (4^2 + 8^2)}$$

$$D_{ab} = 7.1131\text{m}$$

$$D_{bc} = \sqrt[4]{(4\text{m})(5\text{m})(8\text{m})(4\text{m})}$$

$$D_{bc} = 5.6969\text{m}$$

$$D_{ac} = \sqrt[4]{(4\text{m})(4\sqrt{2}\text{m})(11\text{m})(11\sqrt{2}\text{m})}$$

$$D_{ac} = 4.7563\text{m}$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ac}}$$

$$D_{eq} = 5.7630\text{m}$$

$$D_{sc} = \sqrt{D_{\text{bund}} (4\text{m})}$$

$$D_{sc} = .9359\text{m}$$

$$D_{sb} = \sqrt{D_{\text{bund}} (12\text{m})}$$

$$D_{sb} = 1.6211\text{m}$$

$$D_{sc} = \sqrt{D_{\text{bund}} (4\text{m})}$$

$$D_{sc} = .9359\text{m}$$

$$D_s = \sqrt[3]{D_{sc} D_{sb} D_{sc}}$$

$$D_s = 1.1240\text{m}$$

$$L_{ph} = 2 \times 10^{-7} \ln \left( \frac{5.7630\text{m}}{1.1240\text{m}} \right) = 3.2691 \times 10^{-7} \text{H/m}$$

$$L_{ph} = 3.2691 \times 10^{-4} \text{H/km}$$

$$\frac{1.762\text{m}}{12} = r = .02238\text{m}$$

$$D_{\text{bundle}} = \sqrt[4]{(.02238\text{m})(.45\text{m})^3 \sqrt{2}}$$

$$= .2317\text{m}$$

$$D_{sa} = \sqrt{D_{sb} D_c} (11m)$$

$$D_{sb} = \sqrt{D_{sa} D_c} (12m)$$

$$D_{sc} = \sqrt{D_{sa} D_{sb}} (11m)$$

$$D_{sa} = .9627m$$

$$D_{sb} = 1.6675m$$

$$D_{sc} = .9627m$$

$$D_s = \sqrt[3]{D_{sa} D_{sb} D_{sc}}$$

$$D_s = 1.1562m$$

$$C_{ph} = \frac{2\pi \epsilon_0}{\ln\left(\frac{D_{sa}}{D_s}\right)} = 3.4633 \times 10^{-8} \text{ F/km}$$

$$\frac{1}{2\pi k} = \frac{2 \cdot 2\pi k \cdot \epsilon_0}{2\pi k}$$

$$X_{L_{ph}} = 2\pi L_{ph}$$

$$X_{L_{ph}} = .1232 \Omega/\text{km}$$

$$Y_{ph} = 1.3056 \times 10^{-8} \text{ S/km}$$

$$b) \text{ SIL} = \frac{|V_L|^2}{\sqrt{\frac{L}{C}}} = \frac{|500 \text{ kV}|^2}{\sqrt{\frac{3.2691 \times 10^{-4} \text{ H/km}}{3.4633 \times 10^{-8} \text{ F/km}}}}$$

$$\text{SIL} = 2573.1847 \text{ MW}$$

$$c) \lambda = \frac{2\pi}{\beta} \quad \delta = \alpha + j\beta$$

$$Z_s = \left[ \frac{.0241 \Omega/\text{km} + j \cdot .1232 \Omega/\text{km}}{2} \right] 400 \text{ km}$$

$$Z_s = 49.7469 \angle 82.1435^\circ \Omega$$

$$Y = jY_{ph} (400 \text{ km})$$

$$Y = j.005222$$

$$\delta = \sqrt{Z_s Y}$$

$$\delta = \alpha + j.5085$$

$$\lambda = \frac{2\pi}{.5085} = 12.2863 \text{ m}$$

$$d) V_d = (.85)(21.8 \text{ kV/cm}) \left( \frac{.392(74)}{27+273} \right) (2.2377 \text{ cm}) \ln \left( \frac{400 \times 10^5}{2.2377 \text{ cm}} \right) \text{ kV}$$

$$V_d = 66.952 \text{ kV}$$