28.3

$\frac{0.3}{30} = 1\%$

EE351.3 Mid-term test, Signals and Systems

6 Questions. Duration 110 minutes.
You may use a formula sheet.
Attempt all the questions. Justify your answers.

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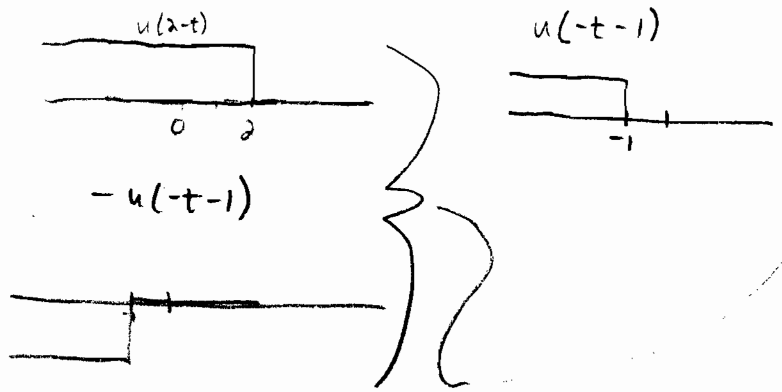
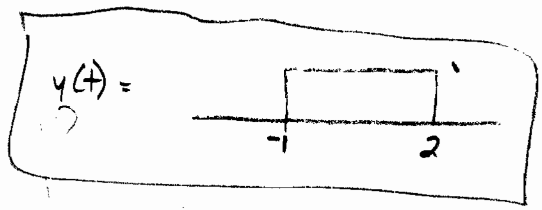
[3 marks] 1) Take the continuous-time signal $x(t) = u(t) - u(t-3)$, where $u(t)$ is the unit step function. Assume that $y(t) = x(2-t)$. Sketch $y(t)$.

2.7

$y(t) = x(2-t) = u(2-t) - u(2-t-3)$
$= u(2-t) - u(-t-1)$



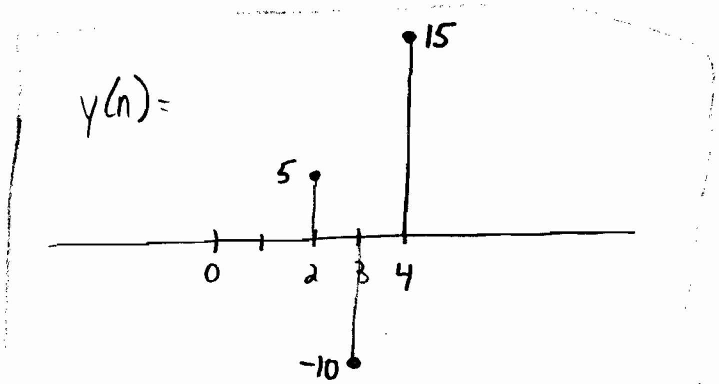


[2 marks] 2) Take the discrete-time signal $x(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2)$, where $\delta(n)$ is the unit impulse. Assume that $y(n) = 5x(n-2)$. Sketch $y(n)$.

$n = n-2$

2.0

$y(n) = 5x(n-2) = 5\left[\delta(n-2) - 2\delta(n-2-1) + 3\delta(n-2-2)\right]$
$= 5\delta(n-2) - 10\delta(n-3) + 15\delta(n-4)$



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9.8

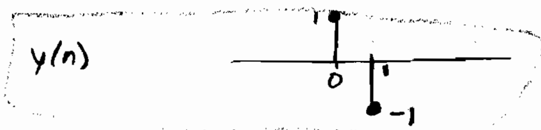
[10 marks] 3) A discrete-time system has the relationship

$$y(n) = x(n) - x(n-1) \qquad h(t) =$$

between its input $x(n)$ and its output $y(n)$.

a) Determine $y(n)$ if the input is the $\delta(n)$. Sketch the output.

$@x(n) = \delta(n), \rightarrow y(n) = \delta(n) - \delta(n-1)$ $\quad \therefore h(n) = \delta(n) - \delta(n-1)$



b) Is this system LTI? $\boxed{\text{Yes}}$

for $\alpha\, x(n)$

$y_2(n) = \alpha\, x(n) - \alpha\, x(n-1)$

$y_2(n) = \alpha\,(x(n) - x(n-1))$

$y_2(n) = \alpha\, y(n)$

Linear

Time Invariant?. Yes, output does not depend on n directly.

for $x(n+a) \rightarrow y_2(n+a) = x(n+a) - x(n+a-1)$

$y_2(n+a) = y(n+a)$

c) Is this system stable? Is it memoryless?

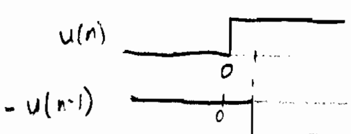
It is not memoryless. It is stable $\rightarrow \sum_{n=-\infty}^{\infty} |h(n)| = 2$ which is $< \infty$

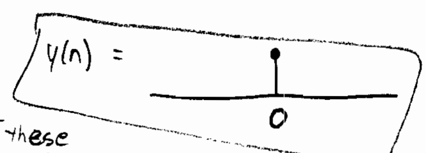
eg: $y(2) = x(2) - x(1)$

* It needs to remember one step in the past

d) Determine $y(n)$ when $x(n)$ is a unit step function $u(n)$?

$y(n) = u(n) - u(n-1)$

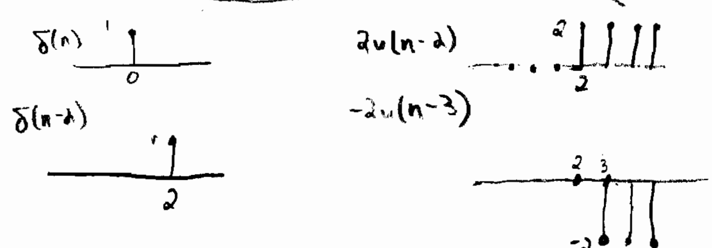


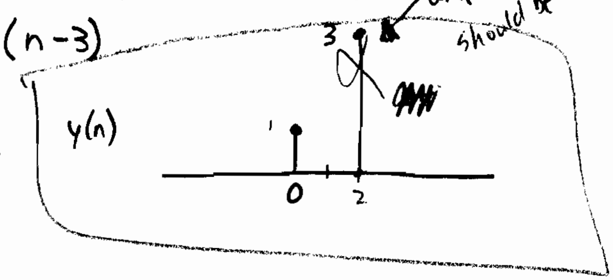


* these are discrete

e) What is the output if the input is $x(n) = \delta(n) + \delta(n-1) + 2u(n-2)$. Sketch $y(n)$

$y(n) = \delta(n) + \cancel{\delta(n-1)} + 2u(n-2) - \cancel{\delta(n-1)} - \delta(n-2) - 2u(n-3)$





amplitude should be 1

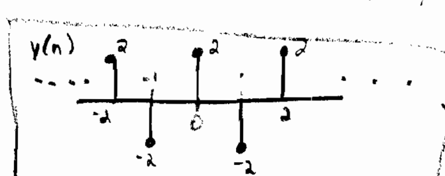
f) What is the steady-state output of the system if $x(n) = (-1)^n$?

$x(n-1) = (-1)^{n-1}$

$y(n) = (-1)^n - (-1)^{n-1}$

$= (-1)^n + (-1)^n$

$y(n) = 2(-1)^n$



g) What does this system do?

Finds the derivative, $\frac{d\,x[n]}{dn}$

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$\omega_0 = \frac{2\pi}{T}$

$\frac{18}{4} = \frac{9}{2} = 4.5$

18
5.5
23.5

6.0

[6 marks] 4) Take the signal $x(t) = 1 + 3\cos(4\pi t) + 6sin(12\pi t)$. Find the period and the Fourier series coefficient of the signal. What is $\int_{t=0}^{1} x^2(t)dt$?

$\omega_0 = 4\pi$

$T = \frac{2\pi}{\omega_0} = \frac{1}{2}$

$x(t) = 1 + \frac{3}{2}e^{j4\pi t} + \frac{3}{2}e^{-j4\pi t} + \frac{6}{2j}e^{j12\pi t} - \frac{6}{2j}e^{-j12\pi t}$

$a_0 = 1$

$a_1 = \frac{3}{2} \qquad a_{-1} = \frac{3}{2}$

$a_3 = \frac{3}{j} \qquad a_{-3} = \frac{-3}{j}$

$\frac{1}{T}\int_0^1 |x(t)|^2\, dt = 2 \cdot \frac{1}{T}\int_0^{1/2} |x(t)|^2\, dt$

$= 2 \cdot \sum_{k=-\infty}^{\infty} |a_k|^2 = 2\left(1 + 2\cdot\frac{9}{4} + 9 + 9\right) = 2(23.5)$

$\therefore \frac{1}{T}\int_0^1 |x(t)|^2 dt = 2 \cdot 23.5 \qquad T = \frac{1}{2}$

$x(t)$ is real
so $|x(t)|^2 = x^2(t)$

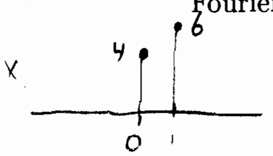
$$\boxed{\int_0^1 x^2(t)\, dt = 2(23.5)\frac{1}{2} = 23.5}$$

5.0

$N=2$

[5 marks] 5)Take the signal x with a period 2. Assume $x(0) = 4$, and $x(1) = 6$. Find the Fourier series coefficient a_k. Write $x(n)$ in terms of a_k.

$\omega_0 = \frac{2\pi}{N} = \pi$



$a_k = \frac{1}{N}\sum_{n=0}^{1} x[n]e^{-jk\omega_0 n}$

$a_0 = \frac{1}{2}(4+6) = 5$

$a_1 = \frac{1}{2}(4 + 6e^{-j\pi}) = \frac{1}{2}(4-6) = -1$

$a_2 = a_0$
$a_3 = a_1$
$\vdots$
repeats

$x[n] = \sum_{k=0}^{1} a_k e^{jk\omega_0 n} = a_0 + a_1 e^{j\pi n} = \underline{a_0 + a_1(-1)^n}$

$[n] = 5 - 1(e^{j\pi})^n$

$= 5 - (-1)^n$

$$\boxed{x[n] = 5 + (-1)^{n+1}}$$

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[4 marks] 6) (a) Take the signal $x(t)$ with a period 10 and Fourier series coefficient a_k. What are the Fourier coefficients of $y(t) = -x(t-5)$? $T = 10$ $\quad \omega = \frac{2\pi}{10} = \frac{\pi}{5}$

let b_K = Fourier coefficients of $y(t) = -x(t-5)$

a_K = FC of $x(t)$

$$b_K = -a_K e^{-jk\omega_0 t_0}$$

$$= -a_K e^{-jk\frac{\pi}{5}(5)}$$

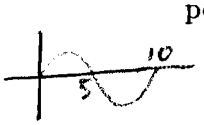
$$= -a_K e^{-j\pi k}$$

$$= -a_K (-1)^K$$

$$\boxed{b_K = a_K (-1)^{K+1}}$$

2.8

(b) Show that if a signal satisfies the condition $x(t) = -x(t-5)$, then it is periodic with period 10. Ex. Sinusoidal wave. take $x(t) = \sin\left(\frac{2\pi}{10} t\right) = \sin\left(\frac{\pi}{5} t\right)$ $\quad \omega_0 = \frac{2\pi}{T}$



does $\sin\left(\frac{\pi}{5}t\right) \overset{?}{=} -\sin\left(\frac{\pi}{5}t - 5\right)$ $\qquad b_K = b_K(-1)^{K+1}$

@ t=5 $\quad \sin(\pi) \overset{?}{=} -\sin(5-5)$ $\qquad -1 = (-1)^K$

$0 = 0$ ✓

See bottom of page

(c) using part (a) and (b) show that if $x(t) = -x(t-5)$, then all of the even harmonics of $x(n)$ are zero. $\quad a_K = a_{-K}^*$

$b_0 = 0$

$b_1 = a_1(-1)^2 = a_1$

$b_2 = a_2(-1)^3 = -a_2 \qquad b_{-2} = -a_2$

$b_{-2} = a_{-2}(-1)^{-1} = -a_{-2}$ $\qquad \left\{ \begin{array}{l} -a_2 = -a_{-2} \\ a_2 = a_{-2} \end{array} \right.$

Periodic wave:

$x(t) = -x(t-5)$

$a_K = a_K(-1)^{K+1}$

$1 = (-1)^{K+1}$ → You shouldn't cancel a_K!

$-1 = (-1)^K$ $\quad$ K must be odd for this equality to hold.

$\therefore$ When K is even $a_K = 0$.

Take generic periodic signal $e^{j\omega_0 t} = e^{j\frac{2\pi}{T}t} \Rightarrow \underbrace{e^{j\frac{2\pi}{10}t}}_{T=10} = e^{j\frac{\pi}{5}t}$

$x(t) \overset{?}{=} -x(t-5)$ → This is not the general form!

$e^{j\frac{\pi}{5}t} \overset{?}{=} -e^{j\frac{\pi}{5}(t-5)}$

$\overset{?}{=} -e^{-j\pi} e^{j\frac{\pi}{5}t}$

$$\boxed{e^{j\frac{\pi}{5}t} \overset{\checkmark}{=} e^{j\frac{\pi}{5}t}}$$

✓

$\therefore$ if a signal satisfies $x(t) = -x(t-5)$ it is peridic with $T = 10$