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EE351.3 Mid-term test, Signals and Systems

7 Questions. Duration 90 minutes.
You may use a formula sheet.
Attempt all the questions. Justify your answers.

[4 marks] 1) Take the discrete-time signal $x(n) = \sin(n\pi/2)(u(n) - u(n-4))$, where $u(n)$ is the unit step function. Sketch $x(n)$.

$x(n) = 0$ for $n < 0$

$x(0) = \sin(0)[u(0) - u(-4)] = 0$

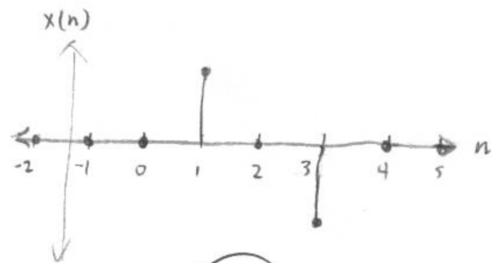
$x(1) = \sin(\frac{\pi}{2})[u(1) - u(-3)] = 1$

$x(2) = \sin(\pi)[u(2) - u(-2)] = 0$

$x(3) = \sin(\frac{3}{2}\pi)[u(3) - u(-1)] = -1$

$x(4) = \sin(2\pi)[u(4) - u(0)] = 0$

$x(n) = 0$ for $n > 4$



4.0

[2 marks] 2) For what values of ω is the signal $x(n) = e^{j\omega n}$ periodic?

$T = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{T}$

assuming a period T of 5 for $e^{jwn} = \cos(wn) + j\sin(wn)$

$\boxed{w = \frac{2}{5}\pi}$ of n intervals

1.0

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[5 marks] 3)Take a system with an impulse response $h(n) = \delta(n) - \delta(n-1)$. Find the response of the system to the input $x(n) = \delta(n) - \delta(n-1) + \delta(n-2)$.

Both $h(n)$, $x(n) = 0$ for $n < 0$

$h(0) = \delta(0) - \delta(-1) = 1$
$h(1) = \delta(1) - \delta(0) = -1$
$h(n) = 0$ for $n > 1$

$h(n) = [1 \ -1 \ 0 \ 0 \ 0]$

$x(0) = \delta(0) - \delta(-1) + \delta(-2) = 1$
$x(1) = \delta(1) - \delta(0) + \delta(-1) = -1$
$x(2) = \delta(2) - \delta(1) + \delta(0) = 1$
$x(n) = 0$ for $n > 2$

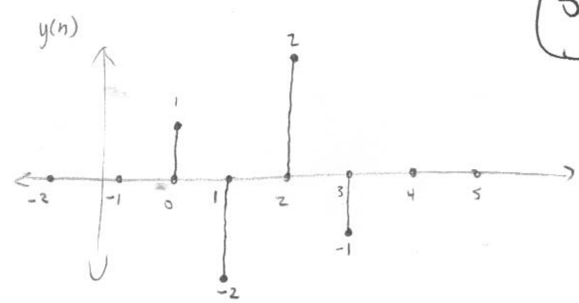
$x(n) = [1 \ -1 \ 1 \ 0 \ 0]$

$y(n) = x(n) * h(n)$

	1	-1	1	0	0
1	1	-1	1	0	0
-1	-1	1	-1	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$y(n) = [1 \ -2 \ 2 \ -1 \ 0]$

5.0



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8.5

[9 marks] 4) A discrete-time system has the relationship

$$y(n) = 0.5x(n) + 0.5x(n-1)$$

between its input $x(n)$ and its output $y(n)$.

a) Determine $y(n)$ if the input is the $\delta(n)$. Sketch the output.

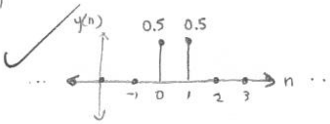
$x(n) = \delta(n)$ $\quad$ $y(n) = 0$ for $n > 1$

$y(n) = 0.5\,\delta(n) + 0.5\,\delta(n-1)$

$y(n) = 0$ for $n < 0$

$y(0) = 0.5\,\delta(0) + 0.5\,\delta(-1) = 0.5$

$y(1) = 0.5\,\delta(1) + 0.5\,\delta(0) = 0.5$



b) Is this system LTI?

linear? $\boxed{\text{YES}}$ because $y(n+a) = 0.5\,x(n+a) + 0.5\,x(n+a-1)$

Time Invariant? $\boxed{\text{YES}}$ because the function $y(n)$ does not depend on ~~continuous~~ time t.

c) Is this system stable? Is it memoryless?

Stable? $\boxed{\text{YES}}$ $\displaystyle\sum_{n=-\infty}^{\infty} |y(n)| = 1$ this value does not run in to $\pm\infty$ so the system has to be stable.

Memoryless? $\boxed{\text{NO}}$ because $y(n) = 0.5x(n) + 0.5x(n-1)$ $y(n)$ calls on a previous value of n

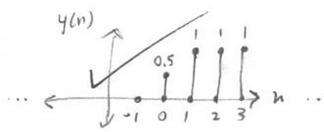
d) Determine $y(n)$ when $x(n)$ is a unit step function $u(n)$?

$x(n) = u(n)$ $\quad$ $y(1) = 0.5\,u(1) + 0.5\,u(0) = 1$

$y(n) = 0.5\,u(n) + 0.5\,u(n-1)$ $\quad$ $y(n) = 1$ for $n > 1$

$y(n) = 0$ for $n < 0$

$y(0) = 0.5\,u(0) + 0.5\,u(-1) = 0.5$



e) What is the output if the input is $x(n) = \delta(n) + \delta(n-1) + 2u(n-2)$. Sketch $y(n)$

$y(n) = 0.5\left[\delta(n) + \delta(n-1) + 2u(n-2)\right] + 0.5\left[\delta(n-1) + \delta(n-1-1) + 2u(n-2-1)\right]$

$y(n) = 0.5\,\delta(n) + 0.5\,\delta(n-1) + u(n-2) + 0.5\,\delta(n-1) + 0.5\,\delta(n-2) + u(n-3)$

$y(n) = 0.5\delta(n) + \delta(n-1) + 0.5\delta(n-2) + u(n-2) + u(n-3)$

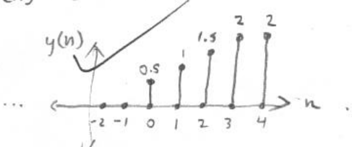
$y(n) = 0$ for $n < 0$ $\quad$ $y(n) = 2$ for $n > 3$

$y(0) = 0.5\,\delta(0) + \delta(-1) + 0.5\,\delta(-2) + u(-2) + u(-3) = 0.5$

$y(1) = 0.5\,\delta(1) + \delta(0) + 0.5\,\delta(-1) + u(-1) + u(-2) = 1$

$y(2) = 0.5\,\delta(2) + \delta(1) + 0.5\,\delta(0) + u(0) + u(-1) = 1.5$

$y(3) = 0.5\,\delta(3) + \delta(2) + 0.5\,\delta(1) + u(1) + u(0) = 2$



f) What is the output of the system if $x(n) = (-1)^n$?

$y(n) = 0.5\,(-1)^n + 0.5\,(-1)^{n-1}$

$y(-2) = 0.5(-1)^{-2} + 0.5(-1)^{-3} = 0$

$y(-1) = 0.5(-1)^{-1} + 0.5(-1)^{-2} = 0$

$y(0) = 0.5(-1)^{0} + 0.5(-1)^{-1} = 0$

$y(1) = 0.5(-1)^{1} + 0.5(-1)^{0} = 0$

$y(2) = 0.5(-1)^{2} + 0.5(-1)^{1} = 0$

No matter what value n is, the answer will be zero

$\boxed{y(n) = 0 \text{ for all } n}$

g) What does this system do?

$y(n)$ finds the ~~derivative~~ value

average!

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[4 marks] 5) Take the signal $x(t) = 1 + 3\sin(3\pi t) + 6\sin(12\pi t)$. Find the period and the exponential Fourier series coefficients of the signal.

$$x(t) = 1 + \frac{3}{2j}e^{j3\pi t} - \frac{3}{2j}e^{-j3\pi t} + \frac{6}{2j}e^{j12\pi t} - \frac{6}{2j}e^{-j12\pi t} \qquad \omega = 3\pi$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j\omega_0 k t} =$$

$$x(t) = \ldots + c_{-4}e^{j12\pi t} + c_{-3}e^{j9\pi t} + c_{2}e^{j6\pi t} + c_{-1}e^{j3\pi t} + c_0 + \ldots$$

$$\ldots + c_1 e^{-j3\pi t} + c_2 e^{-j6\pi t} + c_3 e^{-j9\pi t} + c_4 e^{-j12\pi t} + \ldots$$

$c_{-4} = \frac{3}{j}$ $\quad c_0 = 1$ $\quad c_4 = -\frac{3}{j}$

3.5

$c_{-3} = 0$ $\quad c_1 = -\frac{3}{2j}$

$c_{-2} = 0$ $\quad c_2 = 0$ $\quad T = \frac{2\pi}{\omega} = \boxed{\frac{2}{3}}$

$c_{-1} = \frac{3}{2j}$ $\quad c_3 = 0$

[2 marks] 6) (a) When are the even harmonics of a signal zero?

The even harmonics are zero ~~when~~ the coefficients of that signal satisfy ~~$a_k = -a_k = 0$~~

(b) When are all exponential Fourier series coefficients of a signal real?

The coefficients of an exponential Fourier series are real when they satisfy the proof

0.0

$$a_k = a_{-k}^*$$

where a_k is a coefficient and a_{-k}^* is its conjugate counterpart.

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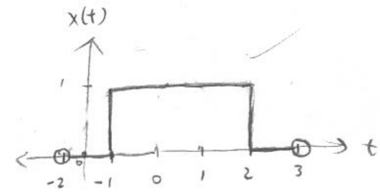
[4 marks] 7) Take the signal with a period 5 as given below

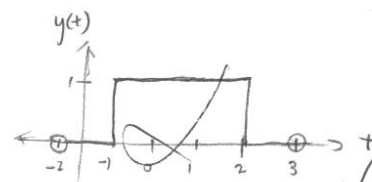
$$x(t) = (u(t+1) - u(t-2)) \text{ for } -2 < t < 3.$$

If $y(t) = \cos(2\pi t)x(t)$. Sketch $x(t)$ and $y(t)$. Find the exponential Fourier series coefficient of $y(t)$.

$x(-1) = u(0) - u(-3) = 1$
$x(0) = u(1) - u(-2) = 1$
$x(1) = u(2) - u(-1) = 1$
$x(2) = u(3) - u(0) = 0$



$y(-1) = \cos(-2\pi)(1) = 1$
$y(0) = \cos(0)(1) = 1$
$y(1) = \cos(2\pi)(1) = 1$
$y(2) = \cos(4\pi)(0) = 0$



0.8

$$a_0 = \frac{1}{T}\int_0^T y(t)\,dt = \frac{1}{5}\int_0^5 \cos(2\pi t)\,dt = \frac{1}{5}\left[-\sin(2\pi t)\right]_0^5$$

$$a_0 = 0$$

$$a_k = \frac{2}{T}\int_0^T y(t)\cos(2\pi k t)\,dt = \frac{2}{5}\int_0^5 \cos(2\pi t)\cos(2\pi k t)\,dt$$

$a_{-1} = 1$ $\quad a_1 = 1$ $\quad a_2 = 0$

$$b_k = \frac{2}{T}\int_0^T y(t)\sin(2\pi k t)\,dt = \frac{2}{5}\int_0^5 \cos(2\pi t)\sin(2\pi k t)\,dt$$

$b_{-1} = 0$ $\quad b_1 = 0$ $\quad b_2 = 0$

the coefficients were solved via HP 50g calculator.