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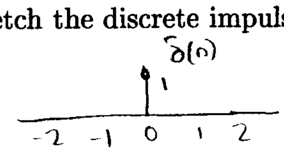
27.3 / 40

EE351.3 Mid-term test, Signals and Systems

8 Questions. Duration 80 minutes.
You may use a formula sheet.
Attempt all the questions. Justify your answers.

[1 marks] 1) Sketch the discrete impulse function $\delta(n)$.

$\delta(n)$



1.0

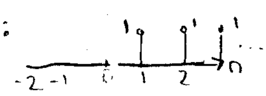
[1 marks] 2) What is the period of the signal $x(t) = e^{4jt}$?

$w_0 = 4 = \frac{2\pi}{T}$ $T = \frac{\pi}{2}$

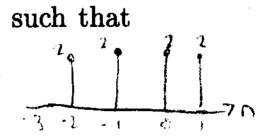
1.0

[3 marks] 3) Take the discrete-time signal $x(n) = (u(n-1) - 2u(n+2))$, where $u(n)$ is the unit step function. Sketch $x(n)$. Find an odd signal x_o and an even signal x_e such that $x(n) = x_o(n) + x_e(n)$.

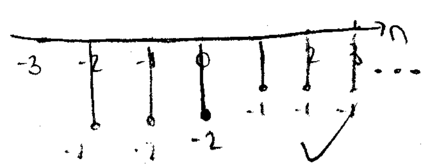
$u(n-1)$:



$2u(n+2)$:



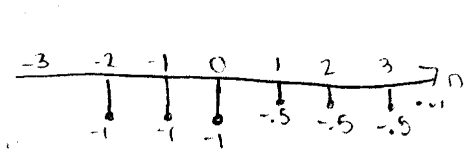
$x(n)$:



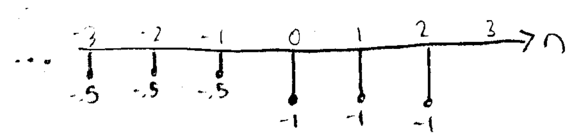
$x_o(n) = \frac{1}{2}x(n) - \frac{1}{2}x(-n)$

$x_e(n) = \frac{1}{2}x(n) + \frac{1}{2}x(-n)$

$\frac{1}{2}x(n)$:

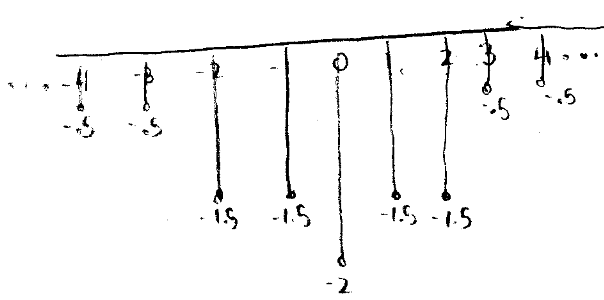


$+ \frac{1}{2}x(-n)$

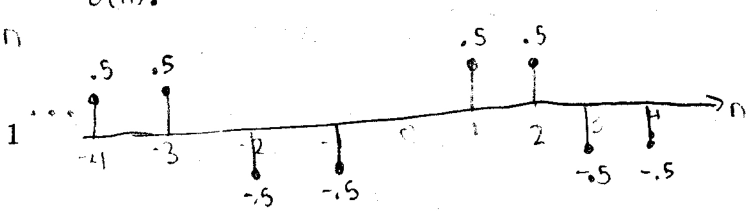


3.0

$x_e(n)$:

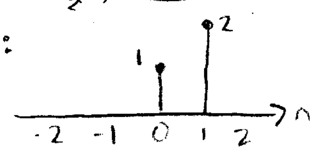


$x_o(n)$:



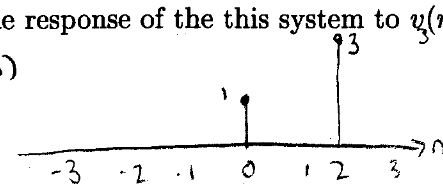
[6 marks] 4)Take a system with an impulse response $h(n) = \delta(n) + 2\delta(n-1)$. Find the response of the system to the input $x(n) = \delta(n) - 3\delta(n-2)$.
What is the response of the this system to $v_1(n) = e^{(j\pi n/2)}$? What is the response of the this system to $v_2(n) = e^{(-j\pi n/2)}$? What is the response of the this system to $v_3(n) = \cos(\pi n/2)$?

$h(n)$:



$\{\underline{1}, 2\}$

$x(n)$



$\{\underline{1}, 0, 3\}$

response of $x(n)$: $y(n) = (1+2z^{-1})(1+3z^{-2})$
$= 1 + 2z^{-1} + 3z^{-2} + 6z^{-3} = \{\underline{1}, 2, 3, 6\}$

5.2

response to $v_2(n)$: $v_2(n) = \{\ldots, e^{\frac{j\pi}{2}}, \underline{1}, e^{-\frac{j\pi}{2}}, e^{-j\pi}, e^{-j\frac{3\pi}{2}} \ldots\}$

$y(n) = v_1(n) * h(n)$
$= (1+2z^{-1})(\ldots + e^{\frac{j\pi}{2}} z^{1} + 1 + e^{-\frac{j\pi}{2}} z^{-1} + e^{-j\pi} z^{-2} + e^{\frac{-j\pi 3}{2}} z^{-3})$
$= \ldots + e^{\frac{j\pi}{2}} z^{1} + 1 + e^{-\frac{j\pi}{2}} z^{-1} + e^{-j\pi} z^{-2} + e^{-\frac{j3\pi}{2}} z^{-3} + \ldots$
$\ldots + 2e^{\frac{j\pi}{2}} + 2z^{-1} + 2e^{-\frac{j\pi}{2}} z^{-2} + 2e^{-j\pi} z^{-3} + 2e^{\frac{-j\pi 3}{2}} z^{-4} + \ldots$

$v_2(n) \to y(n) = \{\ldots, e^{\frac{j\pi}{2}}, \underline{1+2e^{\frac{j\pi}{2}}}, 2+e^{-\frac{j\pi}{2}}, 2e^{-\frac{j\pi}{2}}+e^{-j\pi}, 2e^{-j\pi}+e^{-\frac{j3\pi}{2}}, + \ldots\}$

simplify.

response to $v_1(n)$: $y(n) = \{\ldots, e^{-\frac{j\pi}{2}}, \underline{1+2e^{-\frac{j\pi}{2}}}, 2+e^{\frac{j\pi}{2}}, 2e^{\frac{j\pi}{2}}+e^{j\pi}, 2e^{j\pi}+e^{\frac{j3\pi}{2}}, + \ldots\}$

$\cos\left(\frac{\pi n}{2}\right) = \frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2}$, so $v_3(n) = \frac{v_1(n) + v_2(n)}{2}$

$2\cos x = e^{jx} + e^{-jx}$
$4\cos x = 2e^{jx} + 2e^{-jx}$

$v_3(n) = \frac{1}{2}\{\ldots, 2\cos\left(\frac{\pi}{2}\right), \underline{2 + 4\cos\left(\frac{\pi}{2}\right)}, 4 + 2\cos\left(\frac{\pi}{2}\right), 4\cos\left(\frac{\pi}{2}\right) + 2\cos(\pi), 4\cos(\pi) + 2\cos\left(\frac{3\pi}{2}\right), \}$

6.0

[7 marks] 5) A discrete-time system has the relationship

$$y(n) = 0.5nx(n)$$

between its input $x(n)$ and its output $y(n)$.

a) Determine if the system is linear

try $x(n) = a x(n)$ $\quad y_1(n) = 0.5 a x(n)$

try $x(n) = b x(n)$ $\quad y_2(n) = 0.5 b x(n)$

$y_1(n) + y_2(n) = 0.5 a x(n) + 0.5 b x(n)$

try $x(n) = a x(n) + b x(n)$ $\quad y(n) = 0.5n(a x(n) + b x(n)) = 0.5 n a x(n) + 0.5 b x(n)$

same, therefore linear

b) Is this system is TI?

no, because of $y(n) = 0.5 n x(n)$. The output depends on the time of the input, so it is therefore not TI.

c) Is this system stable? Is it memoryless?

not stable, since $y(n)$ goes up as the n goes up. (BI → X BO). It is memoryless, since output has no memory of past outputs.

d) Is this system causal?

it is causal, since current output does not depend on future inputs

e) What is the inverse of this system

$y^{-1}(n) = \frac{2}{n\, x(n)}$ ~~n=0~~ !

f) Similarly take a continuous-time system with the relationship

$$y(t) = 0.5x(t) + 1$$

Is this system LTI? Is it stable?

Linear? $x(t) = a x(t)$ $\quad y_1(t) = 0.5 a x(t) + 1$

$x(t) = b x(t)$ $\quad y_2(t) = 0.5 b x(t) + 1$

$y_1(t) + y_2(t) = 0.5 a x(t) + 1 + 0.5 b x(t) + 1 = 0.5 a x(t) + 0.5 b x(t) + 2$

try $x(t) = a x(t) + b x(t)$

$y(t) = 0.5(a x(t) + b x(t)) + 1 = 0.5 a x(t) + 0.5 b x(t) + 1$

not equal, so not linear

It is time invariant, since output does not depend on time of input.

it is stable. (BI → BO ✓)

[5 marks] 6) Take the signal $x(t) = 1 + 3\sin(4\pi t) + 2\cos(12\pi t)$. Find the period and the exponential Fourier series coefficients of the signal. Find the Fourier series coefficients of $x(-t)$.

$3\sin(4\pi t)$ $\downarrow$ $\frac{3}{2j}e^{4\pi t} - \frac{3}{2j}e^{-4\pi t}$

$\omega_0 = 4\pi = \frac{2\pi}{T}$, $T = \frac{1}{2}$

$\omega_0 = 12\pi = \frac{2\pi}{T}$, $T = \frac{1}{6}$

$2\cos(12\pi t)$ $\downarrow$ $e^{j12\pi t} + e^{-j12\pi t}$

common period is $\frac{1}{2}$

$x(t)$: $a_0 = 1$

$x(t) = \sum a_k e^{j\omega_0 k t}$

$\omega_0 = 4\pi$

$a_{-3} = 1$ $a_3 = 1$ $a_{-1} = \frac{-3}{2j}$ $a_1 = \frac{3}{2j}$ $a_0 = 1$

5.0

$x(-t)$: $x(t) \to a_k$

$x(-t) \to b_k$

$b_k = a_{-k}$

$a_{-3} = 1$ $a_3 = 1$ $a_{-1} = \frac{3}{2j}$ $a_1 = \frac{-3}{2j}$ $a_0 = 1$

[1 marks] 7) (●) When are all exponential Fourier series coefficients of a signal pure imaginary?

$a_k \to$ imaginary if signal is odd

1.0

$a_0 \to$ not imaginary, unless system is in the imaginary plane

∴, so, all coefficients are imaginary if the signal is odd, and present in the imaginary plane

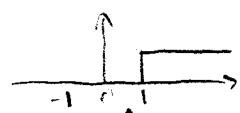
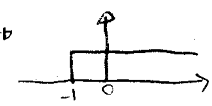
[6 marks] 8) Take the signal with a period 6 as given below

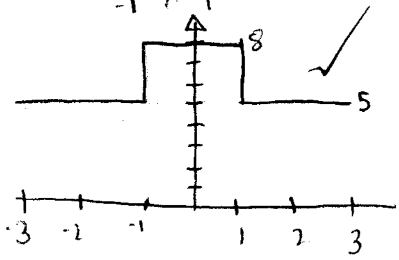
$$x(t) = (5 + 3u(t+1) - 3u(t-1)) \text{ for } -3 < t < 3.$$

Sketch $x(t)$. Find the exponential Fourier series coefficient of $x(t)$. Assume that

$$y(t) = x(t-3) - 6,$$

Plot $y(t)$. what are the Fourier series coefficients of $y(t)$?

$u(t-1) \rightarrow$  $u(t+1) \rightarrow$ 

$x(t)$: 

$T = 6 \quad \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3} \quad a_0 = 5 + \frac{1}{6} 2(1)$

$T_1 = 1 \qquad = 5 + \frac{1}{3} = \boxed{\cancel{\frac{16}{3}} = a_0}$

$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} = \frac{3\sin(\frac{k\pi}{3})}{k\pi} \quad k \neq 0$

$y(t)$: $b_0 = 5 - 6 + \frac{1}{6}(2)$

$= -1 + \frac{1}{3}$

$\boxed{b_0 = \frac{-2}{3}}$

$x(t) \xrightarrow{F.S.} a_k$

$x(t-3) - 6 \xrightarrow{F.S.} b_k$

$b_k = a_k e^{-3jk\omega_0} \quad k \neq 0$

$b_k = \frac{e^{-jk\pi} \sin(\frac{k\pi}{3})}{k\pi} \quad k \neq 0$

$\sin(\frac{k\pi}{3}) = \frac{e^{\frac{jk\pi}{3}} - e^{-\frac{jk\pi}{3}}}{2j}$

$e^{-jk\pi} \sin(\frac{k\pi}{3}) = \frac{e^{\frac{jk\pi}{3} - jk\pi} - e^{-\frac{jk\pi}{3} - jk\pi}}{2j}$

$= \frac{e^{-\frac{2k\pi j}{3}} - e^{-\frac{4}{3}k\pi j}}{2j}$

doesn't really simplify things...

5.1