

Assignment Quiz 7
November 21, 2001

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name:
Student Number:

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1. When the input to an LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n-1],$$

the corresponding output is

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n].$$

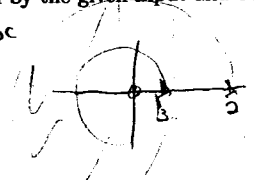
- Find the system function $H(z)$ of the system. Plot the pole(s) and zero(s) of $H(z)$ and indicate the region of convergence.
- Find the impulse response $h[n]$ of the system.
- Write a difference equation that is satisfied by the given input and output.
- Is the system stable? Is it causal? ROC

a)

$$Y(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{-1}{1 - 2z^{-1}}$$

$$= \frac{z - 2z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}$$

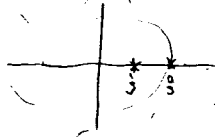
$$= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}$$



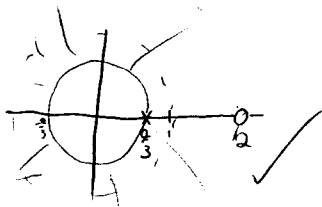
$$y(z) = \frac{5}{1 - \frac{1}{3}z^{-1}} + \frac{-5}{1 - 2z^{-1}}$$

$$= \frac{8 - \frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}$$

$$= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}$$



$$H(z) = \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})} \cdot \frac{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}{-\frac{5}{3}z^{-1}}$$

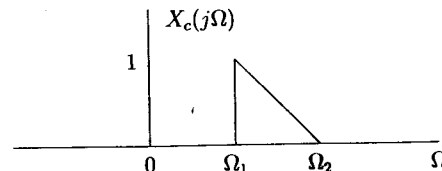


$$H(z) = \frac{1 - 2z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

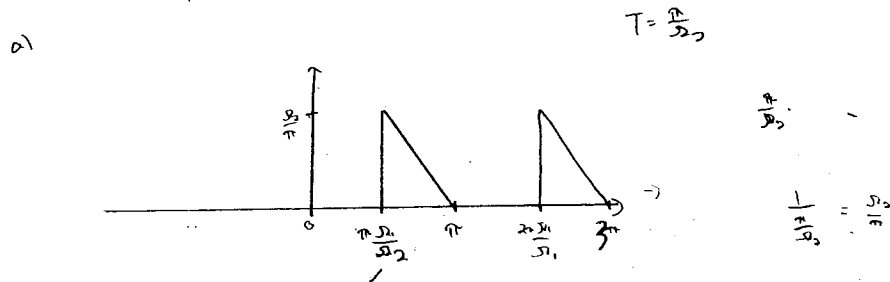
ROC
 $|z| > \frac{2}{3}$

b) $h[n] = \frac{1}{3} - 2^n$

1. A complex-valued continuous-time signal, $x_c(t)$, has the Fourier transform shown in the following figure. The signal is sampled to produce the sequence $x[n] = x_c(nT)$.



- Sketch the Fourier transform, $X(e^{j\omega})$, of the sequence $x[n]$ for $T = \pi/\Omega_2$.
- What is the lowest sampling frequency that can be used without incurring any aliasing distortion, i.e., so that $x_c(t)$ can be recovered from $x[n]$. Show your work. Sketch $X(e^{j\omega})$ using this sampling frequency.
- Draw the block diagram of a system that can be used to recover $x_c(t)$ from $x[n]$ if the sampling rate is greater than or equal to the rate determined in part b). Assume that (complex) ideal filters are available.



b)

$$\frac{1}{T} \geq \frac{\Omega_2}{2\pi}$$

$$\Omega_s \geq \Omega_2 - \Omega_1$$

$$f_s \geq \frac{\Omega_2}{2\pi}$$

