

EE456: Digital Communications
MIDTERM EXAMINATION, 5:00PM–7:00PM, October 22, 2008
(2 hours, closed book)

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Permitted Materials: Non-Programmable Calculator

Note: There are 3 questions. All questions are of equal value (with part marks indicated) but not necessarily of equal difficulty. Full marks shall only be given to solutions that are properly explained and justified.

1. The noise $\mathbf{x}(t)$ applied to the filter in Figure 1 is modeled as a wide-sense stationary random process with power spectral density $S_x(f)$. Let $\mathbf{y}(t)$ denote the random noise process at the output of the filter.

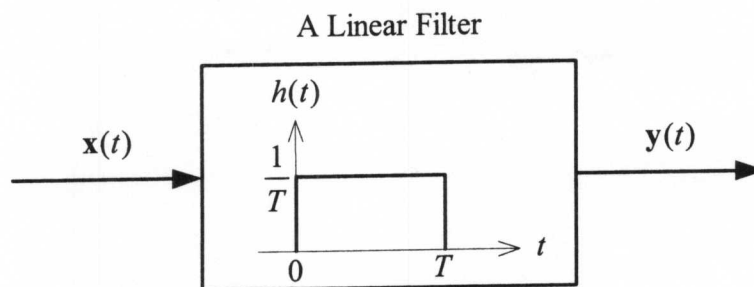


Figure 1: System under consideration in Question 1.

- [1] (a) Is $\mathbf{y}(t)$ a wide-sense stationary noise process? Why?
[3] (b) Show that the frequency response of the filter in Figure 1 is:

$$H(f) = \frac{\sin(\pi f T)}{\pi f T} e^{-j\pi f T}.$$

If the mean value of the input noise is m_x , what is the mean value of the output process?

- [2] (c) If $\mathbf{x}(t)$ is a *white* noise process with power spectral density $N_0/2$, find and sketch the power spectral density of the noise process $\mathbf{y}(t)$.
[2] (d) What frequency components cannot be present in the output process? Explain.
[2] (e) Find the autocorrelation function of the output process. Then suppose that the output noise is sampled every T_s seconds to obtain the noise samples $\mathbf{y}(kT_s)$, $k = 0, 1, 2, \dots$. Find the smallest value of T_s so that the noise samples are *uncorrelated*.
Hint: The $\text{sinc}^2(\cdot)$ and triangular functions form a Fourier transform pair:

$$\begin{cases} 1 - \frac{|t|}{T}, & \text{for } |t| \leq T \\ 0, & \text{for } |t| > T \end{cases} \xleftrightarrow{\mathcal{FT}} T \left[\frac{\sin(\pi f T)}{\pi f T} \right]^2.$$

2. Consider the following signal set for binary data transmission over a channel disturbed by additive white Gaussian noise:

$$s_1(t) = \begin{cases} \sqrt{3}V \cos\left(\frac{2\pi t}{T_b}\right), & 0 \leq t \leq T_b \\ 0, & \text{otherwise} \end{cases}, \quad (1)$$

$$s_2(t) = \begin{cases} V \sin\left(\frac{2\pi t}{T_b}\right), & 0 \leq t \leq T_b \\ 0, & \text{otherwise} \end{cases}. \quad (2)$$

The noise is zero-mean and has two-sided power spectral density $N_0/2$. As usual, $s_1(t)$ is used for the transmission of bit "0" and $s_2(t)$ is for the transmission of bit "1". Furthermore, the two bits are equally likely.

- [2] (a) Show that $s_1(t)$ is *orthogonal* to $s_2(t)$. Then find and draw an orthonormal basis $\{\phi_1(t), \phi_2(t)\}$ for the signal set. *Hint:* You can show the orthogonality by graphically sketching $s_1(t)$ and $s_2(t)$ and explaining, or by performing integration.
- [2] (b) Draw the signal space diagram and the optimum decision regions. Write the expression for the optimum decision rule.
- [2] (c) Let $V = 1$ volt and assume that $N_0 = 10^{-8}$ watts/Hz. What is the maximum bit rate that can be sent with a probability of error $P[\text{error}] \leq 10^{-5}$.
- [2] (d) Draw the block diagram of an optimum receiver that uses only one matched filter. Give the precise expression for the impulse response of the matched filter.
- [2] (e) Assume that, as long as the average energy of the signal set $\{s_1(t), s_2(t)\}$ stays the same, you can freely change (or move) both $s_1(t)$ and $s_2(t)$ in the same signal space. Modify them so that the probability of error is as small as possible. Explain your answer.

