

EE456: Digital Communications
MIDTERM EXAM, 5:30PM–7:30PM, October 21, 2009
(2 hours, open-textbook)
Examiner: Ha H. Nguyen

Note: There are 3 questions. All questions are of equal value (with part marks indicated) but not necessarily of equal difficulty. Full marks shall only be given to solutions that are properly explained and justified.

1. The noise $x(t)$ applied to the filter in Figure 1 is modeled a zero-mean white Gaussian random process with power spectral density $S_x(f) = \frac{N_0}{2}$. Let $y(t)$ denote the noise process at the output of the filter.

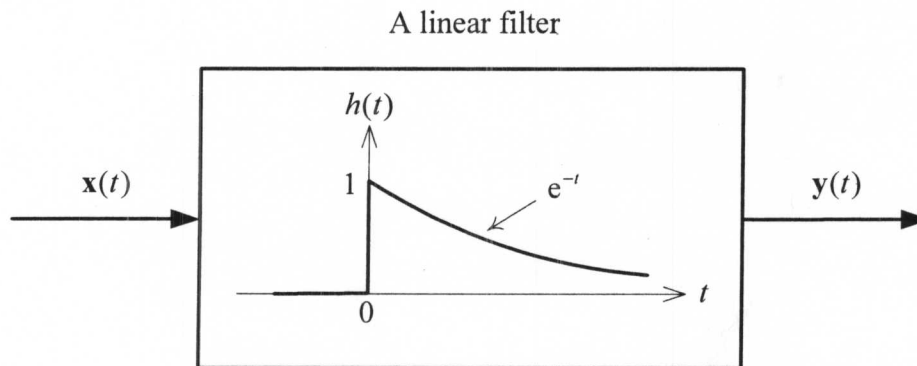


Figure 1: System under consideration in Question 1.

- [2] (a) Show that the frequency response of the filter in Figure 1 is:

$$H(f) = \frac{1}{1 + j2\pi f}$$

- [3] (b) Find and sketch the power spectral density, $S_y(f)$, and the autocorrelation function, $R_y(\tau)$, of the output noise process. *Hint:* $\mathcal{F}^{-1} \left\{ \frac{2a}{a^2 + (2\pi f)^2} \right\} = e^{-a|t|}$, $a > 0$. What is the total power in the output noise process?
- [3] (c) Find the value of “bandwidth” W that captures 99% of the total power in the output noise process. *Hint:* $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$.
- [2] (d) Assume that $N_0 = 1$ watt/Hz. Your friend is especially interested in the output noise amplitude at $t = 100$ msec after the system is turned on. Before her actual observation, what is the probability that she will record an amplitude value between 1 volt and 1.5 volts? Does your answer change if your friend is going to observe output noise at $t = 10$ sec, at $t = 101$ msec? Explain your answer.

Hint:

x	1	2	3	4	5
$Q(x)$	1.59×10^{-1}	1.59×10^{-1}	2.27×10^{-2}	1.35×10^{-3}	3.17×10^{-5}

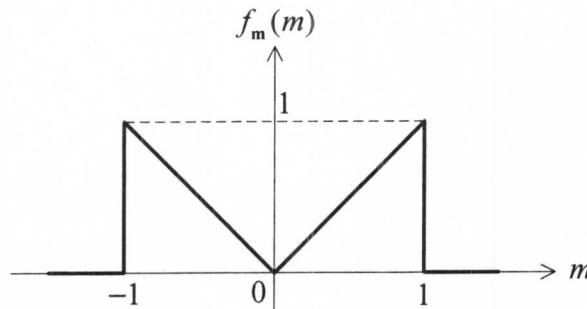


Figure 2: Message pdf considered in Question 2.

2. Consider a message source with the amplitude pdf shown in Figure 2.
 - [2] (a) Determine the average power of the message signal.
 - [3] (b) Determine the target and decision levels and calculate the SNR_q for the one-bit *uniform* quantizer ($L = 2$ levels).
 - [3] (c) Repeat (b) for the one-bit *optimum* quantizer. Compare the SNR_q result with that in (b) and comment.
 - [2] (d) Obtain and solve the set of equations for the target and decision levels of the two-bit *optimum* quantizer ($L = 4$ levels).

3. Consider the signal space diagram in Figure 3, where the waveforms of the two basis functions are also shown. The signal set is used for data transmission over an AWGN channel in which the noise is zero-mean and has two-sided power spectral density of $N_0/2$. As usual, $s_1(t)$ is used for the transmission of bit "0" and $s_2(t)$ is for the transmission of bit "1". Furthermore, the two bits are equally likely and the bit duration is T_b .
 - [2] (a) Determine and neatly sketch the two signals $s_1(t)$ and $s_2(t)$. Without performing the integration but by observing Figure 3, can you tell whether $s_1(t)$ and $s_2(t)$ are *orthogonal*? Why?
 - [2] (b) Clearly draw the optimum decision boundary and label the two decision regions. Write the expression for the optimum decision rule in terms of r_1 and r_2 , the projections of the received signal on $\phi_1(t)$ and $\phi_2(t)$. Simplify the decision rule as much as you can.
 - [2] (c) Let $V = 1$ volt and assume that $N_0 = 10^{-8}$ watts/Hz. What is the maximum bit rate that can be sent with a probability of error $P[\text{error}] \leq 10^{-5}$. *Hint:* $Q^{-1}(10^{-5}) \approx 4.25$.
 - [2] (d) Draw the block diagram of an optimum receiver that uses only one matched filter. Determine and plot the impulse response of the matched filter. *Note:* You do not need to determine the exact value of the threshold for the comparison block.
 - [2] (e) Assume that, as long as the energy of $s_1(t)$ is the same, you can freely change its location (i.e., its shape) in the same signal space. Modify $s_1(t)$ so that the probability of error is as small as possible. Explain your answer.

