

EE461 Midterm:NAME: Craig Bloch-HansenSTUDENT NO.: 10147742

Date: Tuesday, October 21, 2008

Time = 1.5 hours

Two 8.5 by 11 formula sheets (writing both sides)
(i.e. no Text Books or Notes)

Absolutely no worked examples or solved problems

NB: Draw a box around your final answer.

1. 0 (2)
2. 2 (4)
3. 4 (6)
4. 4 (4)
5. 5 (6)
6. 1 (5)
7. 3 (3)
8. 2 (3)
- TOTAL 21 (33)

- (2) 1. Find the steady state output (call it $y(n)$) of a system that has frequency response

$$H(e^{j\omega}) = -((\omega)^2 + j\omega)^2; \quad -2 < \omega < 2,$$

if the input to the system is

$$x(n) = \cos(n) + 2 \sin(\sqrt{2}n)$$

$$x(n) = \cos(n) + 2 \cos(\sqrt{2}n + \pi)$$

$$\omega = 1$$

$$\text{at } \omega = 1 \\ H(e^{j\omega}) = -((1)^2 + j(1))^2$$

$$= -(1 + j)^2$$

$$= -(1 + 2j - 1)$$

$$= -2j = \frac{2}{j}$$

$$y(n) = \frac{2}{j} \cos(n) + \frac{1}{2} \sin(\sqrt{2}n) \quad \times$$

2. A single pole low-pass filter has its pole at $z = 0.85 + j0.0$. The pole location is shown in Figure 1.

- (2) (a) Find the approximate bandwidth of the filter (i.e. the 3 dB down frequency) using a tangential approximation. Answer in the space provided below and give your answer in units of radians/sample.

$$BW \approx 1 - r \quad r \approx 0.85$$

$$BW \approx 1 - 0.85 \approx \underline{0.15 \text{ rad/sample}} \quad \checkmark$$

- (2) (b) Find the actual bandwidth (to the accuracy possible) of the filter graphically using Figure 1. Show the construction lines and clearly mark the point on the unit circle where the filter response is 3 dB down. Give your answer in units of radians/sample.

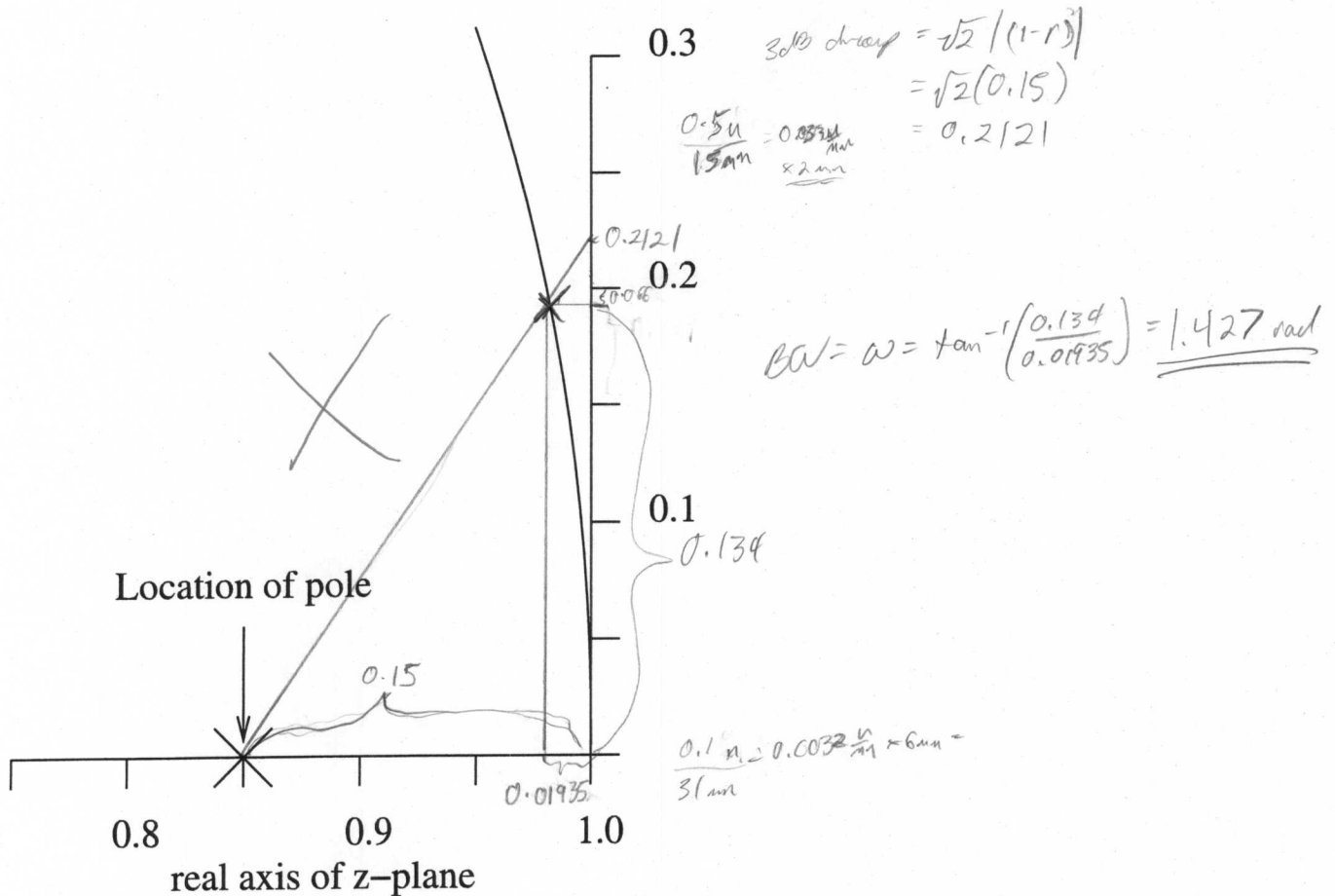


Figure 1: z -plane near $z=1+j0$

3. Questions 3a, 3b and 3c are not really parts of the same question. They are grouped together because they are all from assignment 3.

(2) (a) Suppose you are designing a IIR low-pass filter with a monotonic pass band and ripple in the stop band to the following specification:

- $\omega_p = \pi/4$ radians/sample
- $\omega_s = 3\pi/8$ radians/sample
- $\delta_p = 0.05$
- $\delta_s = 10^{-3}$

What Matlab command would you use to find the order of the filter?

$$\text{ord}(\omega_p, \omega_s, \delta_p, \delta_s)$$

X

(2) (b) Prove the following equality.

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi k}{N}n + \phi\right) = 0; \quad \text{for } k = 1, 2, \dots, N-1$$

$$\begin{aligned} & \frac{1}{2} \sum_{n=0}^{N-1} \frac{e^{j\left(\frac{2\pi k}{N}n + \phi\right)} + e^{-j\left(\frac{2\pi k}{N}n + \phi\right)}}{2} \\ &= \frac{1}{2} \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi k}{N}n + \phi\right)} + \frac{1}{2} \sum_{n=0}^{N-1} e^{-j\left(\frac{2\pi k}{N}n + \phi\right)} \\ &= \frac{1}{2} e^{j\phi} \sum_{n=0}^{N-1} \left(e^{j\frac{2\pi k}{N}n}\right) + \frac{1}{2} e^{-j\phi} \sum_{n=0}^{N-1} \left(e^{j\frac{2\pi k}{N}n}\right) \\ &= \frac{1}{2} e^{j\phi} \left(\frac{e^{j\frac{2\pi k}{N}(0)} - e^{j\frac{2\pi k}{N}(N)}}{1 - e^{j\frac{2\pi k}{N}}} \right) + \frac{1}{2} e^{-j\phi} \left(\frac{e^{j\frac{2\pi k}{N}(0)} - e^{j\frac{2\pi k}{N}(N)}}{1 - e^{j\frac{2\pi k}{N}}} \right) \\ &= \frac{1}{2} e^{j\phi} \left(\frac{1 - e^{j2\pi k}}{1 - e^{j\frac{2\pi k}{N}}} \right) + \frac{1}{2} e^{-j\phi} \left(\frac{1 - e^{j2\pi k}}{1 - e^{j\frac{2\pi k}{N}}} \right) \end{aligned}$$

for any positive integer k $e^{j2\pi k} = 1$
 $\therefore 1 - e^{j2\pi k} = 0 \therefore$ the sum $= 0$

