

**University of Saskatchewan**  
**EE 480.3 Digital Control Systems**  
**Midterm Exam. February 10, 2003**

Note: 2 hour open-book exam.

Instructor: K. Takaya

1. (35) Answer the following questions.

1. Find the z-transform of

$$y_1(k) = a^{k+1}u(k-2) \quad \text{where } a > 0.$$

2. Find the starred transform  $G^*(s)$  of

$$G(s) = \frac{1}{s(s+1)}$$

3. Draw a simulation diagram of the following system  $G(z)$  in standard canonical form. Derive the state equation and output equation from the obtained simulation diagram.

$$G(z) = \frac{z^2 + 0.4z - 0.05}{z^2 - 0.7z + 0.1}$$

2. (35) A servo control system is shown in Fig. 2a, in which  $\theta_r$  and  $\theta$  are reference angle and actual angle, respectively. Fig. 2b shows the frequency response (Bode diagram) of this system showing a small phase margin of approximately  $45^\circ$ . Design a phase lead compensator, which increases the phase margin to  $70^\circ$ . The first order compensator is given by

$$G_c(s) = K_c \frac{s + s_z}{s + s_p}$$

where  $s_z < s_p$ . Allow  $10^\circ$  to account for an expected phase decrease due to the upward shifting of the new cross-over frequency. Determine  $K_c$ ,  $s_z$  and  $s_p$ .

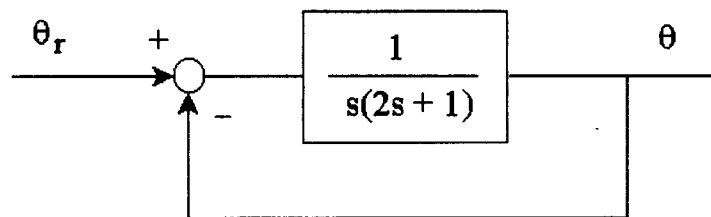


Fig. 2a Block diagram of an uncompensated control system

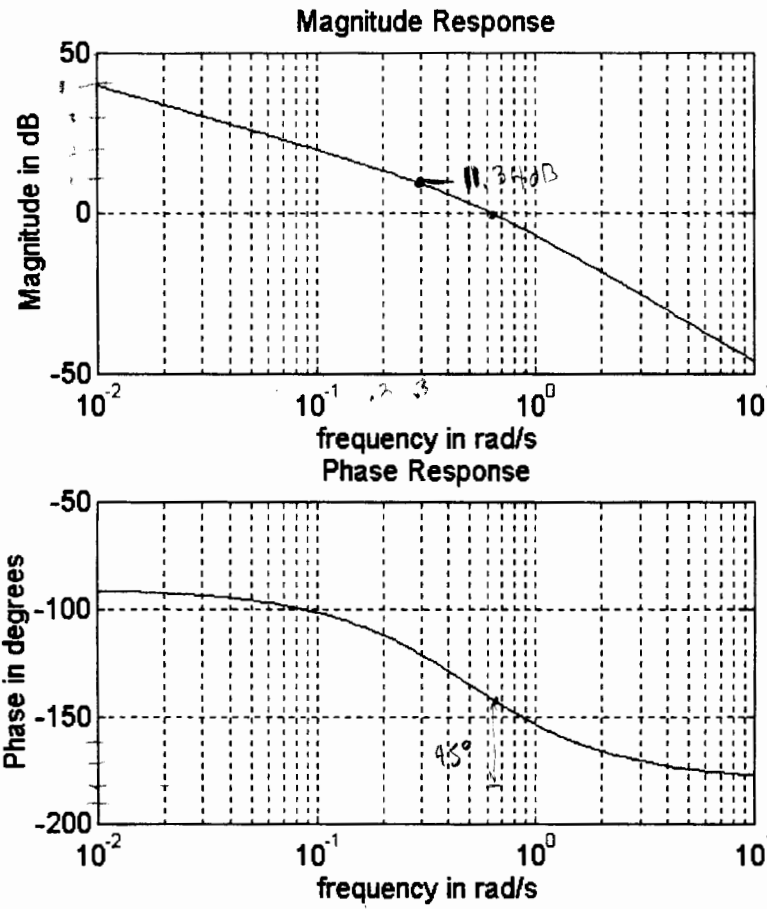


Fig. 2b Frequency Response of the Phase-lead Compensator

3. (30) Given a difference equation,

$$y(k) - y(k - 1) + 0.16y(k - 2) = x(k),$$

which produces output  $y(k)$  from input signal  $x(k)$ ,

1. Obtain the output sequence  $y(k)$  for a unit step input  $x(k) = u(k)$  and  $y(k) = 0$  for  $k < 0$  by using the z-transform.
2. Find the final value of  $y(k)$ , i.e.  $y(\infty)$ , then check if your solution for (1) agrees with the final value.

— The End —