

**University of Saskatchewan**  
**EE 480.3 Digital Control Systems**

**Final Examination, April 14, 2010**

**3 hour open-book exam**

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1. (25) A continuous time system is defined as follows for input  $x(t)$  and output  $y(t)$ .

$$Y(s) = G(s)X(s), \quad G(s) = \frac{1}{s(s+1)(s+2)}.$$

We apply unity gain negative feedback to this system, and adjust gain  $K$ . The discrete time model of this system with zero-order-hold ZOH for a sampling time  $T_s = 0.01$  sec. is calculated as

$$G(z) = \frac{1.6542 \times 10^{-7}(z + 0.2659)(z + 3.7042)}{(z - 1)(z - 0.9900)(z - 0.9802)}.$$

1. Show if the unity gain feedback applied to  $G(s)$ , i.e.  $\frac{KG(s)}{1 + KG(s)}$  is stable or not by a method to check stability.
2. The discrete time transfer function  $G(z)$  was mapped to the  $w$ -domain by the bilinear transformation as given below. Apply Ruth-Hurwitz's stability criterion to find the stable range of the gain  $K$ .

$$G(w) = \frac{-0.005w + 1}{w^3 + 3w^2 + 2w}$$

2. (25) A second order digital control system,  $G(s) = \frac{1}{s(s+1)}$  is shown in Fig. P2 (a). The discrete model of the open loop system combined with a ZOH (Zero Order Hold) has been calculated for  $T = 0.1$  as follows:

$$G(z) = 0.0048 \frac{z + 0.9672}{(z - 1)(z - 0.9048)}.$$

The root locus was plotted for the following compensated system with a phase lead compensator having a pole at  $z = 0$  and a zero at  $z = 0.7$ .

$$G_c(z) = K \frac{(z + 0.9672)(z - 0.7)}{z(z - 1)(z - 0.9048)}.$$

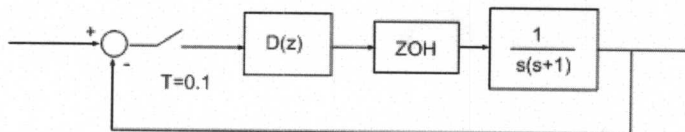


Fig. P2 (a) Block diagram of a control system

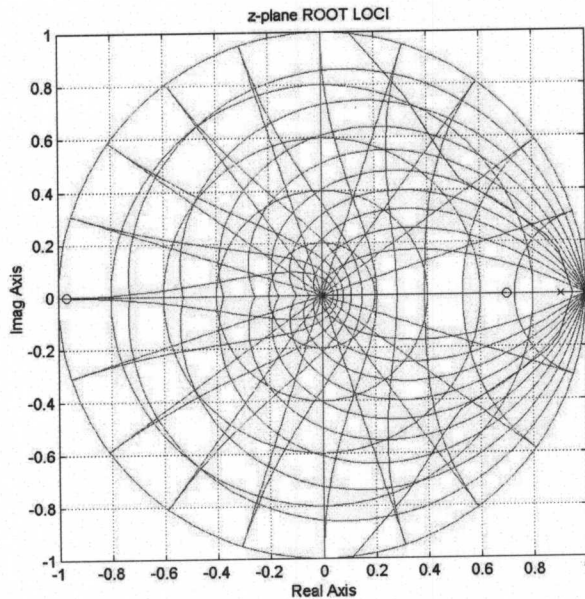


Fig. P2 (b) Root Locus plot of the phase lead compensated system

At the gain  $K = 0.323$ , the closed loop poles found from the root locus are  $z = 0.4608$  and  $z = 0.5605 \pm j0.4005$ . Calculate the following values with respect to the dominant poles of the closed loop system.

1. Damping ratio  $\zeta$
  2. % Overshoot
  3. Time constant  $\tau$
  4. Natural frequency  $\omega_n$
  5. Damped natural frequency  $\omega_d$
  6. Settling time for 3% criterion  $T_s$
3. (25) An analog system with the transfer function,

$$G(s) = \frac{1}{s^2 + s + 1}$$

has state feedback to form a feedback control system as shown in Fig. P3. The state variables are  $x_1$  and  $x_2$ . The sampling rate is 10 samples per second.

1. Using the state variables defined by  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ , obtain the analog state equation of  $G(s)$  which takes the form of

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t)$$

and the output equation which gives  $y(t)$ .

2. Obtain the state transition matrix  $\Phi(t)$  from  $\mathbf{A}$ .
3. Obtain the discrete time state equation expressed in the form of

$$\mathbf{x}(k+1) = \mathbf{P}\mathbf{x}(k) + \mathbf{q}u(k)$$

To calculate  $\mathbf{P}$  and  $\mathbf{q}$ , consider only up to the second order term of the Taylor series expansion,

$$\mathbf{P} = \Phi(T) = \mathbf{I} + \mathbf{A}T + \mathbf{A}^2 \frac{T^2}{2!} + \mathbf{A}^3 \frac{T^2}{3!} + \dots$$

4. Determine the state feedback gain vector  $\mathbf{K} = [K_1, K_2]^T$  that makes the system in Figure 2 critical damping with the two poles at  $z = 0.9$  by using the Ackermann's formula.

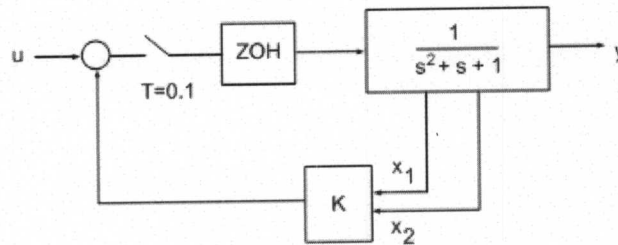


Fig. P3 A state feedback control system

4. (25) An antenna positioning system shown in Fig. P4 (a) consists of a physical system,

$$G(s) = \frac{100}{s(s+6)}$$

and a first order digital compensator given by

$$D(z) = K_d \frac{z - z_0}{z - z_p} \iff D(w) = \frac{a_1 w + a_0}{b_1 w + 1}$$

The digital compensator  $D(z)$  is transformed by the bilinear transformation into the  $w$ -domain as  $D(w)$  to facilitate design.

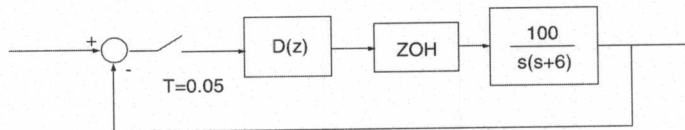


Fig. P4 (a) Block diagram of an antenna control system

The antenna positioning system preceded with a ZOH (Zero Order Hold) is described by the step invariant  $z$ -transform as

$$G(z) = 0.1134 \frac{z + 0.9049}{(z - 1)(z - 0.7408)}$$