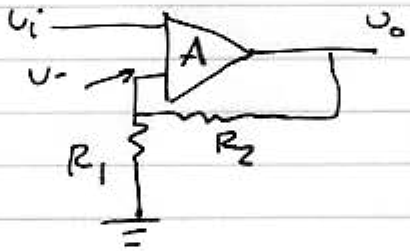
Controls Review

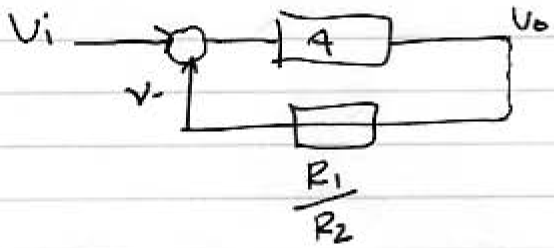
Sensitivity:

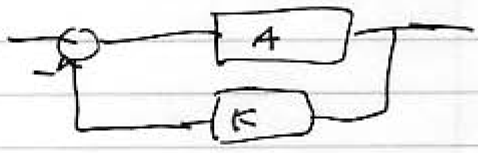
$$S_P^T = \frac{\frac{\partial T}{\partial P}}{\frac{T}{P}}$$



$A_1 = 10\,000$ $\qquad \frac{R_1}{R_2} = 0.1$

$A_2 = 5000$





$$T_1 = \frac{A}{1+Ak} = \frac{10^4}{1+10^4 K{=}0.1} = \frac{10\,00}{1+10^3} \approx \frac{10\,000}{1001} \approx 10$$

$$T_2 = \frac{5000}{501} \approx 10$$

$$\frac{\partial T}{\partial P} = \frac{1+Ak-kA}{(1+kA)^2} = \frac{1}{(1+kA)^2}$$

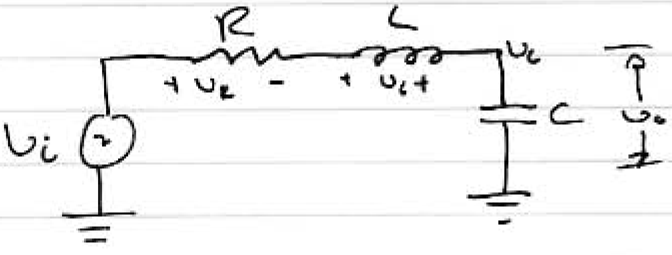
$$\frac{T}{A} = \frac{1}{1+Ak}$$

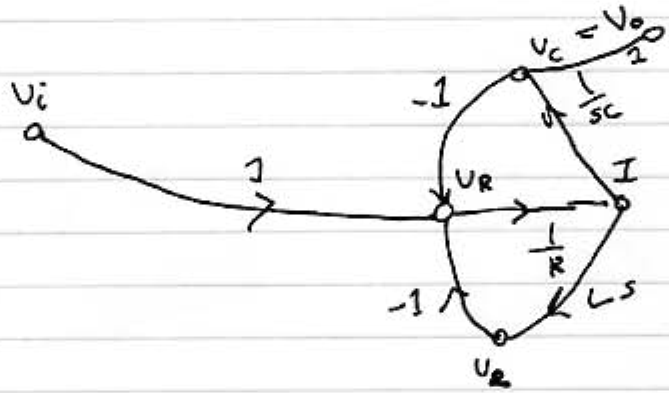
$$S_A^T = \frac{1}{1+kA} = \frac{1}{1+0.1\times10^4} = \frac{1}{1001} \approx 0.001 \qquad \frac{1}{501}$$

$$S_A^T = \frac{\frac{\Delta T}{\Delta A}}{\frac{T}{A}} = \qquad \Delta T = S_A^T\,\Delta A\,\frac{T}{A}$$

$$= 001(-5000)\frac{1}{1+10^4(01)} \approx 0.005$$

It K changes then the Tfcn is changed more $\therefore$ more sensitive.



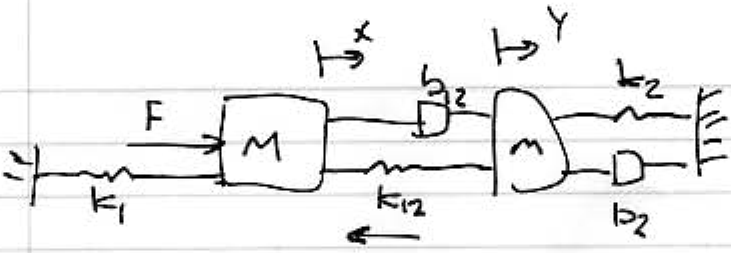


$$P_1 = \frac{1}{RCs} \qquad \Delta_1 = 1$$

$$L_1 = -\frac{Ls}{R} \qquad L_2 = -\frac{1}{sCR}$$

$$\frac{V_c}{V_i} = \frac{\frac{1}{RCs}}{1-\left[\frac{-Ls}{R} - \frac{1}{sCR}\right]} = \frac{1}{RCs + \cancel{R}LCs^2 + 1}$$

$$=$$



M: $F - k_{12}(x-y) - b_{12}(\dot{x}-\dot{y}) - k_1 x = M\ddot{x}$

m: $k_{12}(y-x) - b_{12}(\dot{y}-\dot{x}) - b_2(\dot{y}) - k_2 y = m\ddot{y}$

$\cancel{F = M\ddot{x} + k_{12}(x-y) + b_{12}(\dot{x}-\dot{y}) + k_1 x}$

$F = M\ddot{x} + b_{12}\dot{x} + (k_{12}+k_1)x - (b_{12}\dot{y} + k_{12}y)$

$0 = -(b_{12}\dot{x} + k_{12}x)\, m\ddot{y} + (b_2 + b_{12})\dot{y} + (k_2 + k_{12})y$

$$\begin{pmatrix} \hat{F}(s) \\ 0 \end{pmatrix} = \begin{array}{cc} A_{11} & A_{12} \\ \left(\begin{matrix} Ms^2 + b_{12}s + k_{12} + k_1 & -(b_{12}s + k_{12}) \\ -(b_{12}s + k_{12}) & ms^2 + (b_2 + b_{12})s + k_2 + k_{12} \end{matrix} \right) \\ A_{21} & A_{22} \end{array} \begin{pmatrix} X \\ Y \end{pmatrix}$$

at steady state no change of time $\dot{x}, \dot{y} = 0$ x stays constant

so for steady state remove the dampers

$F - k_{12}(x-y) - k_1 x = 0$ (1) and solve

$-k_{12}(y-x) + k_2 y = 0$ (2)

or

find $T(s)$ from f to x and set $s = 0$

$\frac{X}{F(s)} =$

$$\begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^{-1} \begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} A_{22} & -A_{12} \\ -A_{12} & A_{11} \end{pmatrix} \begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$

$$= \frac{\begin{pmatrix} A_{22}\hat{F}(s) \\ -A_{12}\hat{F}(s) \end{pmatrix}}{\Delta} \qquad X(s) =$$

$$\frac{X(s)}{\hat{F}(s)} = \frac{A_{22}(s)}{A_{11}A_{22} - A_{12}^2} \qquad \frac{Y(s)}{\hat{F}(s)} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2}$$

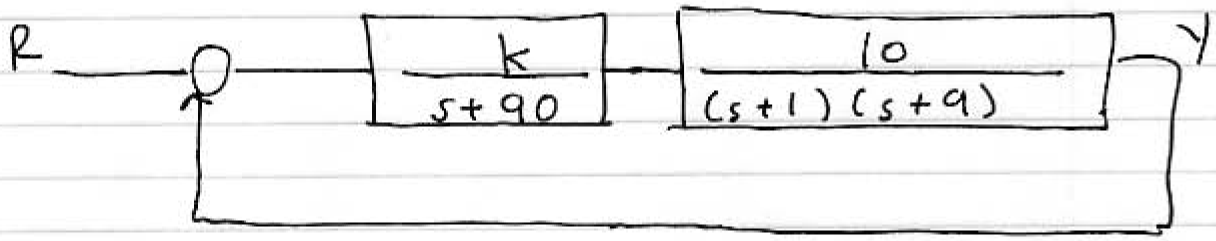
if $F = 1N$ find X at steady state

$$X(s)\big|_{s=0} = \frac{A_{22}(s)\big|_{s=0}}{(A_{11}A_{22} - A_{12}^2)\big|_{s=0}} \quad \Leftarrow \text{steady state value}$$

$$= \frac{k_2 + k_{12}}{(k_2+k_1)(k_{12}+k_1) - k_{12}^2} \qquad \text{if } k_1 = k_2 = k_{12} = 1$$

$$= \frac{2}{4-1} = \frac{2}{3}$$

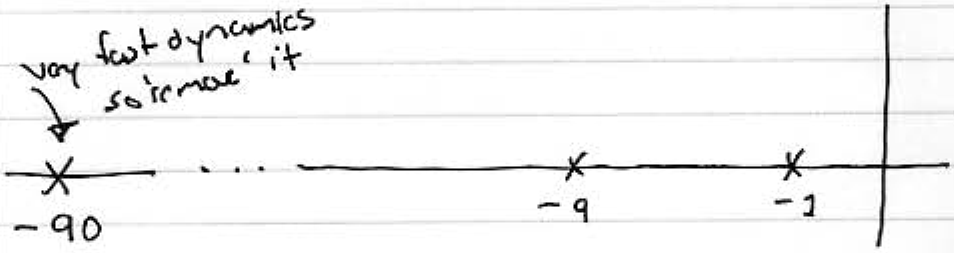
$$= \frac{k_2 + k_{12}}{k_2k_1 + k_2k_{12} + k_1k_{12}}$$



Find k such that $e_{ss} < 0.12$ for unit step input and p.o $\leq 15\%$.

$$\frac{Y}{R} = \frac{10k}{(s+90)(s+1)(s+9)}$$

but we want the DC gain to be the same while removing an s



Find steady state error: uses the plant

$$k_p = \lim_{s \to 0} P(s) = \frac{k}{(s+90)} \frac{10}{(s+1)(s+9)} = \frac{10k}{810} = \frac{k}{81}$$

$$e_{ss}(\infty) = \frac{1}{1+k_p} = \frac{1}{1+\frac{k}{81}} < 0.12$$

$$0.88 < \frac{k}{81} \cdot 0.12$$

$$k > 594$$

~~P.O. = ...~~

now for p.o. $\leq 15\%$ ζ use close loop TF

$\zeta > 0.52$

$$T = \frac{\frac{k}{9}\frac{1}{s^2+10s+9}}{1+\frac{\frac{k}{9}}{s^2+10s+9}} = \frac{\frac{k}{9}}{s^2+10s+(9+\frac{k}{9})}$$

must use this for w_n^2

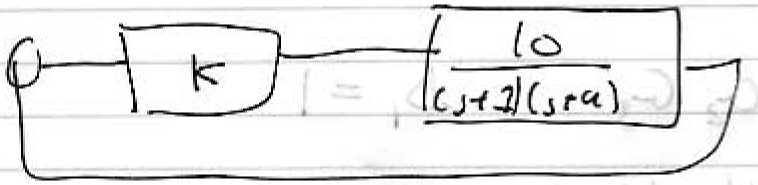
$$2\zeta w_n = 10$$

$$w_n = \frac{10}{2(0.52)} = \sqrt{9+\frac{k}{9}}$$

$$\left(\left(\frac{5}{0.52}\right)^2 - 9\right)9 = k \leq 751$$

$$\boxed{594 < k < 751}$$

Then simulate the real 3rd order system



is TYPE 0
input & output @
steady state

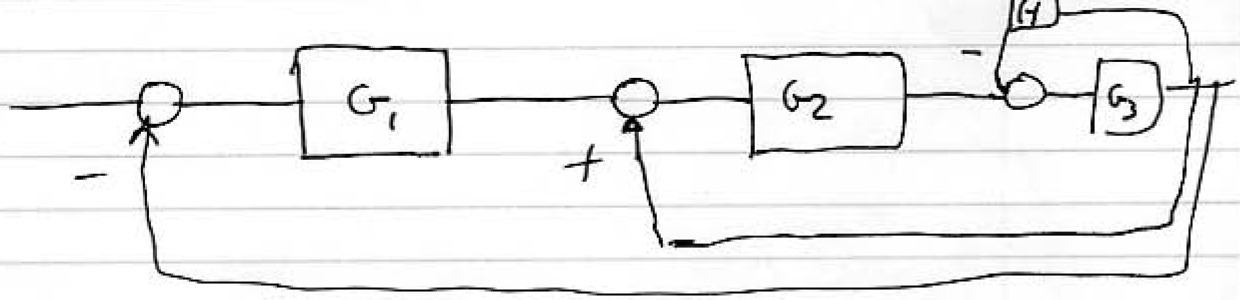
if you have $\frac{k}{s}$ it is a type I system

condition: stable

if you have $\frac{k}{s} + k_2$ is a type I system still

$$\frac{k + k_2 s}{s}$$

if you have $\frac{k_1}{s} + k + k_2 s$ is still type I



$$\frac{\dfrac{G_1 \, G_2 \dfrac{H}{1+HG_3}}{1 - G_2 \dfrac{H}{1+HG_3}}}{1 + \dfrac{G_1 G_2 \dfrac{H}{1+HG_3}}{1 - G_2 \dfrac{H}{1+HG_3}}} = \frac{G_1 \dfrac{G_2 H}{1+HG_3}}{\left[1 - \dfrac{G_2 H}{1+HG_3}\right] + G_1 G_2 \dfrac{H}{1+HG_3}}$$

$$= \frac{G_1 G_2 H}{(1+HG_3)\left[1 - \dfrac{G_2 H}{1+HG_3}\right] + G_1 G_2 H} = \frac{G_1 G_2 H}{1 + HG_3 - G_2 H}$$

crazy!

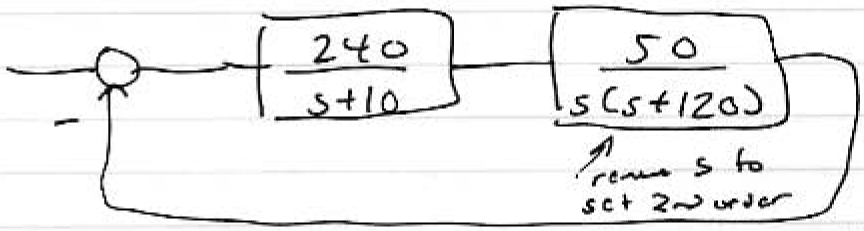
$P1 = G_1 G_2 G_3 \qquad \Delta_1 = 1$

$\Delta = -G_3 H + G_2 G_3 - G_1 G_2 G_3$

$$T(s) = \frac{G_1 G_2 G_3}{1 + G_3 H - G_2 G_3 + G_1 G_2 G_3}$$

they are all touching!

Last Year



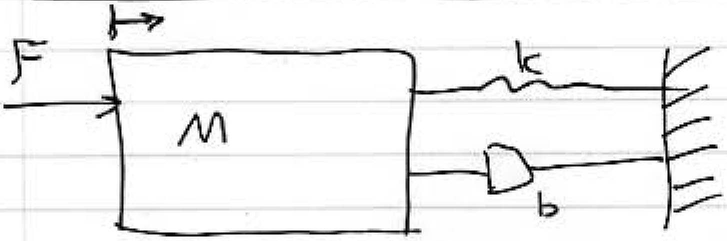
not accurate but right

Find p.o., T_s, T_p, steady state errors to ramp + step inputs

steady state error to step: Type I ⇒ 0 error

$$k_v = \frac{(240)(50)}{(10)(120)} = 10$$

$$T(s) = \;. -$$



What is the steady state value of the output y if $M = 2.4$ $k = 30$ $b = 10\,Ns/m$ $F = 1N$

$$F = ky$$

$$1 = 30y \qquad y = \frac{1}{30}$$

$$(Ms^2 + bs + k)Y = F$$

$$\frac{Y}{F} = \frac{1}{Ms^2 + bs + k} \qquad \left.\frac{Y}{F}\right|_{s=0} = \frac{1}{30}$$

F=1