

24.3

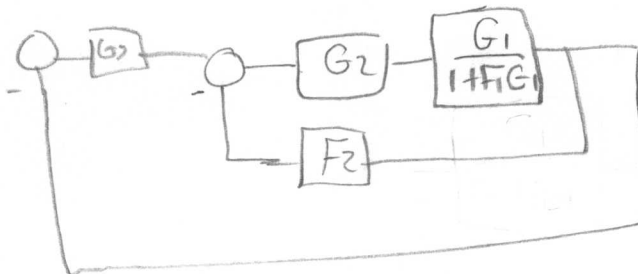
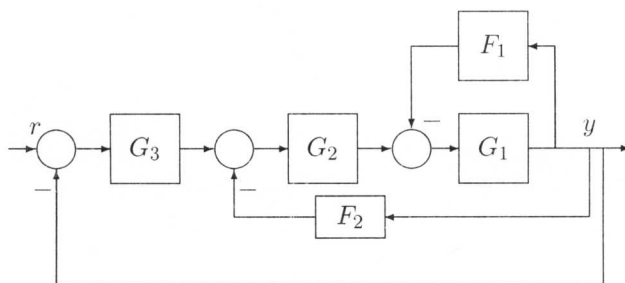
EE481.3 Mid-term test, Control systems

4 Questions, Duration 75 minutes.

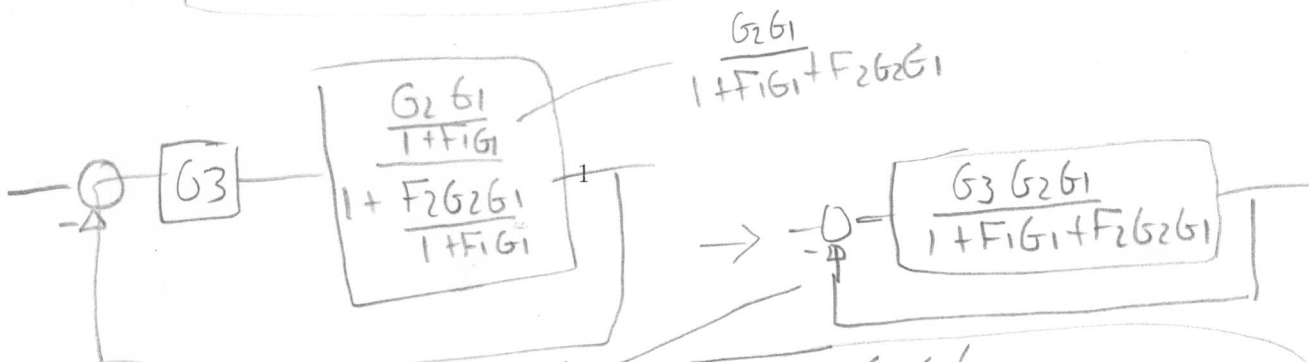
You may use a formula sheet, and a calculator.

Attempt all the questions. Justify your answers.

(6 marks) 1) Find the transfer function of the following system.



0.0

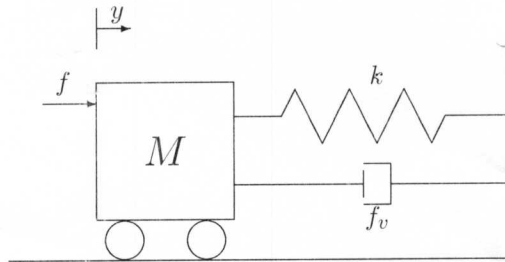


$$\frac{G_3 G_2 G_1}{1 + F_1 G_2 G_3 + F_2 G_2 G_1 + G_3 G_2 G_1}$$

$$\frac{y}{r} = \frac{G_3 G_2 G_1}{1 + F_1 G_2 G_3 + F_2 G_2 G_1 + G_3 G_2 G_1}$$

(6 marks) 2- What is the equation describing the system shown below. Find the transfer function Y/F ?

Take $M = 2.0\text{kg}$, $k = 8\text{N/m}$ and $f_v = 4\text{Ns/m}$. What is the steady state value of the output y , when the input force is 1N . What is the settling time to within 2 percent of the final value. Is the response of the system overdamped or Underdamped? ? If we can replace the shock-absorber(damper), how should we choose a new shock-absorber, so that the resulting mechanical system has a step response with a percent-overshoot of 5%.



INPUT

input force = 1N on (t)
 $= \frac{1}{s}$

$$\Sigma F = 0 = f - y(s)(k + f_v s + M s^2)$$

$$\frac{y(s)}{f} = \frac{1}{k + f_v s + M s^2}$$

$M = 2\text{kg}$
 $k = 8\text{N/m}$
 $f_v = 4\text{Ns/m}$

Final value input
 $y(\infty) = \lim_{s \rightarrow 0} s y(s)$

$$y(s) = \frac{1}{8 + 4s + 2s^2} = \frac{0.5}{s^2 + 2s + 4} = \frac{\frac{1}{8}(4)}{s^2 + 2s + 4}$$

$$y(s) = \frac{1/8}{k + f_v s + M s^2}$$

$$\lim_{s \rightarrow 0} s y(s) = \frac{1}{k} = \frac{1}{8}$$

$\omega_n^2 = 4$
 $\omega_n = 2$
 $\zeta = \frac{2}{2 \times 2} = \frac{2}{4} = 0.5$

$T_s = \frac{4}{3 \times 2} = \frac{4}{6} = \frac{2}{3} \approx 0.67$ (Note: handwritten calculation shows 4 sec)

$0 < \zeta < 1$ underdamped

%OS = 5

$$\zeta = \frac{-\ln(5/100)}{\sqrt{\pi^2 + \ln^2(5/100)}} = 0.69$$

5.7

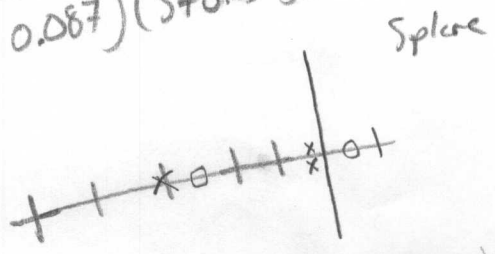
$(s + 1 + j1.73)(s + 1 - j1.73)$
 two complex poles
 Underdamped

$f_v = 23\omega_n$
 $f_v = 2(0.69)(2) = 2.76$

(6 marks) 3. Take the system $G(s) = \frac{(2s+5)(-s+0.5)}{(s+3)(s^2+0.1s+0.01)}$. Discuss the stability of the system. Find a low-order approximation for this system. Sketch the step response of the approximated system and discuss the accuracy of approximating the response of the system using the simplified low-order system.

$$\frac{(2s+5)(-s+0.5)}{(s+3)(s^2+0.1s+0.01)} = \frac{(2s+5)(-s+0.5)}{(s+3)(s+0.05+j0.087)(s+0.05-j0.087)}$$

$$= -\frac{(2s+5)(s-0.5)}{(s+3)(s+0.05+j0.087)(s+0.05-j0.087)}$$



all poles are on left hand side of s-plane
so stable

$\frac{3}{2.5} \geq \frac{0.05 \cdot 10}{0.5}$ So can ignore pole @ -3 but need to consider DC gains from terms
& zero @ -2.5
& zero @ 0.5

$$\frac{5 \frac{2}{3} - \frac{1}{2}}{3} \frac{1}{(s^2+0.1s+0.01)}$$

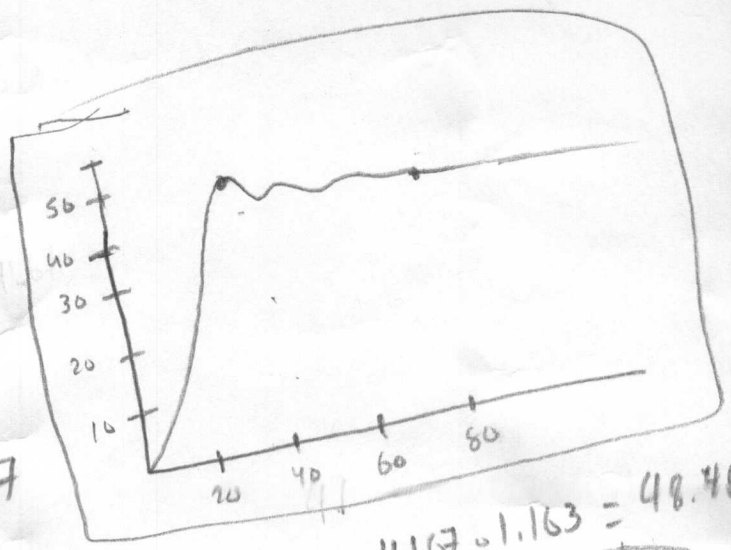
$$\frac{Y}{R} = \frac{5.5}{12(s^2+0.1s+0.01)}$$

$$y(s) = Y(s) = \frac{5.5}{12(0+0+0.01)} = 41.67$$

$$\omega_n^2 = 0.01 \quad \rho_0 = e^{-\left(\frac{0.5\pi}{\pi-0.5^2}\right)}_{100} = 16.3\%$$

$$\omega_n = 0.1 \quad T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{0.1 \sqrt{1-0.5^2}} = 36.28$$

$$0.1 = 2\zeta \omega_n \quad \zeta = \frac{0.1}{0.2} = 0.5 \quad T_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.05} = 80$$



$$41.67 \cdot 1.163 = 48.46$$

With the added poles the system would react a bit quicker but since they were 10 bigger they would hit their S.S way before the two approx S.S poles

5.6

