~~18.3~~
~~18.9~~ 19.0

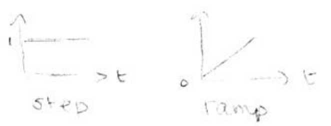
[redacted]

EE481.3 Mid-term test, Control systems

Duration 80 minutes.
You may use a formula sheet, and a calculator.
Attempt all the questions. Justify your answers.

(6 marks) 1) What is the significance of the step input. (Why do we study the response of a system to the step input? What does it signify?) How about the ramp input?

A step input is significant because it is constant in time and good as an input for a system. The ramp increases over time which may ramp the system between input and output which causes noise and perhaps instability.



The step response of a second-order system is shown below. Find the transfer function of the system.

start here →

$T_s \approx 5s$, $T_P \approx 1.75s$

$T_s = \frac{4}{\zeta\omega_n}$

$5s = \frac{4}{\zeta\omega_n}$

$4 = 5\zeta\omega_n$

$\omega_n = \frac{4}{5\zeta}$

$T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$1.75 = \frac{\pi}{\frac{4}{5\zeta}\sqrt{1-\zeta^2}}$

$\frac{\pi}{1.75} = \frac{4}{5\zeta}\sqrt{1-\zeta^2}$

$\left(\frac{\pi}{1.75}\right)^2 = \left(\frac{16}{25\zeta^2}\right)(1-\zeta^2)$

$\left(\frac{\pi}{1.75}\right)^2 = \frac{16}{25\zeta^2} - \frac{16}{25}$

$3.863 = \frac{16}{25\zeta^2}$

$\frac{16}{3.863} = 25\zeta^2$

$\zeta = 0.407$

$\omega_n = \frac{4}{5\zeta}$

$\omega_n = 1.97$

double check:

$T_s = \frac{4}{\zeta\omega_n}$

$5 = \frac{4}{\zeta\omega_n}$

$\zeta = \frac{4}{5\omega_n}$

$T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

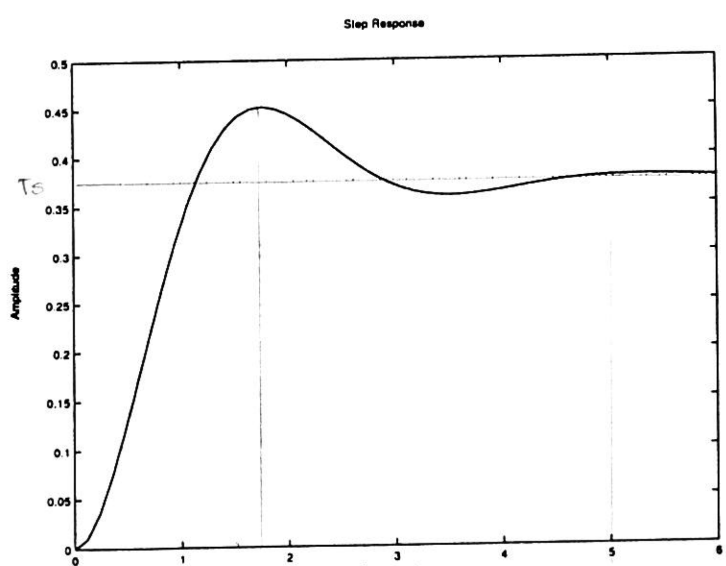
$1.75 = \frac{\pi}{\omega_n\sqrt{1-\frac{16}{25\omega_n^2}}}$

$\left(\frac{\pi}{1.75}\right)^2 = \omega_n^2\left(1-\frac{16}{25\omega_n^2}\right)$

$\left(\frac{\pi}{1.75}\right)^2 = \omega_n^2 - \frac{16}{25}$

$\omega_n = 1.97$ ✓

Ts



Tp

3.0
~~2.5~~

$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\boxed{T(s) = \frac{\cancel{3.96}}{s^2 + 1.6s + 3.96}}$

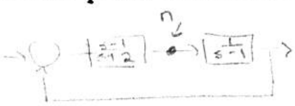
7.~~5~~

(9 marks) 2)What is a bounded ~~signal and~~ what does bounded input, bounded output stability mean? A bounded signal means that over time (in the time domain), the signal will converge to a finite value ~~(won't oscillate or ramp)~~. BIBO means that a signal is stable if every bounded input yields a bounded output. As well, a system is unstable if any input yields an unbounded output.

0.5

1.5

Discuss the concept of internal stability and explain why we should avoid canceling unstable poles with zeros at the same locations?



Internal stability is tested ~~between~~ on the noise, n, and the output. Cancelling unstable poles is avoided because when you try to find the transfer function and you cancel them, you fill find system may be stable when it actually ~~isn't~~. This will cause you problems in design and analysis when the system is not stable to begin with.

1.5

definition → What is the type of a system? Show that a type one system has zero steady state error to a step input. (Give a physical reason for the result to hold and discuss conditions which should hold.) For a system given $G(s) = \frac{(s+z_1)(s+z_2)\ldots}{s^n (s+p_1)(s+p_2)\ldots}$, the type is defined as the integer n. If n = 0, then the type is 0. If n = 1, the type is 1 and so on.

Say that $G(s) = \frac{1}{s(s+1)}$. This is type one

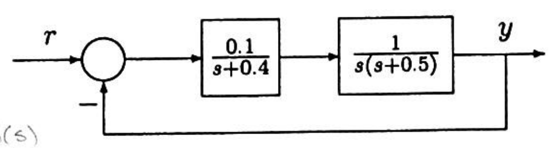
step input → show zero s.s. error

$e_{step}(\infty) = \lim_{s\to 0} \frac{s(1/s)}{1+G(s)} = \frac{1}{1+\lim_{s\to 0}\left(\frac{1}{s(s+1)}\right)} = \frac{1}{\infty} = 0$ ✓

1.5

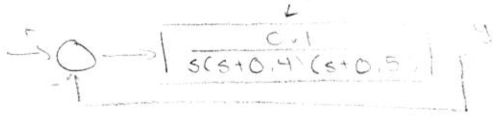
Physically, using a step input for the system will give zero s.s. error because the input can't keep up with the system (like graph). The condition is assuming the system is (marginally) stable. It may have have pole(s) at the origin.

Take the unity feedback system shown below. Discuss the stability and steady-state error to the step input, and the ramp-input in the following system.



System is stable because poles are in LH side of plane.

What are they?



Step input:

$e_{step}(\infty) = \frac{1}{1+\lim_{s\to 0}\left(\frac{0.1}{s(s+0.4)(s+0.5)}\right)} = \frac{1}{\infty} = 0$ ✓

The ss error for a step input is zero b/c G(s) is type 1.

Ramp input:

$e_{step}(\infty) = \lim_{s\to 0} \frac{1}{s\left(\frac{0.1}{s(s+0.4)(s+0.5)}\right)} = \frac{1}{\frac{0.1}{(0.4)(0.5)}} = 2$ ✓ finite ss error

Thus, the ramp input yields a finite value b/c the system is type 1.

3.5

(4 marks) 3) Take the system $G(s) = \frac{s+2}{s^3+3s^2+2s+1.5}$. Give a state space representation for this system. What are the eigen-values of the A matrix in a state-space representation?

significance - answer (written)
what do they represent?

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1.5 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let $x_1 = y$

$x_2 = \dot{x}_1 = \dot{s} = \frac{dy}{dt}$

$x_3 = \dot{x}_2 = \ddot{s} = \frac{d^2y}{dt^2}$

$\frac{d^3y}{dt^3} = \dot{x}_3 = \frac{dr}{dt} + 2r - 3x_3 - 2x_2 - 1.5x_1$

Check?

$$\frac{Y(s)}{R(s)} = \frac{s+2}{s^3+3s^2+2s+1.5}$$

$Y(s)(s^3+3s^2+2s+1.5) = (s+2)R(s)$

Let $s = x_1$

$x_2 = \dot{x}_1 = \dot{s}$

$$\underbrace{Y(s)s^3}_{} + \underbrace{Y(s)3s^2}_{x_3} + \underbrace{Y(s)2s}_{x_2} + \underbrace{1.5Y(s)}_{x_1} = R(s)s + 2R(s)$$

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 1.5y = \frac{dr}{dt} + 2r$$

Okay, in time domain

let $x_1 = y$

$x_2 = \dot{x}_1 = \frac{dy}{dt}$

$x_3 = \dot{x}_2 = \frac{d^2y}{dt^2}$

$\dot{x}_3 = \frac{d^3y}{dt^3} = \frac{dr}{dt} + 2r - 3x_3 - 2x_2 - 1.5x_1$

3.4

The eigenvalues are the poles of a system. Using eigenvalues to solve similarity transformations (given state space, find the similar system) we can find these values to use in a diagonal matrix (eigenvector).

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

($P^{-1}AP$ results in diagonal matrix)

This will make it much easier to solve for each state space variable (each independently). The eigenvalues also have to be negative to have a stable system.

Given $\dot{x} = Ax + Bu$

$y = Cx + Du$

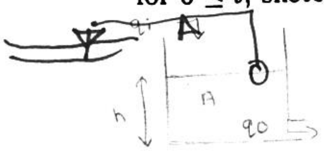
$\dot{z} = P^{-1}APz + P^{-1}Bu$

$y = CPz + Du$

Given an A matrix, can find eigenvalues with $0 = \det(\lambda I - A)$

(6 marks) 4)Take a tank with the cross section $A = 2m^2$. Assume that water flows into the tank through a valve. The rate of the flow of water out of the tank is equal to Kh, where $K = 0.5m^2/hour$ and h is the level of water in the tank. We are interested in controlling the level of water in the tank. Discuss the transfer function of this system. Assume that water is flowing at a rate of $1m^3/hour$ when $t < 0$, and the level of water has reached the steady state at $t = 0$. What is the initial level of water h(0)? If the rate increases to $1.2m^3/hour$ for $0 \leq t$, sketch the level of water as a function of time.

Solve ss value or diff. eqn
Next, increasing (height as function of time)



$q_o = Kh$

$V = Ah$

$q_i - q_o = \frac{dV}{dt} = A\frac{dh}{dt}$

$q_i - Kh = A\frac{dh}{dt}$ (function of h)

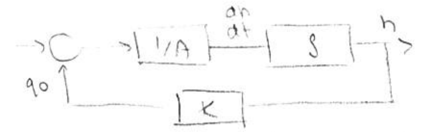
$q_i = Kh + A\frac{dh}{dt}$

no change at = 0, s.s.

$1m^3/hr = (0.5m^2/hr)(h) + (2m^2)(0)$

$h = (1m^3/hr) \div (0.5m^2/hr)$

$\boxed{h(0) = 2m}$



The transfer function relates qi to h (as the diagram shows). It is first a order system.

$AsH(s) + KH(s) = 1/s$

$H(s) = \frac{1}{s(As+K)}$

$H(s) = \frac{1}{A}\left[\frac{1}{s(s+K/A)}\right]$

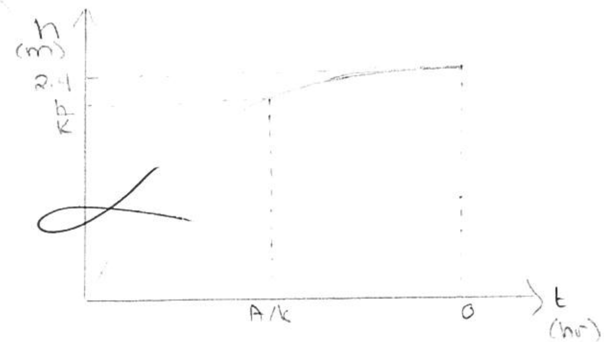
$H(s) = \frac{1}{K}\left[\frac{1}{s} - \frac{1}{s+K/A}\right]$

$q_i = Kh + A\frac{dh}{dt}$

$1.2m^3/hr = (0.5m^2/hr)(h)$

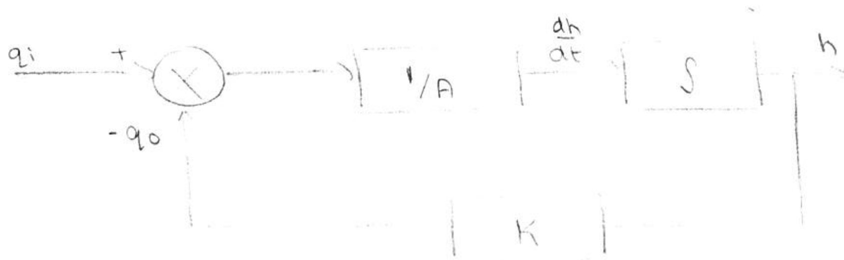
$h_{ss} = 2.4$

time constant A/k



Sketch a closed loop control system that controls the level of water in this system. ?

feedback



4.6

$q_i - q_o = A\frac{dh}{dt}$

$q_i = Kh + A\frac{dh}{dt}$