University of Saskatchewan
Department of Mathematics and Statistics
Math 223 (05, G.Patrick)

28/45

October 17, 2005 | Test #1 | 90 minutes

This examination consists of two parts. Part A contains short, routine questions, which you should answer fully but succinctly in the space provided. The questions in Part B are more difficult, and some are designed to challenge you. Fully answer all questions of Part B in the answer books provided.

You should complete Part A rapidly, and save at least half your time to answer the questions in Part B. Part A is worth 30 points and Part B is worth 20 points. Remember to print your name and student ID in the spaces provided in both Part A and Part B.

The points for each problem are indicated in the right margin.

Permitted resources: none.

This is a midterm test. Cheating on an test is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not bring into the test room any books, resources or papers except at the discretion of the examiner or as indicated on the examination paper. Candidates shall hold no communication of any kind with other candidates within the examination room.

Print your name and student ID here: Craig Blach-Hansen 147742

PART A. Fully answer the following questions in the space provided.

Question A1. Determine whether the point $(2, -1, 2)$ is in the plane $3x - 4y + z - 13 = 0$. **2**

$3(2) - 4(-1) + 2 - 13 = 0$

$6 + 4 + 2 - 13 = 0$

$-1 \neq 0$

Point Not ~~in~~ Plane

3

Question A2. Calculate all the unit vectors which are perpendicular to the two vectors $(1, -2, 1)$ and $(2, 1, 3)$. **2**

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = -7\hat{i} + \hat{j} + 5\hat{k}$$

$|p| = \sqrt{49 + 1 + 25}$

$= \sqrt{55}$

$\sqrt{75}$ $5\sqrt{3}$

$$\hat{p} = \frac{-7}{\sqrt{55}}\hat{i} + \frac{1}{\sqrt{55}}\hat{j} + \frac{5}{\sqrt{55}}\hat{k}$$

2

Question A3. Calculate the projection of the vector $(1, -2, 2)$ onto the vector $(0, 3, -1)$. **2**

$(1, -2, 2) \cdot (0, 3, -1) = 0 - 6 - 2$

$= -8$

$(1, -2, 2) \cdot (0, 3, -1)$

$(0 + 9 + 1) \quad (0\ 3\ -1)$

1

$\left(\frac{U \cdot V}{|V|^2}\right)(V)$

6

Question A4. Calculate the unit tangent vector, in the direction of increasing t, to the curve $\mathbf{r}(t) = (t^2, 1 - t - 2t^2, t)$ at $t = 1$. **2**

$\hat{T} = \frac{\frac{d\mathbf{r}}{dt}}{\left|\frac{d\mathbf{r}}{dt}\right|}$ $\quad \frac{d\mathbf{r}}{dt} = (2t, -1-4t, 1)$ $\quad \left|\frac{d\mathbf{r}}{dt}\right| = \sqrt{4t^2+1+1-8t+16t^2}$ $\quad (-1-4t)(-1-4t)$ $1 - 8t + 16t^2$

$= \sqrt{20t^2 - 8t + 2}$

$\hat{T} = \frac{(2t, -1-4t, 1)}{\sqrt{20t^2 - 8t + 2}}$

$\hat{T}|_1 = \frac{(2, -5, 1)}{\sqrt{16}} = \left|\frac{(2, -5, 1)}{4}\right|$

2

Question A5. If $u(t)$ and $v(t)$ are two curves such that **2**

$$u(0) = (2, -1, 0), \quad v(0) = (1, 1, 1), \quad u'(0) = (1, -1, 1), \quad v'(0) = (-2, 1, 2)$$

then calculate the value of $(u \cdot v)'(0)$.

$(1, -1, 1) \cdot (-2, 1, 2) = -2 - 1 + 2$

$= -1$

Question A6. With exactly the same information as in the previous question, calculate the value of $(u \times v)'(0)$. **2**

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ -2 & 1 & 2 \end{vmatrix} = -3\hat{i} - 0\hat{j} - 1\hat{k}$$

$= -3\hat{i} - \hat{k}$

Question A7. If a particle follows a path in such a way that the curvature of the path at time $t = 1$ is $\kappa = \frac{1}{2}$ and the speed of the particle at time $t = 1$ is 4, then calculate the normal component of the particle's acceleration. **2**

$k(1) = \frac{1}{2}$

$v(1) = 4$

$a_n = v^2(k)$

$= 4\left(\frac{1}{2}\right)$

$= 2$

1

formula

3

learn