

University of Saskatchewan  
 Department of Mathematics and Statistics  
 Math 223 (05, G.Patrick)

23  
30

November 21, 2005

Test #2

90 minutes

This examination consists of two parts. Part A contains short, routine questions, which you should answer fully but succinctly in the space provided. The questions in Part B are more difficult, and some are designed to challenge you. Fully answer all questions of Part B in the space provided.

You should complete Part A rapidly, and save at least half your time to answer the questions in Part B. Part A is worth 30 points and Part B is worth 20 points. Remember to print your name and student ID in the spaces provided in both Part A and Part B.

The points for each problem are indicated in the right margin.

Permitted resources: none. No books, no notes of any kind, no calculators, no electronic devices of any kind.

This is a midterm test. Cheating on an test is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not bring into the test room any books, resources or papers except at the discretion of the examiner or as indicated on the examination paper. Candidates shall hold no communication of any kind with other candidates within the examination room.

PRINT your NAME and STUDENT ID: Craig Bloch-Hansen 147742

**PART A. Fully answer the following questions in the space provided.**

Question A1. Using the method of Lagrange multipliers, find all critical points of the function  $f(x, y, z) = x - y + z$  subject to the constraint  $x^2 - 6xy + y^2 - z = 0$ . 3

$$L(x, y, z, \lambda) = (x - y + z) + \lambda(x^2 - 6xy + y^2 - z)$$

$$\frac{\partial L}{\partial x} = 1 + \lambda(2x - 6) = 2\lambda x - 6\lambda + 1 = 0 \quad x = \frac{7}{2}$$

$$\frac{\partial L}{\partial y} = -1 + \lambda(2y - 6) = 0 = 2\lambda y - 6\lambda - 1 \quad y = \frac{5}{2}$$

$$\frac{\partial L}{\partial z} = 1 + \lambda(-1) = 0 = 1 - \lambda$$

$$\frac{\partial L}{\partial \lambda} = x^2 - 6xy + y^2 - z = 0 \quad \frac{49}{4} - \frac{21 \cdot 5}{4} + \frac{25}{4} = z$$

$$z = \frac{-136}{4} = -34$$

$\left( \frac{7}{2}, \frac{5}{2}, -34 \right)$

Question A2. Using differentials, estimate the value of  $\ln(1 + \sin(2x + y))$  when  $x = .1$  and  $y = -.1$ . 3

$$\frac{df}{dx} dx + \frac{df}{dy} dy \approx f = f(x, y) \quad \begin{matrix} x=0 \\ y=0 \end{matrix} \quad f(x, y) = \ln(1)$$

$$\frac{df}{dx} = \frac{2 \cos(2x+y)}{1 + \sin(2x+y)} \quad \frac{2 \cos(2x)}{1 + \sin(2x)} dx + \frac{\cos(2x+y)}{1 + \sin(2x+y)} dy \approx f(x, y)$$

$$\frac{df}{dy} = \frac{\cos(2x+y)}{1 + \sin(2x+y)} \quad \frac{2 \cos(2(0)+0)}{1 + \sin(2(0)+0)} (.1) + \frac{\cos(2x+y)}{1 + \sin(2(0)+0)} (-.1) \approx f(x, y)$$

$$\frac{2(1)}{1} (.1) + \frac{1}{1} (-.1) \approx f(x, y)$$

$$.2 - .1 \approx .1$$

$f(x, y) \approx .1$

Question A3. Calculate, up to and including terms of order 2, the Taylor series of the function

3

$z = \frac{1 + \sin y}{x}$  at  $(x, y) = (1, \pi)$ .

$$f(x, y) + \frac{\partial f}{\partial x}(x-x_0) + \frac{\partial f}{\partial y}(y-y_0) + \frac{1}{2} \left[ \frac{\partial^2 f}{\partial x^2}(x-x_0) + 2 \frac{\partial^2 f}{\partial x \partial y}(x-x_0)(y-y_0) + \frac{\partial^2 f}{\partial y^2}(y-y_0) \right]$$

$$\frac{\partial f}{\partial x} = \frac{-1 - \sin y}{x^2} = \frac{1 + \sin y}{x} - \frac{1 + \sin y}{x^2}(x-1) + \frac{\cos y}{x}(y-\pi) + \frac{1}{2} \left[ \frac{2 + 2 \sin y}{x^3}(x-1) + \frac{2 \cos y}{x^2}(y-\pi)(x-1) - \frac{1}{x} \sin y (y-\pi) \right]$$

$$\frac{\partial f}{\partial y} = \frac{1}{x} \cos y$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{2 + 2 \sin y}{x^3}$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{x^2} \cos y$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{x} \sin y$$

Sub values  $(1, \pi)$

Question A4. Calculate the value of the double integral  $\iint_R xy^2 dA$  where  $R$  is the region

3

$$R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}.$$

$$\int_0^1 \int_0^2 xy^2 dy dx$$

$$\int_0^1 \left[ \frac{xy^3}{3} \right]_0^2 dx = \int_0^1 x \frac{8}{3} dx = \frac{8}{3} \left[ \frac{x^2}{2} \right]_0^1 = \frac{8}{6} = \frac{4}{3}$$

Question A5. Calculate the iterated double integral  $\int_0^1 \int_0^y (1 + xy) dx dy$ .

3

$$\int_0^1 \int_0^y (1 + xy) dx dy$$

$$\int_0^1 \left[ x + \frac{x^2 y}{2} \right]_0^y dy$$

$$\int_0^1 \left[ y + \frac{y^3}{2} \right] dy = \left[ \frac{y^2}{2} + \frac{y^4}{8} \right]_0^1 = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

Question A6. Calculate the double integral  $\iint_R xy dA$  where  $R$  is the interior of the triangle which has as vertices the three points  $(0, 0)$ ,  $(1, 0)$ , and  $(0, 1)$ .

3

$$y = x + 1 \quad 0 \leq x \leq 1$$

$$y = 1 - x \quad 1 \leq y \leq 1 - x$$

$$(1-x)(1-x)$$

$$= 1 - 2x + x^2$$

$$x - 2x^2 + x^3$$

$$\int_0^1 \int_0^{1-x} xy dy dx$$

$$= \int_0^1 \left[ \frac{xy^2}{2} \right]_0^{1-x} dx = \int_0^1 \frac{x(1-x)^2}{2} - \frac{x}{2} dx = \int_0^1 \frac{-2x^2 + x^3}{2} dx = \left[ -\frac{2x^3}{6} + \frac{x^4}{8} \right]_0^1$$

$$= \left[ -\frac{1}{3} + \frac{1}{8} \right] = -\frac{2}{6} + \frac{1}{8} = -\frac{2}{6} + \frac{3}{24} = \left[ -\frac{5}{24} \right]$$

