B.F.G.

University of Saskatchewan
Department of Mathematics and Statistics
Math 223 (01) (03) (05)

September 19, 2005 | Quiz #1 | 45 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is an formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

Print your name here: Craig Bloch-Hansen

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Print your student number here: 147742

Question 1. Find an equation describing all points equidistant from the points $(-3, 0, 4)$ and $(2, 1, 5)$. What does this equation describe geometrically? **5**

~~a plane~~

$(-3,0,4) - (2,1,5) = (-5,-1,-1)$ $\cancel{\tfrac{1}{2}(2.5, .5, .5)}$

.5
4.5
2.25

$\cancel{(2,1,5)}$ $\sqrt{25+1+1} = \frac{\sqrt{27}}{2}$

$(2,1,5) - (-3,0,4) = (5,1,1)$ $\| \| = \sqrt{27}$

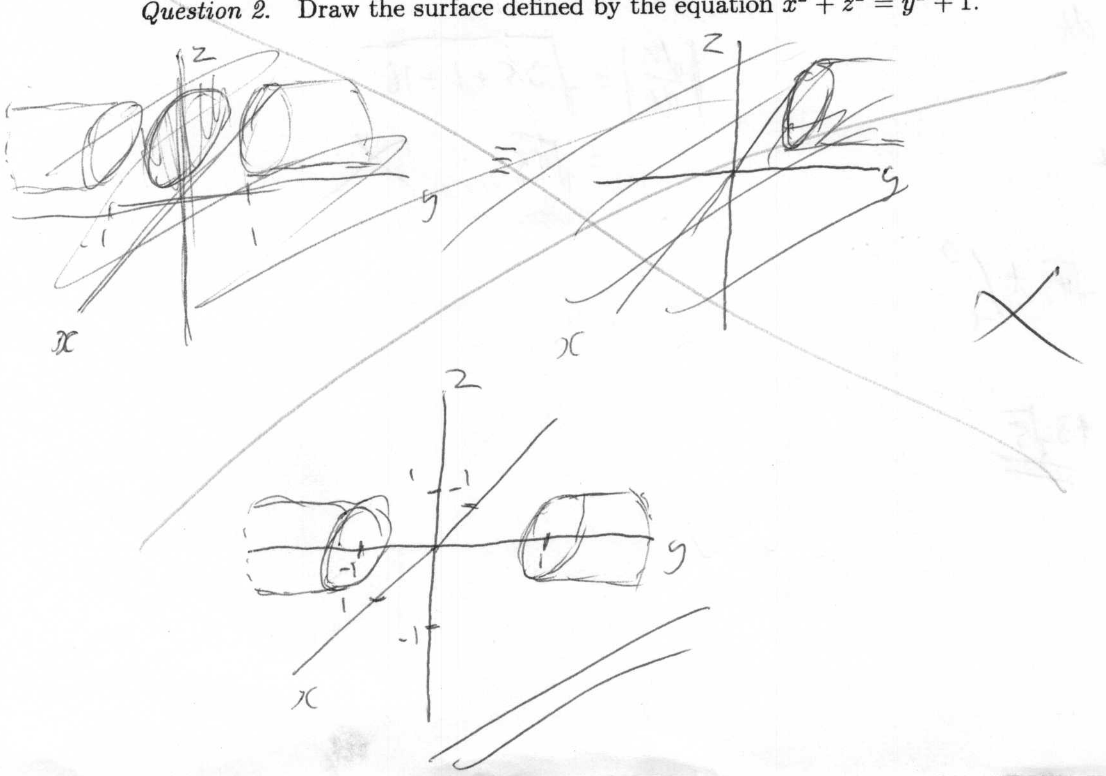
$(-.5, .5, 4.5)$ $\cancel{(-.5x + .5y + 4.5z)}$

$(5,1,1) \times (-.5, .5, 4.5)$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 1 \\ -.5 & .5 & 4.5 \end{vmatrix} = 4\hat{i} - 23\hat{j} + 3\hat{k}$

X

$f(t) = 4\hat{i} - 23\hat{j} + 3\hat{k}$ ~~a plan~~ a straight line

Question 2. Draw the surface defined by the equation $x^2 + z^2 = y^2 + 1$. **5**



Question 3. Prove that if a differentiable function $\mathbf{v}(t)$ has constant length, then at any t for which $d\mathbf{v}/dt \neq 0$, the vector $d\mathbf{v}/dt$ is prependicular to $\mathbf{v}$. **5**

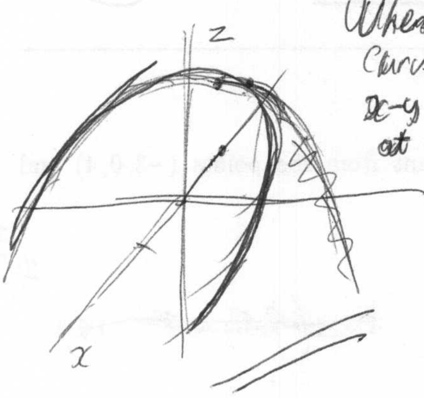
$v(t) = x\hat{i} + y\hat{j} + z\hat{k}$

$|v| = \sqrt{x^2+y^2+z^2} = C$ $\qquad C^2 = x^2+y^2+z^2$ $\qquad 0 = x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}$

$\frac{dv}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

$v \cdot \frac{dv}{dt} = x\frac{dx}{dt}\hat{i} + y\frac{dy}{dt}\hat{j} + z\frac{dz}{dt}\hat{k} = 0$

Question 4. Draw the curve with the position vector $\mathbf{r}(t) = (t^2-1)\hat{\mathbf{i}} + t\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ **5**



Where the Curve is in the x-y plane at $z = 5$

$r(t) = \int \left|\frac{dv}{dt}\right| dt$

$\frac{dv}{dt} = \left|\frac{dv}{dt}\right|$

$2t\hat{i} + 1\hat{j} = \left|\frac{dv}{dt}\right|$

$= \sqrt{x^2+y^2+z^2}$

$4t^2\hat{i} + 1\hat{j} = x^2+y^2+z^2$

Question 5. Find the length of the curve $x = 2-5t, y = 1+t, z = 6+4t, -1 \le t \le 0$. Draw the curve. **5**

$\cancel{\int_{-1}^{0} (2-5t, 1+t, 6+4t)}$

$V(t) = (2-5t)\hat{i} + (1+t)\hat{j} + (6+4t)\hat{k}$

$\frac{dv}{dt} = (-5)\hat{i} + (1)\hat{j} + (4)\hat{k}$

$S = \int_{-1}^{0} \left|\frac{dv}{dt}\right| dt$

$\left|\frac{dv}{dt}\right| = \sqrt{25+1+16}$

$= \sqrt{45}$

$S = \int_{-1}^{0} \sqrt{45}\, dt$

$S = \sqrt{45}\, t \Big|_{-1}^{0}$

$S = +\sqrt{45} = +3\sqrt{5}$