University of Saskatchewan
Department of Mathematics and Statistics
Math 223 (01) (03) (05)

September 26, 2005 | Quiz #2 | 45 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is an formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

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Question 1. If $\mathbf{u} = (1, 3, 6)$ and $\mathbf{v} = (-2, 0, 4)$, find the components of $|3\mathbf{u}|\mathbf{v} - |-2\mathbf{v}|\mathbf{u}$. 5

$|(3,9,18)|(-2,0,4) - |(4,0,-8)|(1,3,6)$

$\sqrt{9+81+324}$ $\sqrt{16+64}$

$= \sqrt{414}(-2,0,4) - \sqrt{80}(1,3,6)$

$= \left(-2\sqrt{414} - \sqrt{80}, -3\sqrt{80}, 4\sqrt{414} - 6\sqrt{80}\right)$

Question 2. If $\mathbf{u} = (3, 1, 4)$, $\mathbf{v} = (-1, 2, 0)$ and $\mathbf{w} = (-2, -3, 5)$, find the components of 5

$$(\mathbf{u} \times \mathbf{w}) - (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times (2\mathbf{u} + \mathbf{v})).$$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ -2 & -3 & 5 \end{vmatrix} = (17, -23, -6)$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & 2 & 0 \end{vmatrix} = (-8, -4, 7)$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 5 & 4 & 8 \end{vmatrix} = (-8, -4, 7)$

$(17, -23, -6) - (-8, -4, 7) + (-8, -4, 7)$

$(17, -23, -6)$

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$\hat{N} = \frac{N}{|N|}$ $N = \frac{d\hat{T}}{dt}$ $\hat{T} = \frac{\dot{r}}{|\dot{r}|}$ $\hat{B} = \hat{N} \times \hat{T}$

Question 3. For the curve 5

$$x = 4\cos t, \quad y = 6\sin t, \quad z = 2\sin t, \qquad -\infty < t < \infty,$$

find $\hat{\mathbf{N}}$ and $\hat{\mathbf{B}}$ at the point $(2\sqrt{2}, 3\sqrt{2}, \sqrt{2})$.

(1)

$r(t) = 4\cos t\,\hat{i} + 6\sin t\,\hat{j} + 2\sin t\,\hat{k}$

$\dot{r} = -4\sin t\,\hat{i} + 6\cos t\,\hat{j} + 2\cos t\,\hat{k}$

$|\dot{r}| = \sqrt{(16\sin^2 t) + (36\cos^2 t) + (4\cos^2 t)} = 2\sqrt{4\sin^2 t + 9\cos^2 t + \cos^2 t}$

$\hat{T} = \dfrac{(-2\sin t,\ 3\cos t,\ \cos t)}{\sqrt{4\sin^2 t + 9\cos^2 t + \cos^2 t}}$

$\hat{N} = \dfrac{(-2\cos t,\ -3\sin t,\ -\sin t)}{\sqrt{4\cos^2 t + 9\sin^2 t + \sin^2 t}}$

$$\hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\cos t & -3\sin t & -\sin t \\ -2\sin t & 3\cos t & \cos t \end{vmatrix}$$

$\sqrt{-2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t + 2e^{2t}(\cos^2 t - \sin^2 t)^2} = 2e^{2t}\sqrt{-1(\cos^2 t - \sin^2 t) + e^{2t}(\cos^2 t + \sin^2 t)^2}$

$2e^{2t}(\cos^2 t - \sin^2 t) = 2e^{2t}\sqrt{-1(\cos^2 t - (1-\cos^2 t)) + (\cos^2 t - 1)^2}$

$(2e^{2t}\cos^2 t - 2e^{2t}\sin t\cos t) + (2e^{2t}\sin^2 t + 2e^{3t}\cos t\sin t)$

Question 4. Find the curvature and the radius of curvature of the curve 5

$$x = e^t\cos t, \quad y = e^t\sin t, \quad z = t, \qquad -\infty < t < \infty.$$

(2)

$\kappa = \dfrac{|\dot{r} \times \ddot{r}|}{|\dot{r}|^3}$

$r(t) = e^t\cos t\,\hat{i} + e^t\sin t\,\hat{j} + t\,\hat{k}$

$\dot{r}(t) = (e^t\cos t - e^t\sin t)\,\hat{i} + (e^t\sin t + e^t\cos t)\,\hat{j} + \hat{k}$

$\ddot{r}(t) = ((e^t\cos t - e^t\sin t) - (e^t\sin t + e^t\cos t))\,\hat{i} + ((e^t\sin t + e^t\cos t) + (e^t\cos t - e^t\sin t))\,\hat{j} + 0$

$= (-2e^t\sin t)\,\hat{i} + (2e^t\cos t)\,\hat{j} + 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ e^t\cos t - e^t\sin t & e^t\sin t + e^t\cos t & 1 \\ -2e^t\sin t & 2e^t\cos t & 0 \end{vmatrix}$$

$= (-2e^t\cos t)\,\hat{i} - (2e^t\sin t)\,\hat{j} + ((e^t\cos t - e^t\sin t)(e^t\cos t) + (2e^t\sin t)(e^t\sin t + e^t\cos t))$

$= (-2e^t\cos t)\,\hat{i} - (2e^t\sin t)\,\hat{j} + 2e^{2t}(\cos^2 t - \sin^2 t)\,\hat{k}$

$|\dot{r} \times \ddot{r}| = 2e^{2t}$ $\quad \boxed{\kappa = \dfrac{2e^{2t}}{e^{3t}\sqrt{9}} = \dfrac{2}{e^t\sqrt{9}}}$ $\quad \boxed{\rho = \dfrac{e^t\sqrt{9}}{2}}$

$|\dot{r}|^3 = e^{3t}(3)^{\frac{3}{2}}$

$|\dot{r}| = \sqrt{(e^t\cos t - e^t\sin t)^2 + (e^t\sin t + e^t\cos t)^2 + 1} = \sqrt{e^{2t}}\sqrt{3} = e^t\sqrt{3}$

Question 5. A particle at $(1, 2, -1)$ starts from rest at time $t = 0$. Find its position as a function of time if its acceleration is $\mathbf{a}(t) = 3t^2\hat{\mathbf{i}} + (t+1)\hat{\mathbf{j}} - 4t^3\hat{\mathbf{k}}$. 5

(5)

$v(t) = t^3\hat{i} + \left(\frac{t^2}{2} + t\right)\hat{j} - t^4\hat{k} + C$

$0 = 0\hat{i} + (0+0)\hat{j} - 0\hat{k} + C$

$C = 0$

$s(t) = \frac{t^4}{4}\hat{i} + \left(\frac{t^3}{6} + \frac{t^2}{2}\right)\hat{j} - \frac{t^5}{5}\hat{k} + C$

$s(0) = (1, 2, -1)$

$\frac{0^4}{4} = 1$

$\frac{0^3}{6} + \frac{0^2}{2} = 2$

$-\frac{0^5}{5} = -1$

$s(t) = \left(\frac{t^4}{4} + 1\right)\hat{i} + \left(\frac{t^3}{6} + \frac{t^2}{2} + 2\right)\hat{j} - \left(\frac{t^5}{5} - 1\right)\hat{k}$

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