

University of Saskatchewan
Department of Mathematics and Statistics
Math 223 (01) (03) (05)

October 3, 2005

Quiz #3

45 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is a formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

Print your name here: Craig Bloch-Hansen

18

Print your student number here: 145742

Question 1. Find the equation of the plane containing the lines

5

$$\frac{x-1}{6} = \frac{y}{8} = \frac{z+2}{2}, \text{ and } \frac{x+1}{3} = \frac{y-2}{4} = z+5.$$

$$r = (6, -8, 2) \quad P_1 = (1, 0, -2) \quad P_2 = (-1, 2, -5)$$

$$P = P_2 - P_1 = (-2, 2, -3)$$

$$r \times P = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -8 & 2 \\ -2 & 2 & -3 \end{vmatrix} = (28)\hat{i} - (-14)\hat{j} + (-28)\hat{k}$$

$$= 28\hat{i} - (-14)\hat{j} + (-28)\hat{k}$$

3

$$P \cdot (x-1, y, z+2) = 0$$

$$= -28x - 28 + 14y - 28z - 56 = 0 \quad -28x + 28 + 14y + 28z + 56 = 0$$

$$= -28x + 14y - 28z - 84 = 0 \quad 28x + 14y + 28z - 84 = 0$$

$$2x - y - 2z = 6$$

$$\begin{aligned} 2x + y - 2z - 6 &= 0 \\ |2x + y - 2z - 6| & \end{aligned}$$

Question 2. Find the area of the parallelogram with the vertices

5

$$(1, -2, 4), (3, 5, 7), (4, 6, 8), (2, -1, 5).$$

$$B = (2, -1, 5) - (4, 6, 8) = (-2, -7, -3).$$

$$P_1 = (3, 5, 7) - (2, -1, 5) = (1, 6, 2)$$

$$P_1 \times P_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ 1 & -2 & 4 \end{vmatrix} = (28)\hat{i} - 2\hat{j} + 8\hat{k}$$

2

$$A = B \cdot h = |(28, -2, 8) \cdot (-2, -7, -3)|$$

$$= |-56 + 14 + 24|$$

$$= 18$$

Question 3. Evaluate the limit $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - 2x - y^2 + 2y}{x^2 - 2x + y^2 - 2y + 2}$.

approach on line

5

approach on $y = x + 1$

$$\lim_{x \rightarrow 1} \frac{x^2 - 2x - (x+1)^2 + (2)(x+1)}{x^2 - 2x + (x+1)^2 - (2)(x+1) + 2}$$

$$= \frac{x^2 - 2x - x^2 - 2x - 1 + 2x + 2}{x^2 - 2x + x^2 + 2x + 1 - 2x - 2 + 2}$$

$$= \frac{-2x + 1}{-2x + 1}$$

$\boxed{1}$

$\textcircled{3}$

$m = 9$

Question 4. Find $\frac{\partial f}{\partial x}$ if $f(x, y, z) = \frac{x^3}{y} + x \sin\left(\frac{yz}{x}\right)$.

5

$$\frac{\partial f}{\partial x} = \frac{3}{y} x^2 + \sin\left(\frac{yz}{x}\right) - \frac{yz}{x^2} \cos\left(\frac{yz}{x}\right)$$

Question 5. Find the gradient of the function $f(x, y, z) = e^{x+y+z}$.

5

$$\nabla f(x, y, z) = e^{x+y+z} (\hat{i} + \hat{j} + \hat{k})$$