University of Saskatchewan
Department of Mathematics and Statistics
Math 223 (01) (03) (05)

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October 24, 2005 | Quiz #4 | 30 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is an formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

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Question 1. Suppose that z is defined by the general functional situation 5

$$z = f(x, y, u), \quad u = h(x, y), \quad x = g(v), \quad v = k(t), \quad y = p(t).$$

Use the chain rule to calculate $\frac{dz}{dt}$ in this general situation, and apply that to the specific case where

$$z = x^2 + y^2 + u^2, \quad u = \frac{1}{x^2 - y^2}, \quad x = v^3 - v^2, \quad v = e^t, \quad y = e^{4t}.$$

$$\frac{dz}{dt} = \frac{\delta z}{\delta x}\frac{\delta x}{\delta v}\frac{\delta v}{\delta t} + \frac{\delta z}{\delta y}\frac{\delta y}{\delta t} + \frac{\delta z}{\delta u}\left(\frac{\delta u}{\delta x}\frac{\delta x}{\delta v}\frac{\delta v}{\delta t} + \frac{\delta u}{\delta y}\frac{\delta y}{\delta t}\right)$$

$\frac{\delta z}{\delta x} = 2x \quad \frac{\delta z}{\delta y} = 2y \quad \frac{\delta z}{\delta u} = 2u$

$\frac{\delta x}{\delta v} = 3v^2 - 2v$

$\frac{\delta v}{\delta t} = e^t$

$\frac{\delta y}{\delta t} = 4e^{4t}$

$\frac{\delta u}{\delta x} = \frac{-2x}{(x^2-y^2)^2}$

$\frac{\delta u}{\delta y} = \frac{2y}{(x^2-y^2)^2}$

$$\frac{\delta z}{\delta t} = (2x)(3v^2-2v)(e^t) + (2y)(4e^{4t}) + (2u)\left[\left(\frac{-2x}{(x^2-y^2)^2}\right)(3v^2-2v)(e^t) + \left(\frac{2y}{(x^2-y^2)^2}\right)(4e^{4t})\right]$$

Question 2. Find the equation of the tangent plane to the surface $x = x^2 - y^3 z$ at $(2, -1, -2)$. 5

$f = x^2 - x - y^3 z$

$\nabla f = (2x - 1, -3zy^2, -y^3) \; @ \; 2, -1, -2$

$= (3, 6, 1)$

$x = 2 - 3t$
$y = 6t - 1$
$z = t - 2$

$\frac{x-2}{-3} = \frac{y+1}{6} = z + 2$

this is not the eq of a plane.

$T = \frac{\partial f}{\partial x}(x-2) + \frac{\partial f}{\partial y}(y+1) + \frac{\partial f}{\partial z}(z+2)$

$= 3x - 6 + 6y + 6 + z + 2$

$0 = 3x + 6y + z + 2$

Question 3. Find all critical points of $f(x, y) = 3xy - x^3 - y^3$ and classify each as yielding a relative maximum, a relative minimum, or a saddle point. 5

$\frac{\partial f}{\partial x} = 3y - 3x^2$

$\frac{\partial f}{\partial y} = 3x - 3y^2$

$\frac{\partial^2 f}{\partial x^2} = -6x \; A$

$\frac{\partial^2 f}{\partial y \partial x} = 3 \; B$

$\frac{\partial^2 f}{\partial y^2} = -6y \; C$

$B^2 - AC$

$\alpha < \alpha \quad \alpha > \alpha \quad \alpha = \alpha$
max min saddle

Critical Points

$0 = 3y - 3x^2$

$0 = 3x - 3y^2$

$(0, 0)$
$(1, 1)$
$(-1, -1)$

Find Rel'n between 2 eqns

$y = \sqrt{x}$

Point	A	B	C	$B^2 - AC$	
0, 0	0	3	0	9	Saddle point ✓
1, 1	-6	3	-6	-25	Maxima
~~-1, -1~~	6	3	6	-25	Minima

$y = x^2 \quad x = y^2 \Rightarrow y = y^4$

$y = x^2$
$x = y^2$
$y = y$
$0 = y^4 - y$
$0 = y(y^3 - 1)$

$\Rightarrow y(y^3 - 1) = 0$

$\Rightarrow y = 0 \text{ or } y = 1$