University of Saskatchewan

Department of Mathematics and Statistics

Math 223 (01) (03) (05)

November 28, 2005 | Quiz #7 | 30 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is an formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

Print your name here: Craig Block-Hansen

10

Print your student number here: 147742

Question 1. Evaluate the triple integral $\iiint_V xy\,dV$ where V is the region enclosed by the surfaces 5

$$z = \sqrt{1 - x^2 - y^2} \quad \text{and} \quad z = 0.$$

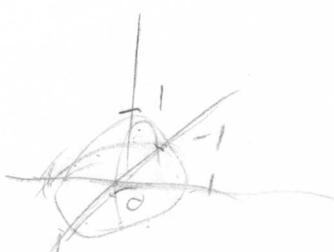
$x^2+y^2+z^2=1$

$z=0$

$-1 \le x \le 1$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le z \le \sqrt{1-x^2-y^2}$



$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} xy\,dz\,dy\,dx$$

$0 \le R \le 1$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \theta \le 2\pi$

2

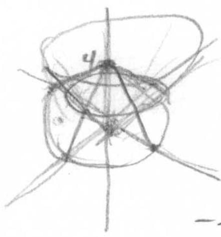
$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{1} (R^2 \sin\phi(\cos\theta))(R^2 \sin\phi \sin\theta)\, R^2 \sin\phi \, dR\, d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{1} R^6 \sin^3\phi \cos\theta \sin\theta \, dR\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{7}\sin^3\phi \cos\theta\sin\theta \, d\phi\, d\theta$$

$$= \frac{1}{7}\int_0^{2\pi}\left[\frac{1}{4}\frac{\sin^5\phi}{\cos\phi}\cos\theta\sin\theta\right]_0^{\pi/2} d\theta = \frac{1}{28}\int_0^{2\pi}\cos\theta\sin\theta\, d\theta = \frac{\pi}{14}$$

Question 2. Find the volume bounded by the surfaces $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$. **5**



$z = x^2+y^2$

$z = 4-x^2-y^2$

$x^2+y^2 = 4-x^2-y^2$

$2x^2+2y^2 = 4$

$x^2+y^2 = 2$

$x = \sqrt{2-y^2}$

$y = \sqrt{2-x^2}$

$-\sqrt{2} \le x \le \sqrt{2}$

$-\sqrt{2-x^2} \le y \le \sqrt{2-x^2}$

$x^2+y^2 \le z \le 4-x^2-y^2$

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{x^2+y^2}^{4-x^2-y^2} 1 \, dz\, dy\, dx$$

?

3

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} 4-2x^2-2y^2 \, dy\, dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[4y - 2x^2y - \frac{2y^3}{3}\right]_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} dx = 2\int_{-\sqrt{2}}^{\sqrt{2}} \left[4\sqrt{2-x^2} - 2x^2\sqrt{2-x^2} - \frac{2(2-x^2)\sqrt{2-x^2}}{3}\right] dx$$

Continued on back

Question 3. By changing to spherical coordinates, evaluate the triple integral **5**

$$\int_0^9 \int_0^{\sqrt{81-y^2}} \int_0^{\sqrt{81-x^2-y^2}} \frac{1}{x^2+y^2+z^2}\, dz\, dx\, dy.$$

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{9} \frac{1}{R^2} R^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} 9\sin\phi \, d\phi\, d\theta$$

$$= \int_0^{\pi/2} \left[-9\cos\phi\right]_0^{\pi/2} d\theta \qquad 0-(-9) = 9$$

$$= \int_0^{\pi/2} 9 \, d\theta$$

5

$$= \frac{9\pi}{2}$$