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UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH. 224.3 (ALL SECTIONS)

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IMPORTANT

- Print your **name** and encode your **student number** on the multiple choice sheet.
- Clearly indicate your **section number**.
- Open Book Examination: Students may use "Calculus" by J. Stewart.
- No Calculators or formula sheets are allowed.
- All questions are of equal value.

Questions 1 and 2 deal with the following DE: $(x+2)\sin y dx + x\cos y dy = 0$.

1. Find an integrating factor of the given non-exact DE.

A

(A) xe^x (B) x (C) e^x (D) x^2 (E) x^2e^x (F) $\ln x$ (G) $x\ln x$ (H) $x^2\ln x$ (I) $e^x\ln x$

2. Find the general solution of the given DE.

B

(A) $\arccos\left(\frac{C}{x^2e^x}\right)$ (B) $\arcsin\left(\frac{C}{x^2e^x}\right)$ (C) $\arctan\left(\frac{C}{x^2e^x}\right)$ (D) $\arccos\left(\frac{C}{xe^x}\right)$

(E) $\arcsin\left(\frac{C}{xe^x}\right)$ (F) $\arctan\left(\frac{C}{xe^x}\right)$ (G) $\arccos\left(\frac{C}{xe^{2x}}\right)$ (H) $\arcsin\left(\frac{C}{xe^{2x}}\right)$

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3. Let $y = y(x)$ be the solution of the IVP $xy' = y + x\cos^2\left(\frac{y}{x}\right)$, $y(1) = 1$. Find $y\left(e^{-\frac{\pi}{4}}\right)$.

B

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3 (F) 4 (G) 5 (H) 6 (I) 7 (J) 8

4. Find an integrating factor of the following first order linear DE

$$\frac{1}{x^2}\frac{dy}{dx} + 2e^{x^2}\frac{y}{x} = \sin x.$$

C

(A) e^{2x} (B) $\ln x^2$ (C) $e^{e^{x^2}}$ (D) e^{x^2} (E) e^{x^2+1}
(F) $e^{e^{x^2}+1}$ (G) $e^{e^{x^2}+1}$ (H) e^{x^3} (I) x^2

5. Which one of the following statements is not true?

T (A) The equation $(xy - y^2)y' = x^2$ is homogeneous.
T (B) The equation $y' = e^x - xy$ is linear.
T (C) The equation $3(x^2 + 2xy) + (3x^2 + 2y)y' = 0$ is exact.
F (D) The equation $y' = 3y - 2x + 6xy - 1$ is separable.
T (E) An integrating factor for the linear equation $xy' + 2y = \sin x$ is x^2.
T (F) If y_1 and y_2 are solutions of $y'' + 6y' + 5y = x$, then $c_1y_1 + c_2y_2$ is also a solution of the equation.

F

6. Solve the IVP $y'' + y' - 2y = e^{-2x}$, $y(0) = 1$, $y'(0) = \frac{5}{3}$.

F

(A) $y = e^{-x} + 2x^2e^{2x} + xe^{-2x}$
(B) $y = xe^{-x} + xe^{2x} + \frac{1}{3}xe^{-2x}$
(C) $y = x - e^{-2x} + xe^{-2x}$
(D) $y = e^x - 2xe^{-2x} + \frac{1}{3}x^2e^{-x}$
(E) $y = 2e^x - e^{-2x} - \frac{1}{3}e^{-2x}$
(F) $y = \frac{4}{3}e^x - \frac{1}{3}e^{-2x} - \frac{1}{3}xe^{-2x}$
(G) $y = 2x - e^{-2x} + xe^{-2x}$
(H) $y = e^x - xe^{-2x} + \frac{1}{3}x^2e^{-x}$
(I) $y = -e^x + 2e^{-2x} - \frac{1}{3}xe^{-2x}$

7. A force of 25 N is required to keep a spring stretched 1 m beyond its natural length. The spring, with mass 5 kg attached to it, is immersed in a fluid with damping constant 10 and is released with velocity 2 m/s from the equilibrium position. No external forces are present. Find the time it takes for the spring to come back to the equilibrium position for the first time after it is released.

G

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{7}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{4}$ (F) $\frac{\pi}{3}$ (G) $\frac{\pi}{2}$ (H) π (I) 2π (J) 3π

8. Solve the DE $y'' + 2y' + y = e^{-x}\ln x$.

(A) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{4} - \frac{x^2}{2}\ln x\right)e^{-x} + (x\ln x - x)xe^{-x}$

(B) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{4} + \frac{x^2}{2}\ln x\right)e^{-x} + (x\ln x - x)xe^{-x}$

(C) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x} - (x\ln x - x)xe^{-x}$

(D) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x + \frac{x^2}{4}\right)e^{-x} - (x\ln x - x)xe^{-x}$

A

(E) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x} + (2x\ln x - x)xe^{-x}$

(F) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x} - (2x\ln x - x)xe^{-x}$

(G) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x} + (3x\ln x - x)xe^{-x}$

(H) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x} - (3x\ln x - x)xe^{-x}$

(I) $c_1e^{-x} + c_2xe^{-x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)e^{-x}$

9. Evaluate the sum of the series $\Sigma_{n=1}^{\infty}\left(\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right)\right)$.

B

(A) 0 (B) $\sin 1$ (C) $\sin 2$ (D) $\sin 3$ (E) $\sin 4$ (F) $\sin 5$ (G) $\sin 6$ (H) $\sin 7$ (I) $\sin 8$

10. Evaluate $\lim\limits_{n\to\infty} a_n$, where $a_n = (1+\sqrt{2}n)^{\frac{1}{n}}$ [**HINT:** consider $\ln a_n$].

A

(A) 1 (B) $\frac{e}{\sqrt{2}}$ (C) e (D) $2\sqrt{2}$ (E) 2 (F) $\sqrt{2}$ (G) $\sqrt{2}e$ (H) $e^{\sqrt{2}}$ (I) $\sqrt{2}e^2$

11. Which of the following functions is represented by the power series $\sum\limits_{n=1}^{\infty}\frac{x^{5n+2}}{n!}$?

J X

(A) $x^2(e^{5^x} - 1)$ (B) $e^{5x+2} - 1$ (C) $x^2((e^x - 5)$ (D) $x^2(e^{5x} - 1)$ (E) $5x^2(e^x - 1)$

(F) x^2e^{5x} (G) $5x^2e^x$ (H) $x^2e^{x^5}$ (I) e^{5x+2} (J) $x^2(e^{x^5} - 1)$

12. Let $f(x) = x^3\cos(x^2)$. Find the coefficient of x^9 in the Maclaurin series for $f(x)$.

I X

(A) $\frac{1}{9!}$ (B) $-\frac{1}{9!}$ (C) $\frac{1}{6!}$ (D) $-\frac{1}{6!}$ (E) 1

(F) $-\frac{1}{6}$ (G) $\frac{1}{6}$ (H) $-\frac{1}{24}$ (I) 0 (J) $\frac{1}{24}$

13. Find a power series expansion for the function $f(x) = \dfrac{x^2}{3(1-x)^2}$, defined over the interval $(-1, 1)$.

H

(A) $\sum_{n=2}^{\infty} \frac{n-1}{6} x^n$ (B) $\sum_{n=2}^{\infty} \frac{n+1}{6} x^n$ (C) $\sum_{n=2}^{\infty} \frac{n-1}{4} x^n$ (D) $\sum_{n=2}^{\infty} \frac{n+1}{4} x^n$

(E) $\sum_{n=2}^{\infty} \frac{n-1}{3} x^{2n}$ (F) $\sum_{n=2}^{\infty} \frac{n+1}{3} x^{2n}$ (G) $\sum_{n=2}^{\infty} \frac{n+1}{3} x^n$ (H) $\sum_{n=2}^{\infty} \frac{n-1}{3} x^n$

(I) $\sum_{n=2}^{\infty} \frac{n-1}{5} x^n$ (J) $\sum_{n=2}^{\infty} \frac{n+1}{5} x^n$

14. Find $(1+i)^{20}$.

H

(A) 1 (B) $512\sqrt{2}(1-i)$ (C) $512\sqrt{2}(1+i)$ (D) $512\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$

(E) $512\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ (F) $512(1+i)$ (G) 512 (H) -1024 (I) 1024 (J) $512(1-i)$

15. It is known that the power series $\sum_{n=0}^{\infty} c_n \left(x - \frac{1}{2}\right)^n$ converges for $x = \frac{3}{2}$ and diverges for $x = -\frac{3}{2}$. Identify the three point set such that the series converges at each of the indicated points.

I

(A) $\left\{-\frac{1}{4}, \frac{1}{8}, \frac{7}{2}\right\}$ (B) $\left\{-\frac{5}{2}, 1, \frac{5}{4}\right\}$ (C) $\left\{-\frac{5}{2}, 0, \frac{1}{4}\right\}$ (D) $\left\{-2, \frac{1}{8}, 1\right\}$ (E) $\left\{-1, 1, \frac{11}{4}\right\}$

(F) $\left\{-\frac{5}{4}, -\frac{1}{8}, 3\right\}$ (G) $\left\{-2, \frac{1}{8}, \frac{5}{4}\right\}$ (H) $\left\{-\frac{7}{4}, \frac{1}{8}, \frac{5}{4}\right\}$ (I) $\left\{-\frac{1}{4}, \frac{1}{8}, \frac{5}{4}\right\}$ (J) $\left\{-\frac{13}{8}, \frac{1}{8}, \frac{5}{4}\right\}$

16. The coefficient of x^3 is the power series expansion at 0 of the function $\int_0^x \ln(t^2+1)\,dt$ is:

F ✗

(A) $\frac{1}{8}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$ (F) $\frac{1}{3}$ (G) $\frac{1}{2}$ (H) 1 (I) 0 (J) -1

17. If $y = \sum_{k=0}^{\infty} c_k x^k$ is a power series solution of the differential equation $x^2 y' = y - 2x - 1$, then c_3 equals to:

C

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8 (F) 10 (G) 12 (H) 14 (I) 16

18. Assume that a clean water is flowing into a polluted lake at a constant rate 114 km^3/year and that water flows out at the same rate. Assume also that the total volume of water in the lake is 300km^3 and that no further pollutants are being dumped in the lake. How long (in yeras) will it take for 99.9% of the pollution to be removed from the lake? (Assume, in your calculations, that $\ln 10 \approx 2.28$)

I

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14 (F) 15 (G) 16 (H) 17 (I) 18 (J) 19

19. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{(n+1)4^n}$ is :

D

(A) $[-1,3]$ (B) $[-1,3)$ (C) $[-1,3)$ (D) $[-1,7)$ (E) $[-1,7]$
(F) $(-1,7]$ (G) $(-1,7)$ (H) $(0,1)$ (I) $(-\infty,\infty)$ (J) $[-1,\infty)$

20. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 10$ and $a_{n+1} = \sqrt{a_n + 6}$ for each $n \geq 1$. Find $\lim_{n\to\infty} a_n$.

C

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 7 (H) 8 (I) 9 (J) 10

THE END