University of Saskatchewan
Department of Mathematics and Statistics
Math 224 (02) (04) (06)

13/15

January 13, 2006 | Quiz #1 | 30 minutes

Fully answer the following questions in the space provided. The points for each problem are indicated in the right margin.

Permitted resources: None. Closed book. No calculators.

This is an formal assessment. Cheating on an assessment is considered a serious offense by the University and can be met with disciplinary action, including suspension or expulsion. Candidates shall not make use of any books, resources or papers except at the discretion of the examiner or as indicated on the assessment paper. Candidates shall hold no communication of any kind with other candidates within the assessment room.

Print your name here: Craig Bloch-Hansen

Print your student number here: 147742

Question 1. Show that each function in the family 5

$$y(x) = C_1 \cos(2\ln x) + C_2 \sin(2\ln x) + \frac{1}{4}$$

satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 1.$$

5

$\frac{dy}{dx} = C_1(-\sin(2\ln x))\frac{1}{x} + C_2\cos(2\ln x)\frac{1}{x}$

$\frac{d^2y}{dx^2} = C_1(-\cos(2\ln x))\frac{1}{x^2} + C_1(+\sin(2\ln x))\frac{1}{x^2} + C_2(-\sin(2\ln x))\frac{1}{x^2} + (-\cos(2\ln x))\frac{1}{x^2}$

$x^2\left[8C_2(-\cos(2\ln x)\frac{1}{x^2}\right] + x\left[C_1(-\sin(2\ln x))\frac{1}{x} + C_2\cos(2\ln x)\frac{1}{x}\right] + 4\left[C_1\cos(2\ln x) + C_2\sin(2\ln x) + \frac{1}{4}\right] = 1$

$= \frac{4}{4} = 1$

$\boxed{1 = 1}$ Q.E.D.

turn over

Question 2. Find the general solution of the differential equation 5

$$\frac{dy}{dx} = 6x^2 + 2x.$$

$\int dy = \int 6x^2 + 2x \, dx$

$y = 2x^3 + x^2 + C$

5

Question 3. Find the solution of the differential equation 5

$$(xy + y)\,dx - (xy - x)\,dy = 0$$

that also satisfies the condition $y(1) = 2$.

$y(x+1)\,dx = x(y-1)\,dy$

$\frac{x+1}{x}\,dx = \frac{y-1}{y}\,dy$

$\int 1 + \frac{1}{x}\,dx = \int 1 - \frac{1}{y}\,dy$

$x + \ln|x| + C = y - \ln|y|$

$x + \ln|x| + (1 - \ln|2|) = y - \ln|y|$

3

$1 + \ln|1| + C = 2 - \ln|2|$

$1 + C = 2 - \ln|2|$

$C = 1 - \ln|2|$