

University of Saskatchewan
Department of Mathematics and Statistics
Math 225 Spring 2003

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Test #1

2 hours
George Patrick

The examination consists of two parts, Part A and Part B, each worth 20 points. The points however, will be used in a formula to calculate your test grade.

- *Encode* your student number correctly on your opscan sheet.
- *Print* your name and student number on your opscan sheet.
- Answer all questions of Part A *in pencil* on your opscan sheet. There is no penalty for a wrong answer in Part A.
- Answer all questions in Part B in the answer book provided.
- One formula sheet is permitted. No calculators.

PART A

Fill in the bubbles on your opscan sheet corresponding to the correct answers. Each problem in this section is worth 1 point.

Question 1. The value of $2\vec{a} - 3\vec{b}$ where $\vec{a} = (1, -2, 3)$ and $\vec{b} = (1, 1, -1)$ is

- (A) $(0, -2, 1)$ (B) $(-1, 3, 0)$ (C) $(-1, -7, 9)$ (D) $(1, -1, 2)$ (E) $(1, -2, 3)$
(F) $(2, -4, 6)$ (G) $(0, 0, 0)$ (H) $(3, 3, -3)$

Question 2. The value of $\vec{a} \cdot \vec{b}$ where $\vec{a} = (1, 0, 3)$ and $\vec{b} = (3, 1, 2)$ is

- (A) 3 (B) 4 (C) 0 (D) $\sqrt{10}$ (E) 10
(F) 9 (G) -3 (H) 6

Question 3. The length of the vector $(-1, 3, 1, 2)$ is

- (A) $\sqrt{10}$ (B) 3 (C) 15 (D) 4 (E) 225
(F) 0 (G) $\sqrt{15}$ (H) 1-

Question 4. The value of $\vec{a} \times \vec{b}$ where $\vec{a} = (1, -2, 1)$ and $\vec{b} = (-1, 1, 1)$ is

- (A) $(-3, -2, 3)$ (B) -3 (C) $(1, -1, 0)$ (D) $(1, 1, 0)$ (E) $\sqrt{3}$
(F) $(2, -3, 0)$ (G) $(-3, 0, 0)$ (H) $(0, 1, 0)$

Question 5. The distance between the two points $(-2, 1, 2)$ and $(1, 4, 0)$ is

- (A) 484 (B) 4 (C) 6 (D) 2 (E) $\sqrt{22}$
(F) 16 (G) $\sqrt{8}$ (H) 22

Question 6. The value of t such that $(t, 1, 2t)$ and $(1, 1, -2)$ are orthogonal vectors is

- (A) $\frac{1}{5}$ (B) 1 (C) 5 (D) 0 (E) 3
(F) $\frac{1}{3}$ (G) $\frac{2}{3}$ (H) -1

No right answers.

Question 7. The line which contains the points $(1, 0, 2)$ and $(3, 0, 3)$ meets the plane $z = 0$ at the point

- (A) $(-3, -2, 3)$ (B) -3 (C) $(1, -1, 0)$ (D) $(1, 1, 0)$ (E) 0
 (F) $(2, -3, 0)$ (G) $(-3, 0, 0)$ (H) $(0, 1, 0)$

Question 8. The equation of the plane containing the points $(1, 0, 2)$, $(2, 2, 1)$, and $(-1, 1, 0)$ is

- (A) $3x - 4y - 5z + 7 = 0$ (B) $3x - 4y - 5z + 1 = 0$ (C) $3x - y - 5z + 1 = 0$
 (D) $3x + y - 5z - 7 = 0$ (E) $-3x - 4y - 5z + 7 = 0$ (F) $3x - 4y - 5z = 0$
 (G) $3x - 4y - 2z + 2 = 0$ (H) $x - y - z + 1 = 0$

Question 9. The component of $\vec{b} = (3, -2, 2)$ on $\vec{a} = (2, 0, 1)$, or $\text{comp}_{\vec{a}} \vec{b}$, is

- (A) 6 (B) $\frac{8}{\sqrt{17}}$ (C) $\frac{8}{5}$ (D) 8 (E) 0
 (F) $\frac{8}{17}$ (G) $\frac{8}{\sqrt{5}}$ (H) 1

Question 10. The projection of $\vec{b} = (3, -2, 2)$ on $\vec{a} = (1, 0, 1)$, or $\text{proj}_{\vec{a}} \vec{b}$, is

- (A) $(1, 0, 1)$ (B) $(\frac{15}{17}, -\frac{10}{17}, \frac{10}{17})$ (C) $(\frac{\sqrt{5}}{2}, 0, \frac{\sqrt{5}}{2})$ (D) $(\frac{5}{2}, 0, \frac{5}{2})$ (E) $\frac{5}{\sqrt{2}}$
 (F) 0 (G) $(3, -2, 2)$ (H) $(\frac{15}{17}, \frac{10}{17}, \frac{10}{17})$

Question 11. The tangent line to the curve $\vec{r}(t) = (1 + t, \sin t, t^2 - 2)$ at $t = 0$ meets the plane $x = 0$ at

- (A) $(0, 2, -2)$ (B) $(0, 1, 1)$ (C) $(0, -2, -1)$ (D) $(0, -1, -2)$ (E) $(0, 1, 3)$
 (F) $(0, 3, 1)$ (G) $(0, 0, 0)$ (H) $(1, 0, 1)$

Question 12. If $\vec{u}(t)$ and $\vec{v}(t)$ are two curves such that

$$\vec{u}(0) = (1, -2, 1), \quad \vec{v}(0) = (0, 1, 1), \quad \vec{u}'(0) = (1, -2, 0), \quad \vec{v}'(0) = (-2, 1, 0)$$

then the value of $(\vec{u} \cdot \vec{v})'(0)$ is

- (A) 4 (B) -4 (C) 1 (D) 0 (E) 3
 (F) -6 (G) -1 (H) -3

Question 13. With exactly the same information as in the previous question, the value of $(\vec{u} \times \vec{v})'(0)$ is

- (A) $(0, 0, 4)$ (B) $(1, 3, -4)$ (C) $(4, 1, -2)$ (D) $(1, 1, 1)$ (E) $(0, 0, -4)$
 (F) $(0, 0, 0)$ (G) $(-3, -3, -2)$ (H) $(1, 0, 0)$

Question 14. If $\vec{r}(t) = (t, 1, t^2)$ then $\int_0^1 \vec{r}(t) dt$ is

- (A) $(1, -1, 1)$ (B) $(1, 1, 1)$ (C) $(0, 0, 1)$ (D) $(0, 0, 0)$ (E) $\frac{11}{6}$
 (F) $(\frac{1}{2}, 1, \frac{1}{3})$ (G) $(1, 0, 2)$ (H) $(1, 1, 0)$

Question 15. The velocity of a particle moving by the curve $\vec{r}(t) = (\sin t, \cos t, t^2)$ at $t = \frac{\pi}{4}$ is

- (A) $(1, 0, 0)$ (B) $(0, 0, 0)$ (C) $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{\pi}{2})$ (D) $(\sqrt{2}, \sqrt{2}, 1)$ (E) $(0, \sqrt{2}, \frac{\pi}{4})$
 (F) $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{\pi}{4})$ (G) $(0, 0, \frac{1}{3})$ (H) $(\sqrt{2}, \sqrt{2}, \frac{\pi}{4})$

Question 16. The unit tangent vector to the helix $\vec{r}(t) = (\cos t, \sin t, -t)$ at $t = \frac{\pi}{4}$ is

- (A) $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ (B) $(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{\sqrt{2}})$ (C) $(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$ (D) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$ (E) $(1, 0, 0)$
 (F) $(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2})$ (G) $(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ (H) $(0, 1, 0)$

Question 17. If $z = x^2y + 3xy$ then $\frac{\partial z}{\partial x}$ is

- (A) 0 (B) $x^2 + 3x$ (C) $2xy$ (D) $2xy + 3$ (E) $3xy$
(F) $x^2y + 3y$ (G) $2xy + 3y$ (H) $2x + 3$

Question 18. If $z = \sin(x^2 + xy)$ then $\frac{\partial z}{\partial y}$ is

- (A) $y \cos(x^2 + xy)$ (B) $\cos(x^2 + xy)$ (C) $(2x + y) \cos(x^2 + xy)$ (D) $3x \cos(x^2 + xy)$
(E) $\sin(x^2 + xy)$ (F) $x \cos(x^2 + xy)$ (G) $2x + y$ (H) $y \cos(x^2 + xy)$

Question 19. The equation of the tangent plane to the surface $z = xy$ at $x = 1$ and $y = 2$ is

- (A) $z = -3 + x + 2y$ (B) $z = x + 2y$ (C) $z = 2$ (D) $z = -4 + 2x + 2y$
(E) $z = -2 + 2x + y$ (F) $z = -1 + x + y$ (G) $z = y$ (H) $z = 1 + x$

Question 20. If $z = x^2y^3$ and $x = -2$, $y = -1$, $dx = .1$, $dy = -.1$, then dz is

- (A) 1.6 (B) .016 (C) 2.8 (D) .28d (E) 16
(F) 1.6d (G) 0 (H) 28

PART B

Show all your work in the booklets provided.

Question 21.

2 × 5 = 10

- (a) Find the symmetric equations of the line of intersection of the planes $x + 2y + z = 1$ and $2x - y + z = 2$.
(b) Determine the intersection of the two lines

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{2}, \quad \frac{x-\frac{1}{4}}{1} = \frac{y+\frac{1}{4}}{3} = z.$$

- (c) Find the distance from the point $(1, 0, 1)$ to the plane $2x + y - z = -2$.
(d) Find the plane perpendicular to the plane $x - 2y - z = 1$ and containing the two points $(0, 1, 0)$ and $(1, -2, 1)$.
(e) Find the area of the triangle with vertices $(1, 1, 0)$, $(1, 1, -2)$ and $(0, 2, 0)$.

Question 22. Calculate the curvature, unit tangent, and normal vectors at $t = 0$ for the curve $\vec{r}(t) = (t - \cos t, \sin 2t, t)$.

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Question 23. Find all (a, b) such that the tangent plane to $z = x^2 + 2xy + 3y^2$ at (a, b) is normal to the unit vector $(-\frac{2}{3}, -\frac{2}{3}, 1/3)$.

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