May 30, 2003 **Math 226 Midterm # 1** George Patrick

Fully answer all the questions in the booklets provided. Closed book. No calculators. No communication of any kind with your fellow students. The only aid allowed is a single $8\frac{1}{2} \times 11$ hand written sheet. Time: 2 hours.

Question 1. **5 + 3 + 2 = 10**

(a) Calculate $\bar{z}$, $|w|$, $z+w$, zw, and z/w if $z = 3+i$ and $w = 1-i$.

(b) Find the imaginary part of $(1+\sqrt{3}i)^{1001}$

(c) By writing the compex number z in the form $a+bi$, prove that $|z^2| = |z|^2$ for all $z \in \mathbb{C}$.

Question 2. Determine if the sequence converges. If it does then calculate the limit. **2 × 5 = 10**

(a) $\dfrac{6n^2+2n}{4n^2+5n+2}$

(b) $(-1)^n \dfrac{1+4^n}{3^n}$

(c) $n\left(1-e^{\frac{1}{2n}}\right)$

(d) $\dfrac{2^{2n-1}+3^n}{4^{n+2}}$

(e) $(-1)^n \dfrac{3n^2+2}{n^3+2}$

Question 3. **2 × 6 = 12**

(a) Define $\lim_{n\to\infty} a_n = L$.

(b) Find the fourth partial sum of the series $\sum_{n=3}^{\infty} (-1)^n n^2$.

(c) State the integral test.

(d) Prove that the n^{th} partial sum s_n of the series $1 + r + r^2 + r^3 + \cdots$ is

$$s_n = \frac{1-r^{n+1}}{1-r}.$$

(e) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ such that $\lim_{n\to\infty} a_n = 0$.

(f) State the ratio test.

Question 4. Determine which of the following series converge and which diverge. Give a complete explanation for your answers. **3 × 10 = 30**

(a) $\sum_{n=1}^{\infty} \dfrac{n!n!}{(2n)!}$

(b) $\sum_{n=1}^{\infty} \dfrac{\cos n}{n^2}$

(c) $\sum_{n=1}^{\infty} \dfrac{1}{n^2} \tan n$

(d) $\sum_{n=2}^{\infty} \dfrac{n^2}{n^4-1}$

(e) $\sum_{n=1}^{\infty} \dfrac{n^2 3^n}{2^n}$

(f) $\sum_{n=1}^{\infty} (-1)^n (e^{\frac{1}{n}} - 1)$

(g) $\sum_{n=1}^{\infty} 1 - \cos\left(\dfrac{1}{n}\right)$

(h) $\sum_{n=1}^{\infty} n \tan \dfrac{1}{n}$

(i) $\sum_{n=1}^{\infty} \dfrac{n^3}{n^3+1}$

(j) $\sum_{n=1}^{\infty} \dfrac{1}{n(1+\ln n)}$

Question 5. **2 × 4 = 8**

(a) State the alternating series test.

(b) Using the alternating series test, show that the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^4}{2+n^2}$ converges.

(c) How many terms of the series in part (b) do you need to sum the series to an accuracy of .0005?

(d) Define absolute convergence. Does the series in Part (b) converge absolutely?