Wave Energy Power Transmission Lines: Electric and Magnetic Field Propagation

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Experimental Background
Experimental Setup
Measuring $B_\phi$
Modelling $B_\varphi$

Ampere’s Circuital Law can be written in integral or differential form via Stokes theorem (Time independent).

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\int_{C} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_{S} \mathbf{J} \cdot d\mathbf{S} = \mu_0 I_{\text{enc}}$$

By integrating around the closed loop C, the expression below is obtained.

$$\frac{B_\varphi}{I} = e^{i\omega t} \frac{\mu_0}{2\pi R}$$
Modelling $B_\varphi$
Measuring $E_R$
Modelling $E_R$

Gauss’s Law can be written in integral or differential form via the divergence theorem.

$$\int_S E \cdot dA = \frac{Q}{\varepsilon_0} \quad \nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad E_R = \frac{\lambda}{2\pi\varepsilon_0 R}$$

A model by Assis allows the charge to be calculated.

$$Q_B = 2\pi a L \sigma_B \quad \sigma_B = \varepsilon_0 \left( \frac{\Omega I + 2\varphi_R}{2a \ln (L/a)} \right) \quad \frac{E_R}{I} = \frac{L/\sigma \pi a^2}{2R \ln (L/a)}$$
Modelling $E_R$

![Graph showing Radial distance vs. $E_R$ with data points and curves for different currents.]

$E_R$ [V]m$^{-1}$

Radial distance [m]

- $E_R$ (I = 10A)
- $E_R$ (I = 7A)
- Assis (I = 10A)
- Assis (I = 7A)
Measuring $E_Z$
Modelling $E_z$

The Maxwell-Faraday equation can be written in differential form and the magnetic field theory can be inserted.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{B_\varphi}{I} = e^{i\omega t} \frac{\mu_0}{2\pi R}$$

The induced electric field discussed by Gauthier and Shakur in the 80s is shown below.

$$\frac{E_z}{I} = -i\omega e^{i\omega t} \frac{\mu_0}{2\pi} \ln \left( \frac{R}{a} \right) + K$$

$$\tilde{E}_z = -\frac{\mu_0 \omega}{2\pi} \ln \left( \frac{R}{a} \right) + K$$
Modelling $E_Z$
Modelling $E_z$

The Hertz vector formulation is another method of recasting the problem in terms of potentials.

$$\Pi_z = \frac{I}{4\pi\sigma} \int_{\ell_1}^{\ell_2} \frac{e^{-\gamma r}}{r} \, d\ell \quad \gamma = (i\mu\omega\sigma)^{1/2}$$

The Hertz vector in terms of the scalar and vector potentials.

$$\phi = -\nabla \cdot \Pi \quad A = \mu\varepsilon \frac{\partial \Pi}{\partial t}$$

$$E = -\nabla \phi - \frac{\partial A}{\partial t} \quad B = \nabla \times A$$
Modelling $E_z$

The electric and magnetic fields can now be written in terms of a single ‘super potential’ the Hertz vector.

$$E = -\gamma^2 \Pi + \nabla \nabla \cdot \Pi \quad \quad B = \mu \sigma \nabla \times \Pi$$

The electric field can be written in analytical form.

$$\frac{E_z}{I} = \frac{i \mu \omega}{4\pi} \left[ \sinh^{-1} \left( \frac{l_2 - z}{\rho} \right) - \sinh^{-1} \left( \frac{l_1 - z}{\rho} \right) \right]$$

$$+ \frac{z - l_1}{4\pi \sigma r_1^3} - \frac{z - l_2}{4\pi \sigma r_2^3}$$
Modelling $E_Z$
Modelling $E_z$

![Graph showing $E_z$ vs. Radial distance with data points and lines representing different models and data sets.](image-url)
Correlation $E_z$ and $B_\phi$

![Graph showing the correlation between $E_z$ and $B_\phi$.](image)

- $E_z$ Data (I = 10A)
- $E_z$ Data (I = 7A)
- Sommerfeld and Stratton (I = 10A)
- Sommerfeld and Stratton (I = 7A)
Future Work

Explore electromagnetic fields generated by three phase transmission cables in a controlled system.

Examine the effects of frequency and current on the electromagnetic fields.

Construct a larger system and explore behavior. Use an actuator to input controlled wave pulses in the system.

Derive a predicative model using first principles.
Future Work
Future Work

Explore the effects of boundary conditions. Does the confined system impact the electromagnetic measurements at the boundaries?

Measure the electromagnetic fields with the tri-axial cable buried under various ocean floor materials. Explore field behavior at the boundary between ocean floor materials and seawater.

Thank you all very much for your attention!
References
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[1] O. 2nd and 2008, “New unknowns, new buoy, new funds put OSU on wave energy crest,” LIFE@OSU.
[Accessed: 15-Apr-2014].