Dynamic Phasors for Small Signal Stability Analysis

Udaya Annakkage (University of Manitoba)

Chandana Karawita (Transgrid Solutions)
Outline

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   - Typical Outputs
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2. Dynamic Phasors

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Power System Simulation and Analysis

- Electromagnetic Transient Simulation - Time Domain Technique
- Transient Stability Simulation - Time Domain Technique
- Small Signal Stability Analysis - Frequency Domain Technique
Sample responses of a four-generator power system after a three phase fault

The amplitude of the 60 Hz voltage waveform is modulated by the low frequency of oscillations of the rotor.
Sample responses of a four-generator power system after a three phase fault

Rotor angle of generator 2 and the rms voltage of Bus 2 show low frequency oscillations around 1 Hz.
Typical Output of a Small Signal Analysis

**Structural Information**

- Oscillation modes (frequencies and corresponding damping).
- Mode Shapes of oscillation frequencies.
- Participation of state variables in oscillation modes.
- Observability of oscillation modes.
- Controllability of oscillation modes.
- Residues for input-output pairs.
Typical Output of a Small Signal Analysis: Participation Factors

Participation Factors show the relative participation of state variables when a mode is excited.
Mode Shape shows whether the state variables are oscillating together or not.
Machine Models

Transient Stability and Small Signal Stability

- Rotor fluxes are modelled as state variables.
- Stator fluxes are NOT modelled as state variables.

Electromagnetic Transient Simulation

- Rotor fluxes and stator fluxes are modelled as state variables.

Common to both

- Dynamics of the rotor and that of auxiliary controllers are modelled using differential equations.
Transmission Line Models

Transient Stability and Small Signal Stability

- Series inductance and shunt capacitance are modelled as constant impedances (admittances) calculated at the nominal frequency $\omega_0$.

Electromagnetic Transient Simulation

- Transmission line is modelled using differential equations (telegraphic equations).
Small Signal Stability: Frequency domain technique

- Only accurate in the vicinity of nominal frequency.
- Structural Information relevant to the system is available.

Transient Stability: Time domain technique

- Only accurate in the vicinity of nominal frequency.
- Large integration time step is used $\Rightarrow$ simulation is fast.

Electromagnetic Transient Simulation: Time domain technique

- Accurate over a wide frequency range.
- Integration time step is small $\Rightarrow$ simulation is slow.
Dynamic Phasors

Instantaneous Current Waveform

\[ i_{ac} = A_m e^{j\phi} e^{j\omega_0 t} = [A_m \cos(\phi) + jA_m \sin(\phi)] e^{j\omega_0 t} \]

\( A_m \) is the magnitude of the current, \( \phi \) is the phase of the current, and \( \omega_0 \) is the nominal system frequency.

In Rectangular Coordinates

\[ i_{ac} = (I_R + jI_I) e^{j\omega_0 t} \]
Modelling a Transmission Line using Dynamic Phasors

Series Branch

Series R-L circuit connected between nodes 1 and 2.

\[ v_{12} = L \frac{di_{12}}{dt} + Ri_{12} \]

Using the Complex rotating phasor relationships

\[ (V_R + jV_i)e^{j\omega_0 t} = L \frac{d(I_R + jI_i)e^{j\omega_0 t}}{dt} + R(I_R + jI_i)e^{j\omega_0 t} \]
Assuming that the nominal system frequency \((\omega_0)\) is constant

\[ V_R + jV_I = L \frac{d(I_R + jI_I)}{dt} + (R + j\omega_0 L)(I_R + jI_I) \]

Since \(L\) is in pu, \((\omega_0/L)\) terms appear instead of \((1/L)\)

\[
\begin{bmatrix}
\Delta I_R \\
\Delta I_I
\end{bmatrix}
= \begin{bmatrix}
-R\omega_0/L & \omega_0 \\
-\omega_0 & -R\omega_0/L
\end{bmatrix}
\begin{bmatrix}
\Delta I_R \\
\Delta I_I
\end{bmatrix}
+ \begin{bmatrix}
\frac{\omega_0}{L} & 0 & -\frac{\omega_0}{L} & 0 \\
0 & \frac{\omega_0}{L} & 0 & -\frac{\omega_0}{L}
\end{bmatrix}
\begin{bmatrix}
\Delta V_{1R} \\
\Delta V_{1I} \\
\Delta V_{2R} \\
\Delta V_{2I}
\end{bmatrix}
\]
Modelling a Transmission Line using Dynamic Phasors

Parallel Branch

\[
\begin{bmatrix}
\Delta \dot{V}_{1R} \\
\Delta \dot{V}_{1I}
\end{bmatrix} = \begin{bmatrix}
-\frac{\omega_0}{RC} & \omega_0 \\
-\omega_0 & -\frac{\omega_0}{RC}
\end{bmatrix} \begin{bmatrix}
\Delta V_{1R} \\
\Delta V_{1I}
\end{bmatrix} + \begin{bmatrix}
\frac{\omega_0}{C} & 0 \\
0 & \frac{\omega_0}{C}
\end{bmatrix} \begin{bmatrix}
\Delta I_R \\
\Delta I_I
\end{bmatrix}
\]
Other Interpretations of Dynamic Phasors

**d-q Components of Network Voltages and Currents**

Network voltages and currents are represented by their d-q components which are modelled as state variables.

**Fourier Components of Network Voltages and Currents**

Network voltages and currents are represented by their Fourier components which are modelled as state variables.
Power System Signals as Amplitude Modulated Signals

If \( R \) and \( I \) components are constants
The instantaneous waveforms are sinusoidal.

If \( R \) and \( I \) components are oscillating at frequency \( \omega \)
The instantaneous waveforms are amplitude modulated waveforms with carrier frequency \( \omega_0 \). This results in two sidebands of \( \omega_0 - \omega \) and \( \omega_0 + \omega \)
Introduction
Dynamic Phasors
Applications
Current Research Work

Power System Signals as Amplitude Modulated Signals

Example
If \( f_0 = 60 \text{ Hz} \) and \( f = 5 \text{ Hz} \), the two sideband frequencies are \( f_1 = 55 \text{ Hz} \) and \( f_2 = 65 \text{ Hz} \). Both are close to 60 Hz and the constant admittance representation of transmission network is acceptable.

Example
If \( f_0 = 60 \text{ Hz} \) and \( f = 25 \text{ Hz} \), the two sideband frequencies are \( f_1 = 35 \text{ Hz} \) and \( f_2 = 85 \text{ Hz} \). Both are significantly different to 60 Hz and the constant admittance representation of transmission network is NOT acceptable.
Interactions Between Nearby HVDC Converters

A simple Network for model Validation

- Two HVDC lines, ac filters, ac transmission line, and a generator.
- A pulse of magnitude of 5% and duration 0.3s was applied to the rectifier current controller input.
Comparison of EMT, SS-traditional, and SS-Dynamic Phasor Approach.

(a) Change in Idcr of HVDC1

(b) Change in Idcr of HVDC2
Rotor Oscillations: SS-traditional and SS-Dynamic Phasors give same results
Frequency Response of the Model – EMT Vs SS-Dynamic Phasor

Magnitude (input: $\alpha$, output: $V_{cap}$)

Phase (input: $\alpha$, output: $V_{cap}$)
Changes in Rectifier side DC currents for a 5 %, 200Hz sinusoidal change of the HVDC1 rectifier side AC source voltage (VS1).

(a) Change in Idcr of HVDC1
(b) Change in Idcr of HVDC2

PSCAD/EMTDC Model  ~  Model 1  ~  Model 2
Participation Factors $\Rightarrow$ presence of an interaction between the two HVDC converters
Mode Shape $\Rightarrow$ state variables of the two converters oscillate against each other
The CIGRE benchmark HVDC test system with some modifications.

A synchronous generator is connected at rectifier side AC bus to supply half of the P-Q requirement of rectifier.
SS-Dynamic-Phasor provides accurate results in the frequency range of interest

10 % change in rectifier current reference for 10 ms
Torsional Interaction Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq. (Hz)</th>
<th>D (%)</th>
<th>Major Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16.24</td>
<td>-0.03</td>
<td>HVDC-Generator-Turbine</td>
</tr>
<tr>
<td>B</td>
<td>16.36</td>
<td>1.05</td>
<td>HVDC-Generator-Turbine</td>
</tr>
</tbody>
</table>
Participating states are identified using Participation Factors
Publications


Current Research Work

- SSR between DFIG based Wind Power Plant and series compensated transmission lines (Hiranya).
- SSI between nearby LCC-HVDC and VSC-HVDC terminals (Kevin – MH).
- SSR mitigation using FACTS controllers (TGS).
- Transient Stability Simulation using Dynamic Phasors (Rae – MH).

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